University of Mosul

College of Petroleum and Mining Engineering

Department of Petroleum Reservoir Engineering

Applied Reservoir Engineering II

Third Year

Lecture 8

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Fundamentals of Reservoir Fluid Flow

A more convenient form of the MBE can be determined by introducing the concept of the total (two-phase) formation volume factor B_t into the equation. This oil PVT property is defined as:

The primary reservoir characteristics that must be considered include:

Types of fluids in the reservoir
Flow regimes
Reservoir geometry
Number of flowing fluids in the reservoir

TYPES OF FLUIDS

The isothermal compressibility coefficient is essentially the controlling factor in identifying the type of the reservoir fluid. In general, reservoir fluids are classified into three groups:

_ Incompressible fluids
_ Slightly compressible fluids
_ Compressible fluids

The isothermal compressibility coefficient c is described mathematically by the following two equivalent expressions:

- In terms of fluid volume:

$$c = \frac{-1}{V}\frac{\partial V}{\partial p} \tag{6-1}$$

- In terms of fluid density:

$$c = \frac{1}{\rho}\frac{\partial \rho}{\partial p} \tag{6-2}$$

where V and ρ are the volume and density of the fluid, respectively.

1- Incompressible Fluids

An incompressible fluid is defined as the fluid whose volume (or density) does not change with pressure, i.e.:

$$\frac{\partial V}{\partial p} = 0$$

$$\frac{\partial \rho}{\partial p} = 0$$

Incompressible fluids do not exist; this behavior, however, may be assumed in some cases to simplify the derivation and the final form of many flow equations.

2- Slightly Compressible Fluids

These "slightly" compressible fluids exhibit small changes in volume, or density, with changes in pressure. Knowing the volume V_{ref} of a slightly compressible liquid at a reference (initial) pressure p_{ref}, the changes in the volumetric behavior of this fluid as a function of pressure p can be mathematically described by integrating Equation 6-1 to give:

$$-c \int_{p_{ref}}^{p} dp = \int_{V_{ref}}^{V} \frac{dV}{V}$$

$$e^{c(p_{ref} - p)} = \frac{V}{V_{ref}}$$

$$V = V_{ref} e^{c(p_{ref} - p)} \qquad (6\text{-}3)$$

Where:

p = pressure, psia
V = volume at pressure p, ft^3
p_{ref} = initial (reference) pressure, psia
V_{ref} = fluid volume at initial (reference) pressure, psia

The e^x may be represented by a series expansion as:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^2}{3!} + \cdots + \frac{x^n}{n!} \qquad (6\text{-}4)$$

Because the exponent x [which represents the term c (p_{ref}–p)] is very small, the e^x term can be approximated by truncating Equation 6-4 to:

$$e^x = 1 + x \qquad (6\text{-}5)$$

Combining Equation 6-5 with Equation 6-3 gives:

$$V = V_{ref}[1 + c(p_{ref} - p)] \qquad (6\text{-}6)$$

A similar derivation is applied to Equation 6-2 to give:

$$\rho = \rho_{ref}[1 - c(p_{ref} - p)] \qquad (6\text{-}7)$$

where:

V = volume at pressure p
ρ = density at pressure p
V_{ref} = volume at initial (reference) pressure p_{ref}
p_{ref} = density at initial (reference) pressure p_{ref}

It should be pointed out that crude oil and water systems fit into this category.

3- Compressible Fluids

These are fluids that experience large changes in volume as a function of pressure. All gases are considered compressible fluids. The truncation of the series expansion, as given by Equation 6-5, is not valid in this category and the complete expansion as given by Equation 6-4 is used. As shown previously in Chapter 2 in Equation 2-45, the isothermal compressibility of any compressible fluid is described by the following expression:

$$c_g = \frac{1}{p} - \frac{1}{z}\left(\frac{\partial z}{\partial p}\right)_T \qquad (6\text{-}8)$$

Figures 6-1 and 6-2 show schematic illustrations of the volume and density changes as a function of pressure for the three types of fluids.

