جامعة الموصل – كلية الهندسة
قسم الهندسة المدنية
2020 -2021

University of Mosul
College of Engineering
Department of Civil Engineering
Dr. Oday Asal Salih
Dr. Jasim Ali Abdulla

Theory of Structures
3rd Class 2020 - 2021

Introduction to Structural Analysis

(A Structure)

The definition of a structure is clearly projected in Figure 1.1.

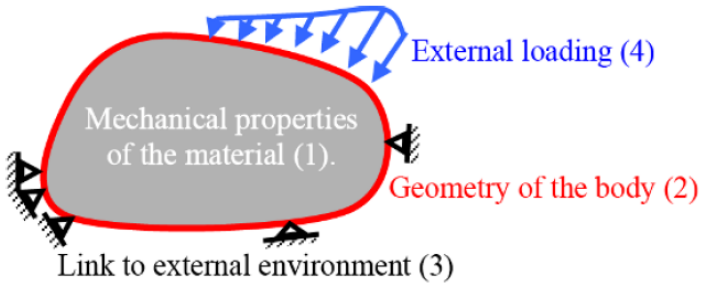
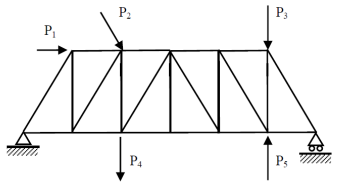


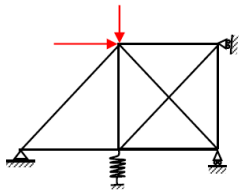
Figure 1.1. *The conditions for defining a structure*[1]

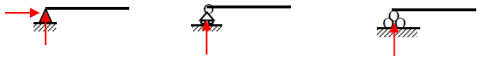
(Classification of structures)

- The structures used in civil engineering can be divided into four categories.

1) Plane trusses

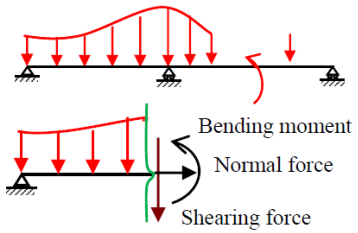




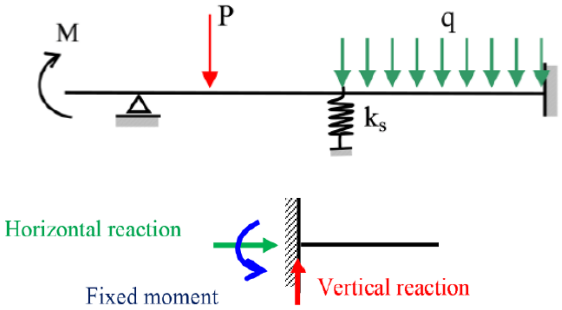


(Classification of structures)

2) Beams

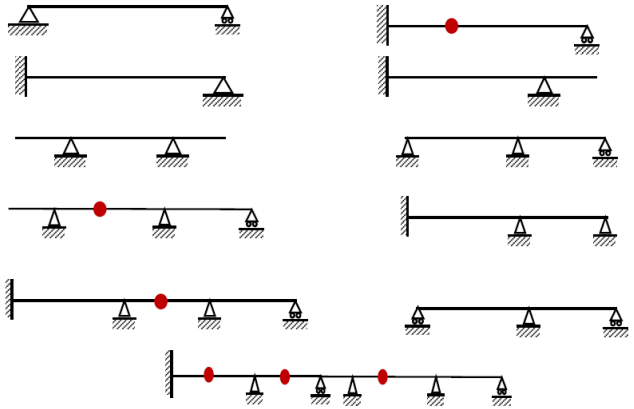


Internal actions



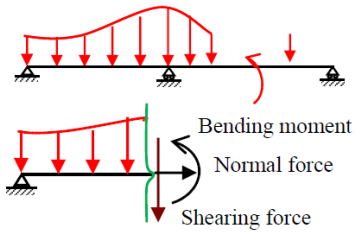
(Classification of structures)

