University of Mosul

College of Petroleum and Mining Engineering

Department of Petroleum Reservoir Engineering

# *Applied Reservoir Engineering II*

## *Third Year*

## *Lecture 6*

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## The material balance equation:

The material balance equation (MBE) is one of the basic tools of reservoir engineers for interpreting and predicting reservoir performance. The MBE can be used to:

● Estimate initial hydrocarbon volumes in place.

● Predict future reservoir performance.

● Predict ultimate hydrocarbon recovery under various types of primary driving mechanisms.

The concept of the material balance equation was presented by Schilthuis in 1941. In its simplest form, the equation can be written on volumetric basis as:

Initial volume = volume remaining + volume removed

Since oil, gas, and water are present in petroleum reservoirs, the material balance equation can be expressed for the total fluids or for any one of the fluids present.

Several of the material balance calculations require the total pore volume (P.V) as expressed in terms of the initial oil volume N and the volume of the gas cap. The expression for the total pore volume can be derived by introducing the parameter m into the relationship as follows:

Define the ratio m as:

$$m = \frac{\text{Initial volume of gas cap}}{\text{Volume of oil initially in place}} = \frac{G\,B_{gi}}{N\,B_{oi}}$$

Where:

G: Initial gas-cap gas (scf).

$B_{gi}$: Initial gas formation volume factor (bbl/STB).

N: Initial (original) oil in place (STB).

$B_{oi}$: Initial oil formation volume factor.

Solving for the volume of the gas cap gives:

Initial volume of the gas cap = $G\,B_{gi} = m\,N\,B_{oi}$

The total volume of the hydrocarbon system is then given by:

Initial oil volume + initial gas cap volume = (P.V) $(1 - S_{wi})$

$$N B_{oi} + m N B_{oi} = (P.V)(1 - S_{wi}) \qquad \text{……………………. (11-1)}$$

or

$$PV = \frac{N B_{oi}(1 + m)}{1 - S_{wi}} \qquad \text{……………………………… (11-2)}$$

Where:

$S_{wi}$ = initial water saturation

N = initial oil in place (STB)

P.V = total pore volume (bbl)

m = ratio of initial gas-cap-gas reservoir volume to initial reservoir oil volume (bbl/bbl).

Treating the reservoir pore as an idealized container as illustrated in Figure 1, volumetric balance expressions can be derived to account for all volumetric changes which occur during the natural productive life of the reservoir.