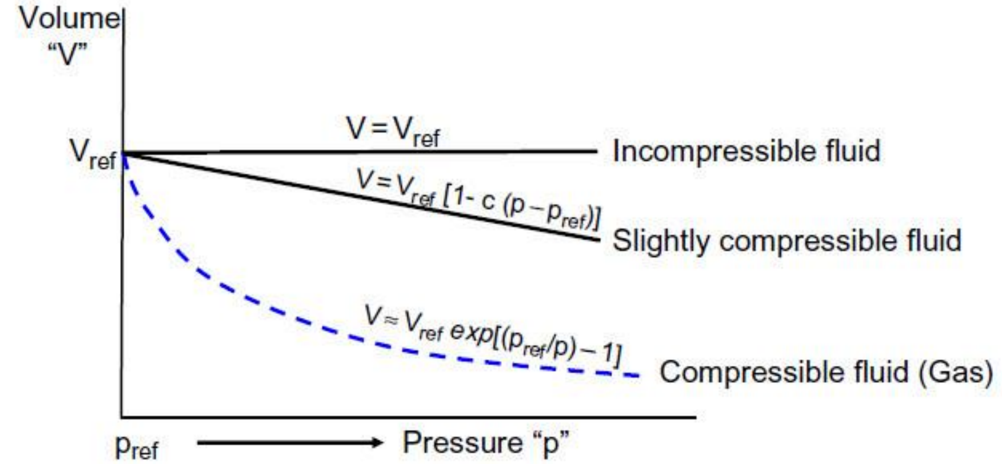

FIGURE 6-1 Pressure-volume relationships.

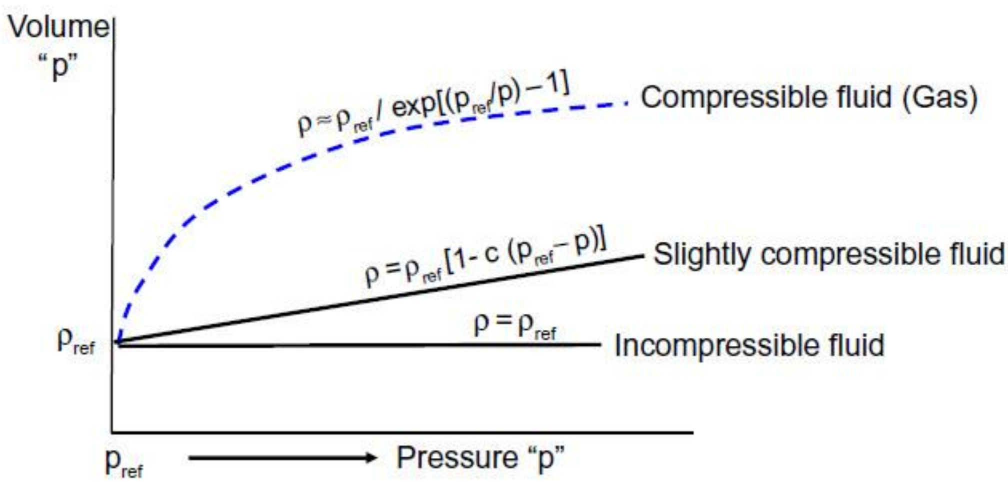

FIGURE 6-2 Fluid density versus pressure for different fluid types.

1- Steady-State Flow

The flow regime is identified as a steady-state flow if the pressure at every location in the reservoir remains constant, i.e., does not change with time. Mathematically, this condition is expressed as:

$$\left(\frac{\partial p}{\partial t}\right)_i = 0 \qquad (6\text{-}9)$$

The above equation states that the rate of change of pressure p with respect to time t at any location "i" is zero. In reservoirs, the steady-state flow condition can only occur when the reservoir is completely recharged and supported by strong aquifer or pressure maintenance operations.

2- Unsteady-State Flow

The unsteady-state flow (frequently called transient flow) is defined as the fluid flowing condition at which the rate of change of pressure with respect to time at any position in the reservoir is not zero or constant. This definition suggests that the pressure derivative with respect to time is essentially a function of both position "i" and time "t", thus:

$$\left(\frac{\partial p}{\partial t}\right) = f(i, t) \qquad (6\text{-}10)$$

3- Pseudosteady-State Flow

When the pressure at different locations in the reservoir is declining linearly as a function of time, i.e., at a constant declining rate, the flowing condition is characterized as the pseudosteady-state flow. Mathematically, this definition states that the rate of change of pressure with respect to time at every position is constant, or

$$\left(\frac{\partial p}{\partial t}\right)_i = \text{constant} \qquad (6\text{-}11)$$

It should be pointed out that the pseudosteady-state flow is commonly referred to as semisteady-state flow and quasisteady-state flow.