2) Beams

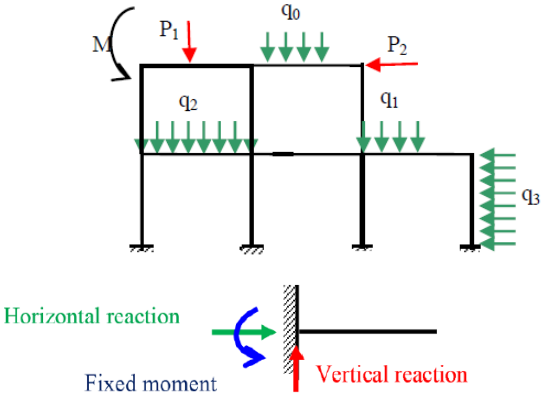


(Classification of structures)

3) Frames



Internal actions



(Classification of structures)

3) Frames

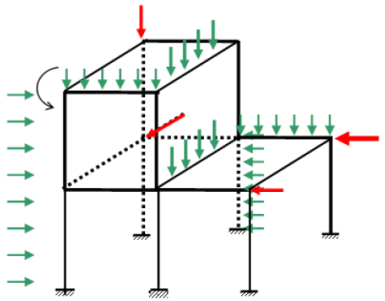
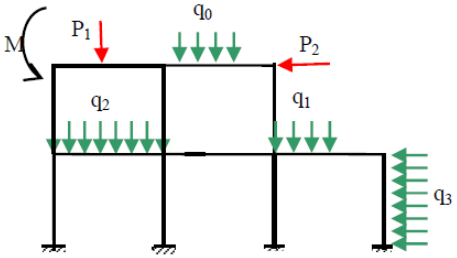
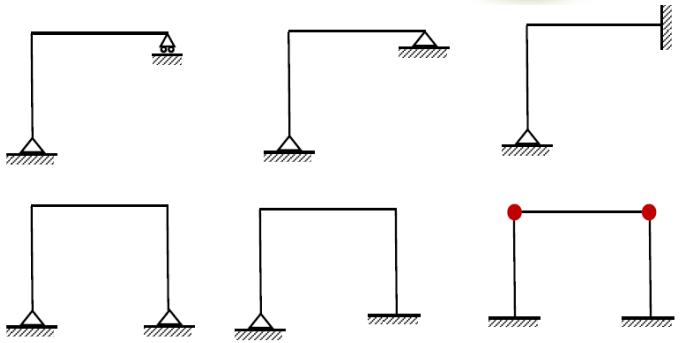


Figure 1.8. *Three-dimensional frames*



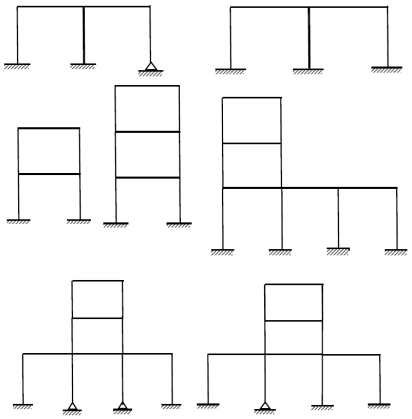
(Classification of structures)

3) Frames



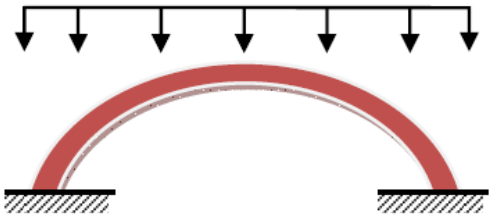
(Classification of structures)

3) Frames



(Classification of structures)

4) Arches

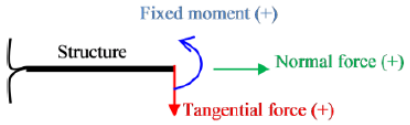


Sign convention

– normal force is positive when it causes traction. On the contrary, it is negative if it generates a compression;

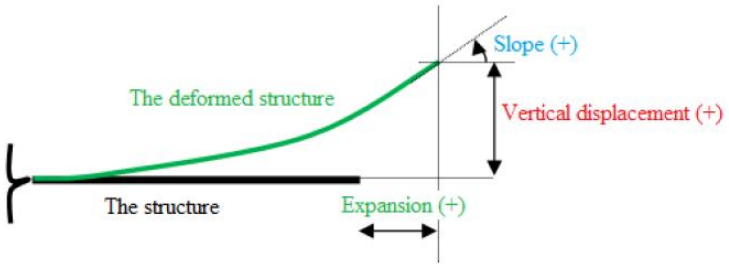
– the bending moment is considered positive if its rotation is counterclockwise;

– the shear force is assumed to be positive if it is oriented downwards to the left side.