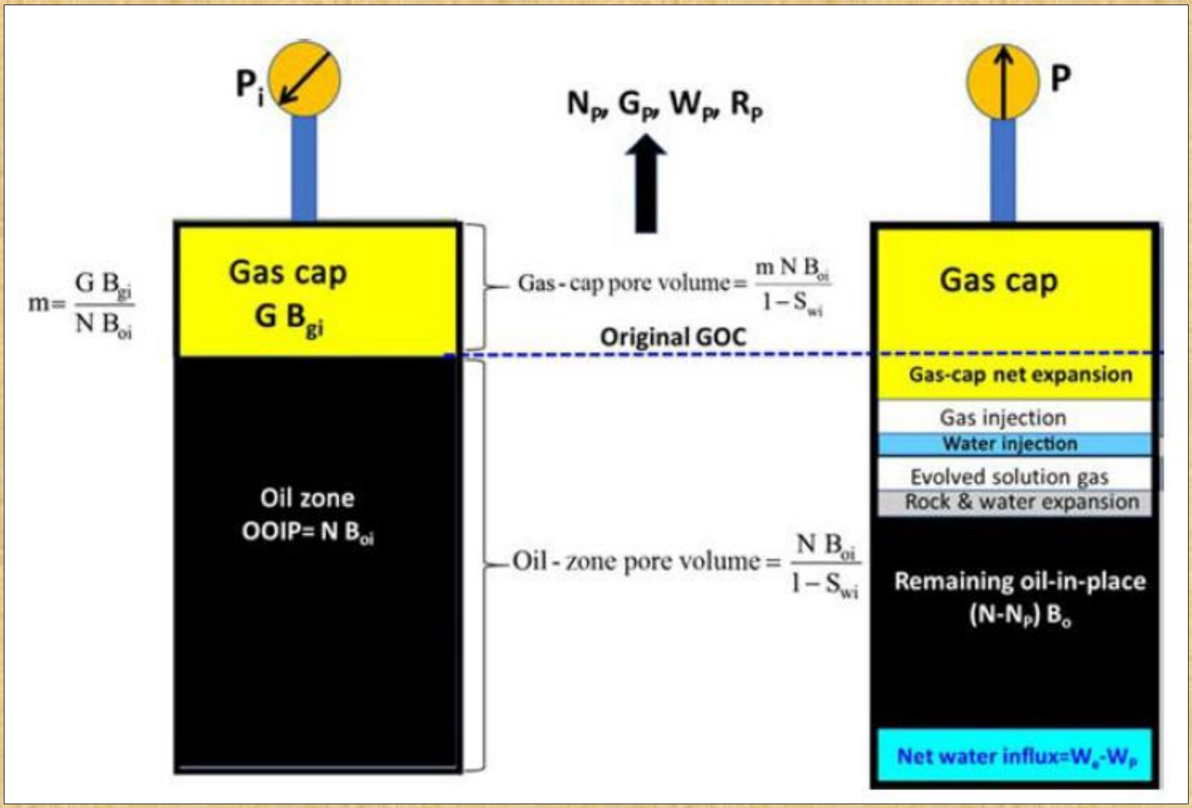


Figure 1: The concept of the tank model.

The MBE can be written in a generalized form as follows:

Pore volume occupied by the oil initially in place at Pi

+

Pore volume occupied by the gas in the gas cap at Pi

=

Pore volume occupied by the remaining oil at P

+

Pore volume occupied by the gas in the gas cap at P

+

Pore volume occupied by the evolved solution gas at P

+

Pore volume occupied by the net water influx at P

+

Change in pore volume due to connate water expansion and pore volume reduction due to rock expansion

+

Pore volume occupied by the injected water at P

The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

***Pore Volume Occupied by the Oil Initially in Place***

Volume occupied by initial in place = N $B_{oi}$ ……….(11-3)

Where:

N = oil initially in place, STB

$B_{oi}$ = oil formation volume factor at initial reservoir pressure $P_i$, bbl/STB

***Pore Volume Occupied by the Gas in the Gas Cap***

Volume of gas cap = m N $B_{oi}$ …………………… (11-4)

Where m is a dimensionless parameter and defined as the ratio of gas-cap volume to the oil zone volume.

***Pore Volume Occupied by the Remaining Oil***

Volume of the remaining oil = (N – $N_p$) $B_o$ …………….. (11-5)

Where:

$N_p$ = cumulative oil production, STB

$B_o$ = oil formation volume factor at pressure p, bbl/STB

### *Pore Volume Occupied by the Gas Cap at Reservoir Pressure p*

As the reservoir pressure drops to a new level p, the gas cap expands and occupies a larger volume. Assuming no gas is produced from the gas cap during the pressure decline, the new volume of the gas cap can be determined as:

$$\text{Volume of the gas cap at p} = \left[\frac{m\ N\ B_{oi}}{B_{gi}}\right] B_g \tag{11-6}$$

Where:

$B_{gi}$ = gas formation volume factor at initial reservoir pressure, bbl/scf
$B_g$ = current gas formation volume factor, bbl/scf

### *Pore Volume Occupied by the Evolved Solution Gas*

This volumetric term can be determined by applying the following material balance on the solution gas:

$$\begin{bmatrix}\text{volume of the evolved}\\ \text{solution gas}\end{bmatrix} = \begin{bmatrix}\text{volume of gas initially}\\ \text{in solution}\end{bmatrix} - \begin{bmatrix}\text{volume of gas}\\ \text{produced}\end{bmatrix} - \begin{bmatrix}\text{volume of gas}\\ \text{remaining in solution}\end{bmatrix}$$

or

$$\begin{bmatrix}\text{volume of the evolved}\\ \text{solution gas}\end{bmatrix} = \left[N R_{si} - N_p R_p - \left(N - N_p\right) R_s\right] B_g \tag{11-7}$$