Figure 6-3 shows a schematic comparison of the pressure declines as a function of time of the three flow regimes.

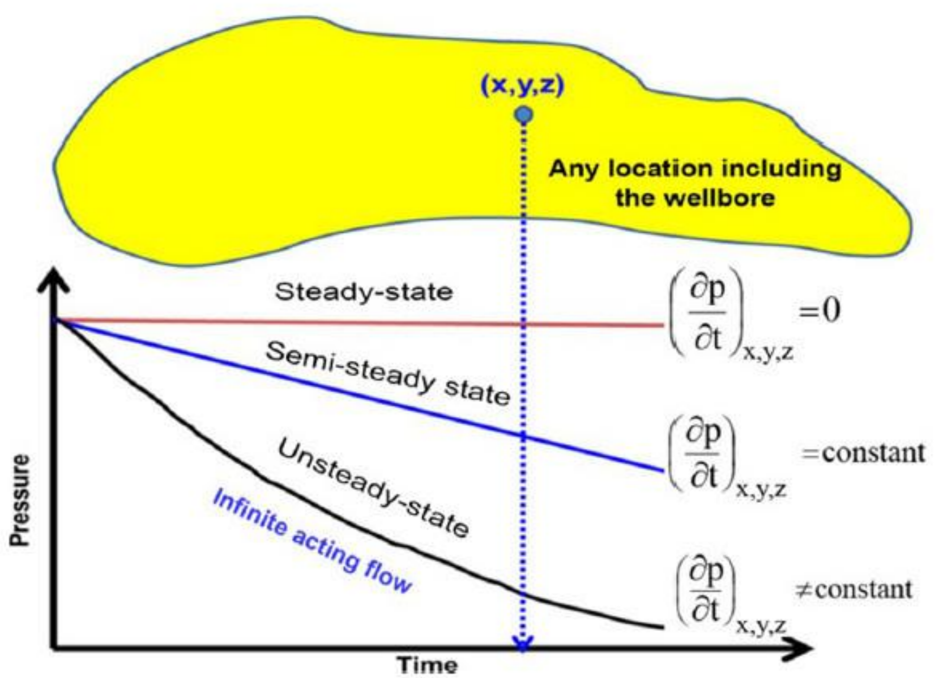

FIGURE 6-3 Flow regimes.

RESERVOIR GEOMETRY

The shape of a reservoir has a significant effect on its flow behavior. Most reservoirs have irregular boundaries, and a rigorous mathematical description of geometry is often possible only with the use of numerical simulators. For many engineering purposes, however, the actual flow geometry may be represented by one of the following flow geometries:

1- Radial flow
2- Linear flow
3- Spherical and hemispherical flow

1- Radial Flow

In the absence of severe reservoir heterogeneities, flow into or away from a wellbore will follow radial flow lines from a substantial distance from the wellbore. Because fluids move toward the well from all directions and coverage at the wellbore, the term “radial flow” is given to characterize the flow of fluid into the wellbore. Figure 6-4 shows idealized flow lines and iso-potential lines for a radial flow system.