Sign conventions of internal actions

Sign conventions of deflections



Loads

Dead loads all actions that do not depend on time and position.

Live loads Live loads are actions that vary in time and position.

Snow loads Snow loads are considered to be variable, fixed and static actions.

Wind loads Wind loads act directly on the walls of a construction

Seismic loads The seismic force at the base of a construction

Thermal loads describe the temperature variations on the structural elements

Hydrostatic loads When structures are designed to retain a fluid, soil or granular material,

D = dead loads
E = seismic or earthquake load effects
F = loads due to the weight and pressure of fluids
W = wind load

L_r = roof live loads
R = rain loads
T = total effects of temperature, creep, shrinkage, and differential settlement

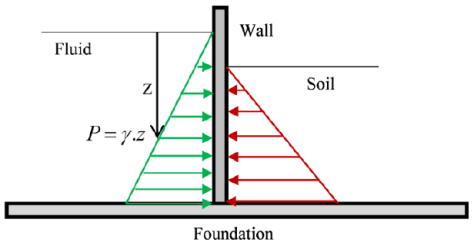
L = floor live load
S = snow loads
H = loads due to weight and lateral earth pressure of soils, groundwater, or bulk materials

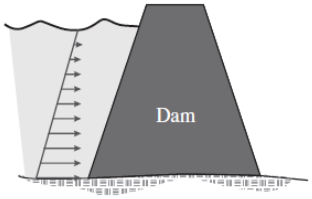
To determine the most severe loadings that the structure must be able to safely support, it is necessary to consider which of the loads (dead, live, and environmental) can occur simultaneously—or rather which simultaneous loads are reasonable to consider. In accordance with Chapter 2 of ASCE 7-16, the following possible simultaneous load situations may occur and should be considered for determining the *most severe yet reasonable situations*. These equations are called *load combinations*.

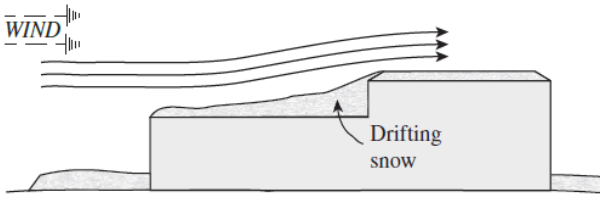
1. D
2. D + L
3. D + (L_r or S or R)
4. D + 0.75L + 0.75(L_r or S or R)
5. D + 0.6W
6. D + 0.75L + 0.75(0.6W) + 0.75(L_r or S or R)
7. 0.6D + 0.6W
8. D + 0.7E_v + 0.7E_h
9. D + 0.525E_v + 0.525E_h + 0.75L + 0.75S
10. 0.6D − 0.7E_v + 0.7E_h

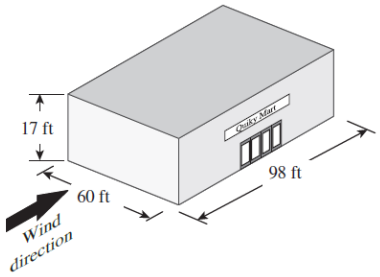
1. U = 1.4D
2. U = 1.2D + 1.6L + 0.5(L_r or S or R)
3. U = 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)
4. U = 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)
5. U = 0.9D + 1.0W
6. U = 1.2D + 1.0E_v + 1.0E_h + 1.0L + 0.2S
7. U = 0.9D − 1.0E_v + 1.0E_h