Where:

$N_p$ = cumulative oil produced, STB
$R_p$ = net cumulative produced gas-oil ratio, scf/STB
$R_s$ = current gas solubility factor, scf/STB
$B_g$ = current gas formation volume factor, bbl/scf
$R_{si}$ = gas solubility at initial reservoir pressure, scf/STB

### *Pore Volume Occupied by the Net Water Influx*

$$\text{net water influx} = W_e - W_p B_w \tag{11-8}$$

Where:

$W_e$ = cumulative water influx, bbl
$W_p$ = cumulative water produced, STB
$B_w$ = water formation volume factor, bbl/STB

### Change in pore volume due to connate water expansion and pore volume reduction due to rock expansion

$$\text{Expansion of connate water} = \frac{N\, B_{oi}(1+m)}{1-S_{wi}} S_{wi}\, c_w\, \Delta p \tag{11-9}$$

Where:

$\Delta p$ = change in reservoir pressure, $(p_i - p)$
$c_w$ = water compressibility coefficient, $psi^{-1}$
m = ratio of the volume of the gas-cap gas to the reservoir oil volume, bbl/bbl

Similarly, the reduction in the pore volume due to the expansion of the reservoir rock is given by:

$$\text{Change in pore volume} = \frac{N\, B_{oi}(1+m)}{1-S_{wi}}\, c_f\, \Delta p \tag{11-10}$$

Combining the expansions of the connate water and formation as represented by Equations 11-9 and 11-10 gives:

Total changes in the pore volume

$$= N\, B_{oi}(1+m)\left(\frac{S_{wi}c_w + c_f}{1-S_{wi}}\right)\Delta p \tag{11-11}$$

**Pore volume occupied by the injection of gas and water**

Assuming that $G_{inj}$ volumes of gas and $W_{inj}$ volumes of water have been injected for pressure maintenance, the total pore volume occupied by the two injected fluids is given by the following:

$$\text{Total volume} = G_{inj}\, B_{ginj} + W_{inj}\, B_w \qquad (11\text{-}12)$$

Where:

$G_{inj}$ = cumulative gas injected, scf
$B_{ginj}$ = injected gas formation volume factor, bbl/scf
$W_{inj}$ = cumulative water injected, STB
$B_w$ = water formation volume factor, bbl/STB

Combining Equations 11-3 through 11-12 with Equation 11-2 and rearranging gives:

$$N = \frac{N_pB_o + (G_p - N_pR_s)B_g - (W_e - W_p\,B_w) - G_{inj}B_{ginj} - W_{inj}B_w}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p} \qquad (11\text{-}13)$$

Where:

N = initial oil in place, STB
$G_p$ = cumulative gas produced, scf
Np = cumulative oil produced, STB
$R_{si}$ = gas solubility at initial pressure, scf/STB
m = ratio of gas-cap gas volume to oil volume, bbl/bbl
$B_{gi}$ = gas formation volume factor at $p_i$, bbl/scf
$B_{ginj}$ = gas formation volume factor of the injected gas, bbl/scf

The cumulative gas produced $G_p$ can be expressed in terms of the cumulative gas-oil ratio $R_p$ and cumulative oil produced $N_p$ by:

$$G_p = R_p\, N_p \qquad (11\text{-}14)$$

Combining Equation 11-14 with Equation 11-13 gives:

$$N = \frac{N_p\left[B_o + (R_p - R_s)B_g\right] - (W_e - W_p\,B_w) - G_{inj}B_{ginj} - W_{inj}B_{wi}}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + m\,B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p} \qquad (11\text{-}15)$$

The above relationship is referred to as the ***material balance equation (MBE).***