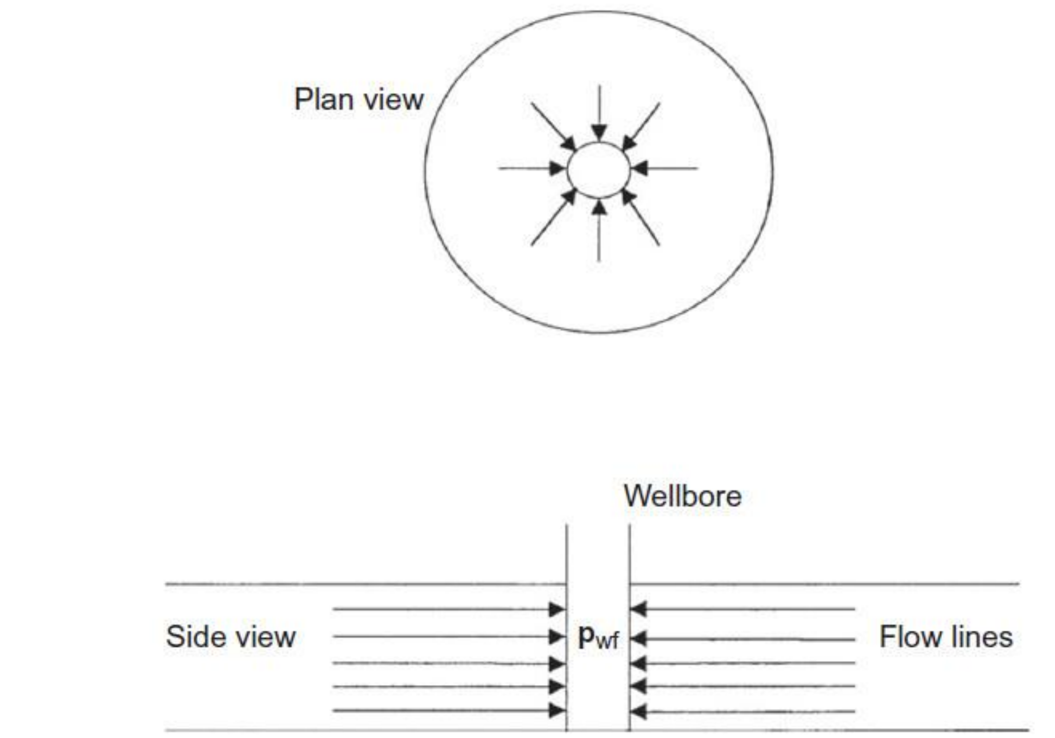

FIGURE 6-4 Ideal radial flow into a wellbore.

2- Linear Flow

Linear flow occurs when flow paths are parallel and the fluid flows in a single direction. In addition, the cross-sectional area to flow must be constant.
Figure 6-5 shows an idealized linear flow system. A common application of linear flow equation is the fluid flow into vertical hydraulic fractures as illustrated in Figure 6-6.

Spherical and Hemispherical Flow

Depending upon the type of wellbore completion configuration, it is possible to have a spherical or hemispherical flow near the wellbore. A well with a limited perforated interval could result in spherical flow in the vicinity of the perforations, as illustrated in Figure 6-7. A well that only partially penetrates the pay zone, as shown in Figure 6-8, could result in hemispherical flow. The condition could arise where coning of bottom water is important.

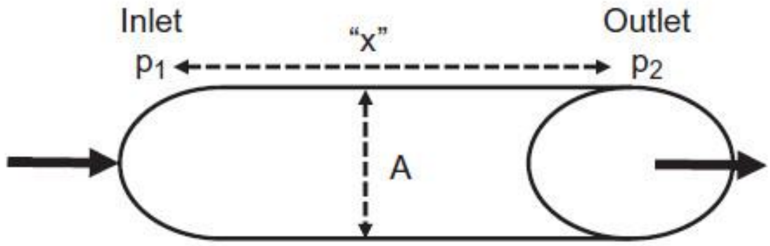

FIGURE 6-5 Linear flow.