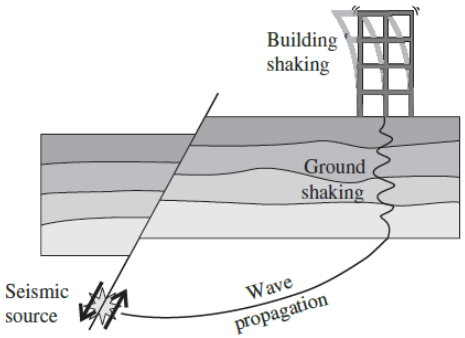
Hydrostatic loads

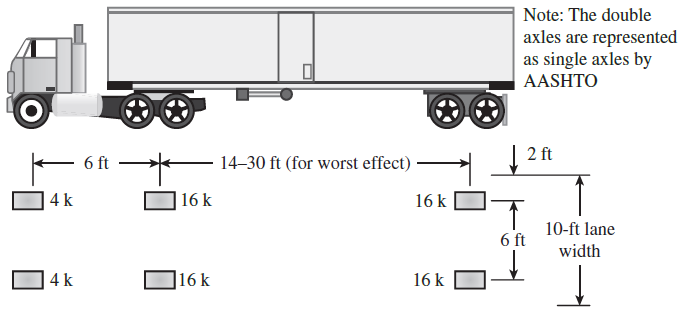












Highway Bridge Loads

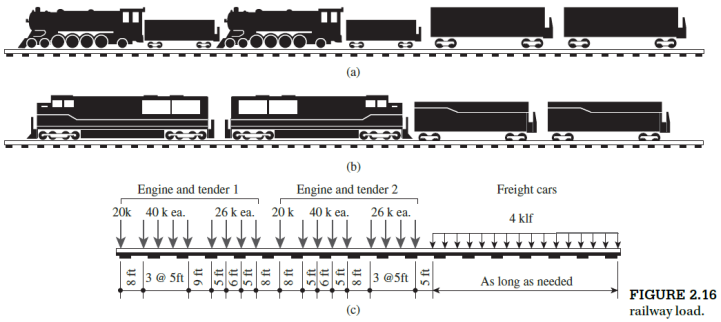
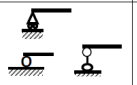


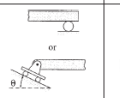
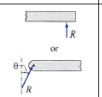
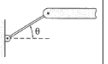
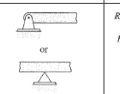
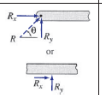
FIGURE 2.16 Cooper E-40 railway load.

Railway Bridge Loads

Types of plane structure supports

Type of support	Graphic representation	Blocked displacements	Support reactions
Roller		Vertical $\delta_y = 0$	Vertical
Hinge		Vertical and horizontal $\delta_x = \delta_y = 0$	Vertical and horizontal
Support on an inclined plane		Perpendicular and horizontal to the inclined plane $\delta_t = \delta_n = 0$	Perpendicular and horizontal to the inclined plane
Fixed		Vertical, horizontal and rotating	Vertical, horizontal and moment
Hinge		Rotation	/

Types of plane structure supports

Category	Type of support	Symbolic representation	Reactions	Number of unkowns
I	Roller			1 The reaction force R acts perpendicular to the supporting surface and may be directed either into or away from the structure. The magnitude of R is the unknown.
	Rocker			
	Link			1 The reaction force R acts in the direction of the link and may be directed either into or away from the structure. The magnitude of R is the unknown.
II	Hinge			2 The reaction force R may act in any direction. It is usually convenient to represent R by its rectangular components, R_x and R_y. The magnitudes of R_x and R_y are the two unknowns.
III	Fixed			3 The reactions consist of two force components R_x and R_y and a couple of moment M. The magnitudes of R_x, R_y, and M are the three unknowns.

Types of plane structure supports

FIG. 3.4 Roller Support
Patrick Feller

FIG. 3.5 Rocker Support
Patrick Feller, https://www.flickr.com/photos/nakrsum/11577558824/

FIG. 3.6 Hinged Support
Frank Rietschkaus/Rainy Stock Photo

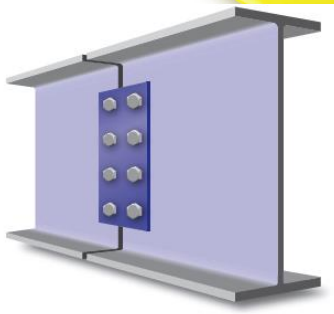
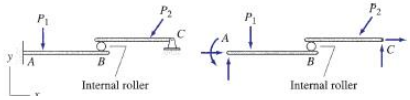


