Sequential Unconstrained minimization (SUMT)

This problem is converted into an unconstrained minimization problem by constructing a function of the form

$$\phi(x_k, \mu) = f(x) + r_k \boxed{C_i(x)}$$

the non-linear programming has form

$$\min f(x)$$
$$s.t \begin{cases} g(x) \le b_i \\ h(x) = b_i \end{cases}$$

there are three type of converted:

1- Type-one (interior point method)
if the Constrained inequality constrained if method is called interior point method or interior penalty function or Barrier method

Let $\min f(x)$, $g(x) \le b_i$ } constrained optimization
the converted into unconstrained optimization by the form

$$cp = f(x) + r_k B(x)$$

The penalty function formulation for inequality

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in the formulations some populaly used forms of B_j are given by

$$B_j(x) = -\frac{1}{g_j(x)}$$

$$B_j(x) = \log[-g(x)]$$

In the case of exterior penalty function formulation are

$$B_j = \max[0, g_j(x)]$$

$$B_j = \max[0, g_j(x)]^2$$

Iterative process

Step 1 Start with an initial feasible point X ~~sta~~ satisfying all the constraints with strict inequality sign that is $g_j(X_j) \leq 0$ for $j = 1, 2 \ldots n$ and an intial value of $r_k > 0$ set $k = 1$

Step 2 - min $\phi(X, r)$ by using any of the unconstrained minimization methods and obtain the solution X_j^*

step 3 Test whether X_1^* is the optimum solution of the original problem. if X_1^* is found to be optimum terminate the process otherwise, go to be next step

step 4 Find the value of the next penalty parameter r_{k+1} as

$$r_{k+1} = c\,r_k$$

wher $c < 1$

step 5 set the new value of $k = k+1$ take the new starting point at $X_1 = X_i^*$ and go to step 2

Ex:- Find the min

$$f(x_1, x_2) = \frac{1}{2}(x_1+1)^3 + x_2$$

$$\text{s.t.} \quad -x_1 + 1 \leq 1$$

$$-x_2 \leq 0$$

Sol:-

$$\phi(x, r) = \frac{1}{3}(x_1+1)^3 + x_2 - r\left(\frac{1}{-x_1+1} - \frac{1}{x_2}\right)$$

Find the min of uncoustaned we used the necessary conditions

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$$\frac{\partial \phi}{\partial x_1} = (x_1+1)^2 - \frac{r}{(1-x_1)^2} = 0 \Longrightarrow (x_1^2-1)^2 = r$$

$$\frac{\partial \phi}{\partial x_2} = 1 - \frac{r}{x_2^2} = 0 \Longrightarrow x_2^2 = r$$

$$x_1(r) = (r^{1/2}+1)^{1/2}$$

$$x_2(r) = r^{1/2}$$

$$\phi_{min} = \frac{1}{3}\left[(r^{1/2}+1)^{1/2}+1\right]^3 + 2r^{1/2} - \frac{1}{\left(\frac{1}{r}\right) - \left(\frac{1}{r^{3/2}} + \frac{1}{r^2}\right)}$$

To obtain the solution of the original problem we know that

$$f_{min} = \lim_{r \to 0} \phi_{min}(r)$$

$$x_1^* = \lim_{r \to 0} x_1(r)$$

$$x_2^* = \lim_{r \to 0} x_2(r)$$