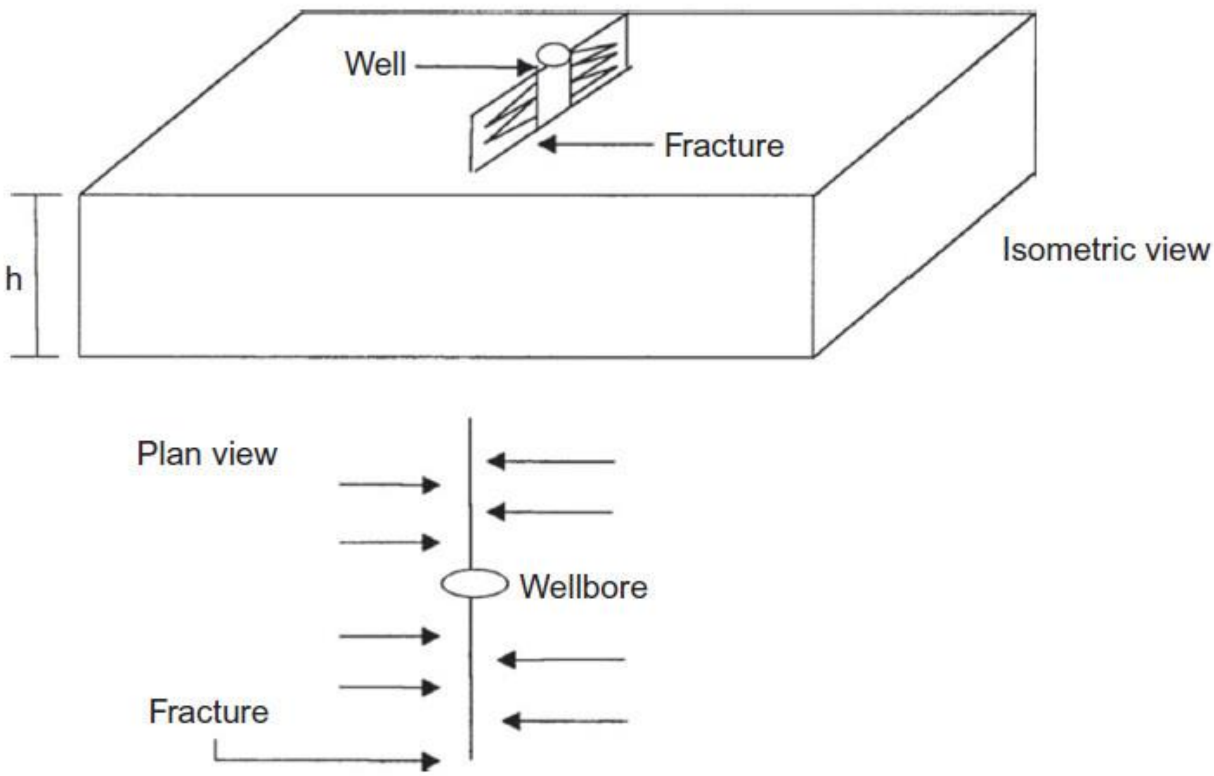

FIGURE 6-6 Ideal linear flow into vertical fracture.

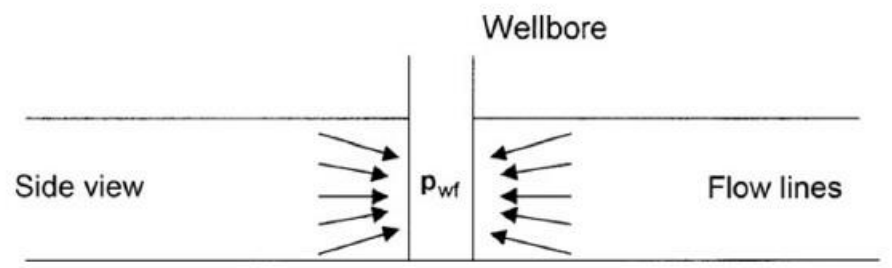

FIGURE 6-7 Spherical flow due to limited entry.

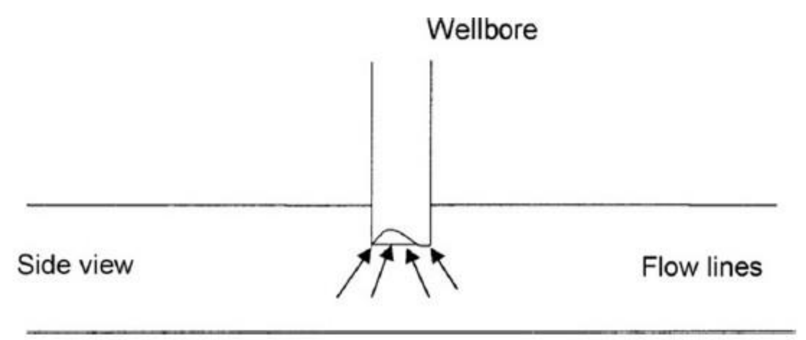

FIGURE 6-8 Hemispherical flow in a partially penetrating well.

NUMBER OF FLOWING FLUIDS IN THE RESERVOIR

The mathematical expressions that are used to predict the volumetric performance and pressure behavior of the reservoir vary in form and complexity depending upon the number of mobile fluids in the reservoir. There are generally three cases of flowing systems:

Single-phase flow (oil, water, or gas)
Two-phase flow (oil-water, oil-gas, or gas-water)
Three-phase flow (oil, water, and gas)

The description of fluid flow and subsequent analysis of pressure data becomes more difficult as the number of mobile fluids increases.