FIG. 3.14 Shear Splice



Thanks for your Attention

Determinacy and Stability of Structures

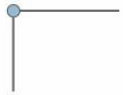
Theory of Structure

Dr. Oday Asal
Dr. Jasim Ali

Support connection

- *Pin support and pin connection*

pin support



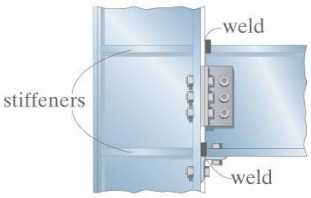
pin-connected joint

typical "pin-supported" connection (metal)

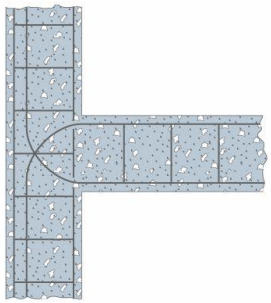
- *Fixed support & fixed connection*

fixed support

fixed-connected joint

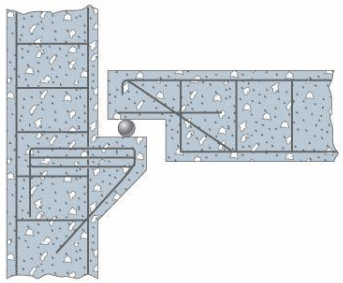


typical "fixed-supported" connection (metal)



typical "fixed-supported" connection (concrete)

- *Roller support*

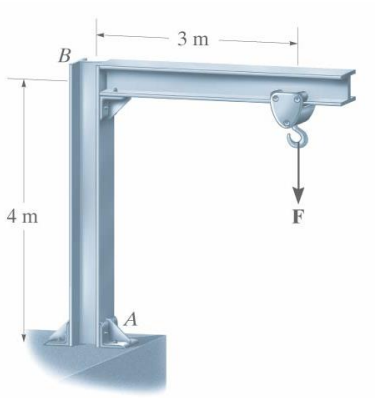


typical "roller-supported" connection (concrete)

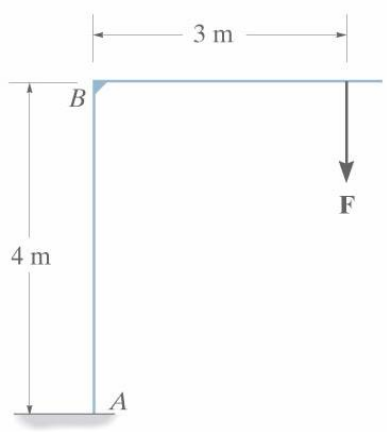
Support for coplanar structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1) light cable; weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2) rollers; rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5) smooth pin or hinge			Two unknowns. The reactions are two force components.
(7) fixed support			Three unknowns. The reactions are the moment and the two force components.

