1 Newton's method

consider the quadratic approximation of the function $f(\lambda)$ at $\lambda = \lambda_i$ using Taylor's series expansion

$$f(\lambda) = f(\lambda_i) + f'(\lambda_i)(\lambda - \lambda_i) + \frac{1}{2} f''(\lambda)(\lambda - \lambda_i)^2 \quad (1)$$

By using the derivative of eq (1) equal to zero for the min of $f(\lambda)$ we obtain

$$f'(\lambda) = f'(\lambda_i) + f''(\lambda_i)(\lambda - \lambda_i) = 0$$

eq (1) can be rearranged to obtain in improved approximation as

$$\lambda_{i+1} = \lambda_i - \frac{f'(\lambda_i)}{f''(\lambda_i)}$$

في طريقة نيوتن نحتاج الى ايجاد المشتقة الاولى والثانية لايجاد النقطة الجديدة طريقة نيوتن تحتاج الى دالة هدف ونقطة بداية لكي نبدأ بالطريقة

Remark

1) The method requires both the first and second order derivative of $f(\lambda)$

2) if $f''(\lambda_i) \neq 0$ the eq (1) the newton itera method has apowerful (fastest) convergence known (quadratic convergence)

3) if the starting point for the iterative proce is not close to the true solution λ^*

Find the min. of the function

$$f(\lambda) = 0.65 - \frac{0.75}{1+\lambda^2} - 0.65 \tan^{-1} \frac{1}{\lambda}$$

$\lambda_1 = 0.1$ using $\epsilon = 0.01$

$$f'(\lambda) = \frac{1.5\lambda}{(1+\lambda^2)^2} + \frac{0.65\lambda}{1+\lambda^2} - 0.65 \tan^{-1} \frac{1}{\lambda}$$

$$f''(\lambda) = \frac{1.5(1-3\lambda^2)}{(1+\lambda^2)^3} + \frac{0.65(1-\lambda^2)}{(1+\lambda^2)^2} + \frac{0.65}{1+\lambda^2} = \frac{2.8-3.2\lambda^2}{(1+\lambda^2)^3}$$

Iter. 1

$\lambda_1 = 0.1$ $\quad f(\lambda_1) = -0.188197$

$f'(\lambda_1) = -0.744832$

$f''(\lambda_1) = 2.68659$

Test convergence $|f'(\lambda_1)| \nless \epsilon$

$$\lambda_2 = \lambda_1 - \frac{f'(\lambda_1)}{f''(\lambda_1)} = 0.377241$$

Test Convergence

$|f'(\lambda_2)| \doteq |-0.138230| > \epsilon$

iter 2

$f(\lambda_2) = -0.303279$ $\quad f'(\lambda_2) = -0.138230$

$f''(\lambda_2) = 1.57296$

$$\lambda_3 = \lambda_2 - \frac{f'(\lambda_2)}{f''(\lambda_2)} = 0.465119$$

Test. $|f'(\lambda_3)| = |-0.0179078| > \epsilon$

iter. 3

$f(\lambda_3) = -0.309881$ $\quad f'(\lambda_3) = -0.0179078$

$f''(\lambda_3) = 1.17126$

$$\lambda_4 = \lambda_3 - \frac{f'(\lambda_3)}{f''(\lambda_3)} = 0.480409$$

Test $|f'(\lambda_4)| = |0.0005033| < \epsilon$

STOP

$\lambda = \lambda_4 = 0.480409$