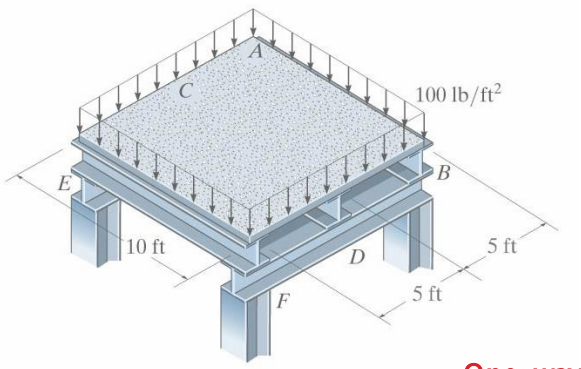
Idealized Structure

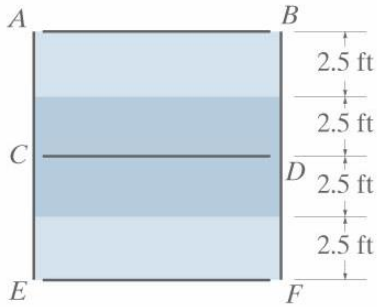


actual structure



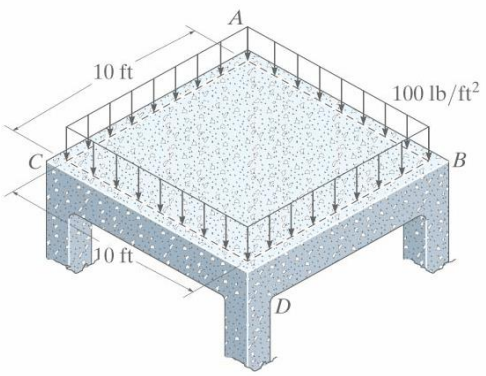
idealized structure

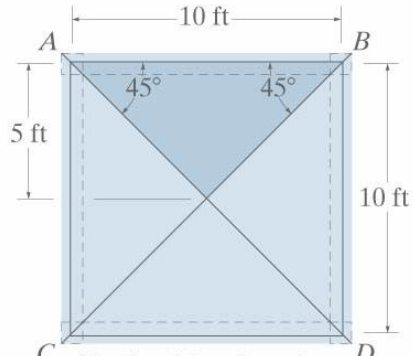




idealized framing plan

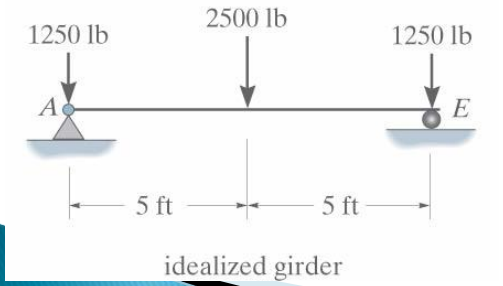
One-way slab



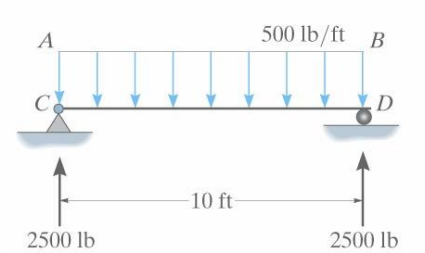


idealized framing plan

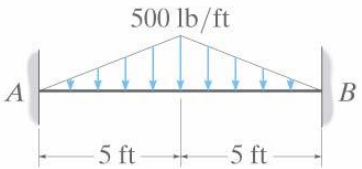
Two-way slab



idealized girder



idealized beam

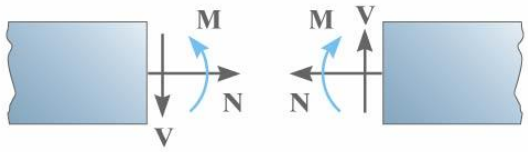


idealized beam

Equation of Equilibrium

- *In x-y plane*

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_o = 0$$



internal loadings

Determinacy & Stability

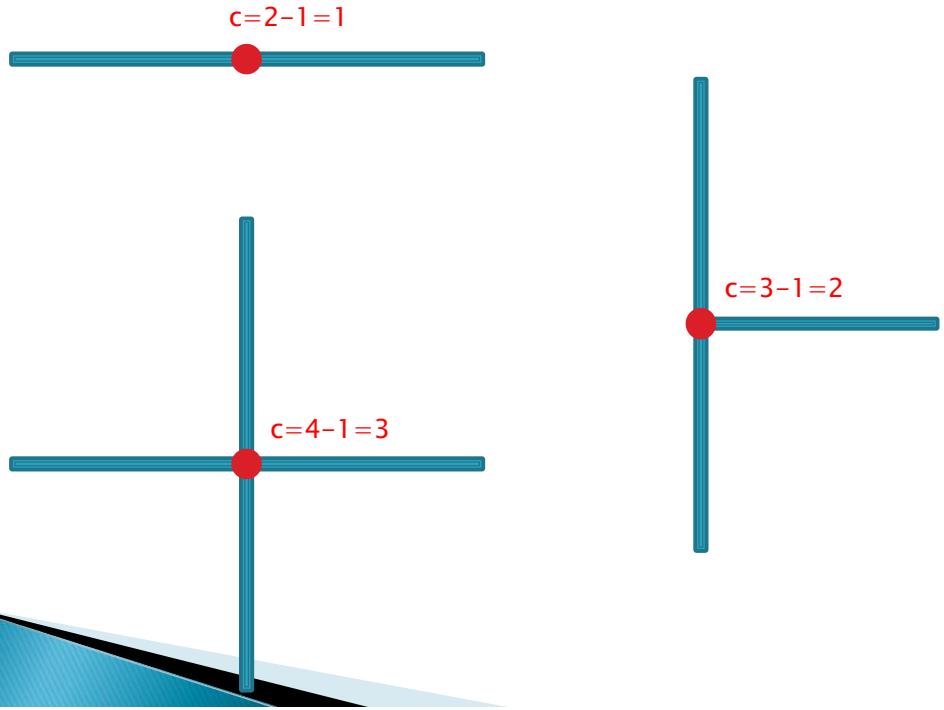
- **Determinacy**: when all the forces in structure can be determined from equilibrium equation , the structure is referred to as *statically determinate*. Structure having more unknown forces than available equilibrium equations called *statically indeterminate*
- If 3 is number of equilibrium equations, c is number of equilibrium equations from internal hinges and rollers (c=number of members-1)& r is number of unknown forces:

r:3+c

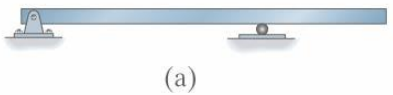
$r = 3+c$, statically determinate

$r > 3+c$, statically indeterminate

$r < 3+c$, unstable



Classify determinate & indeterminate structure



(a)

$r = 3$

$c = 0$

$r : 3+c$

$3 = 3+0$ Statically Determinate

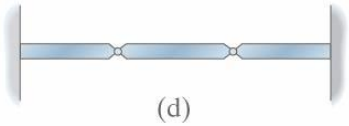
$r = 5$

$c = 0$

$r : 3 + c$

$5 > 3 + 0$ Statically indeterminate

2^{nd} degree



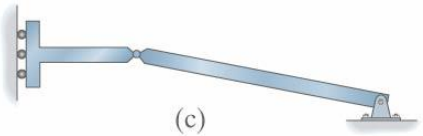
$r = 6$

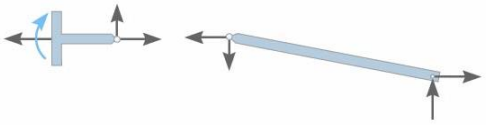
$c = 2$

$r : 3 + c$

$6 > 3 + 2$ Statically indeterminate

1^{st} degree



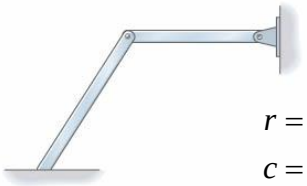


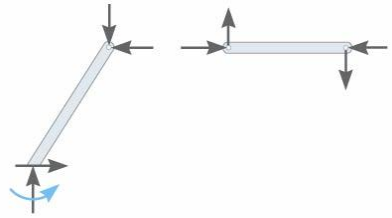
$r = 4$

$c = 1$

$r : 3 + c$

$4 = 3 + 1$ Statically determinate



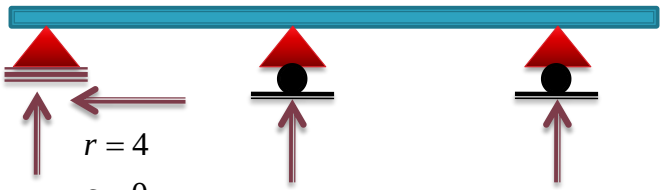


$r = 5$

$c = 1$

$r : 3 + c$

$5 > 3 + 1$ Statically indeterminate, 1^{st} degree



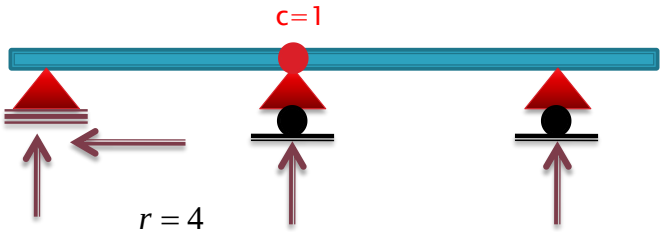
$r = 4$

$c = 0$

$r : 3 + c$

$4 > 3 + 0$ Statically indeterminate

1^{st} degree

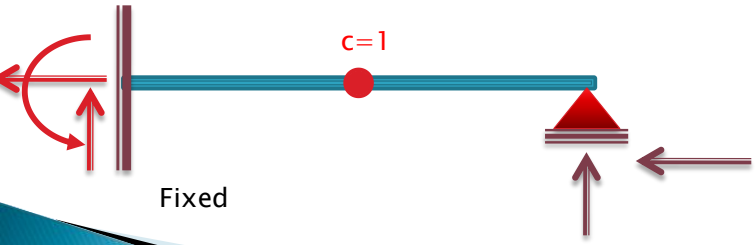


$r = 4$

$c = 1$

$r : 3 + c$

$4 = 3 + 1$ Statically determinate



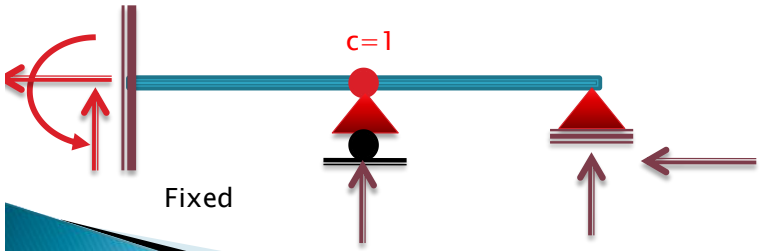
$r = 5$

$c = 1$

$r : 3 + c$

$5 > 3 + 1$ Statically indeterminate

1^{st} degree



$r = 6$

$c = 1$

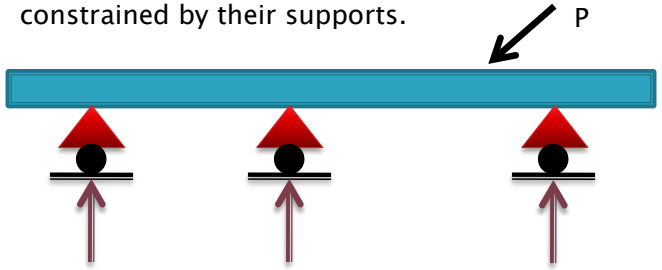
$r : 3 + c$

$6 > 3 + 1$ Statically indeterminate

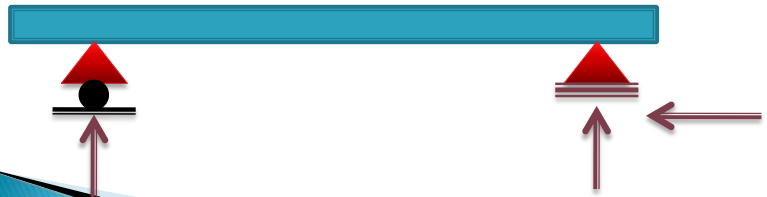
2^{nd} degree

Stability:

Stability of Structures: To ensure the equilibrium of a structure or its members, it is not only necessary to satisfy the equations of equilibrium, but the members must also be properly held or constrained by their supports.

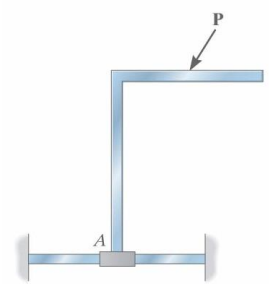


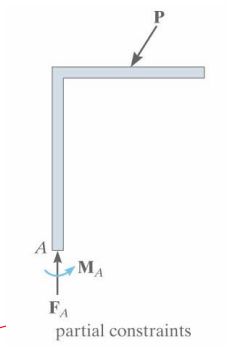
Unstable



stable

Partial Constraints

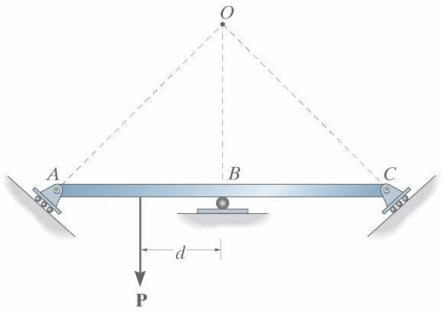




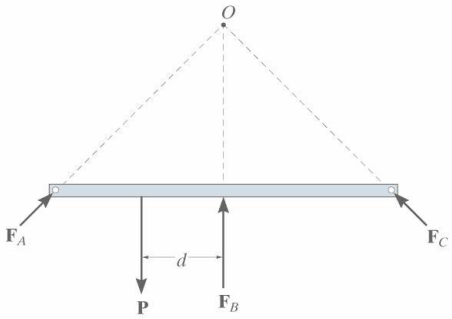
$\sum F_x = 0$ will not be satisfied $\Rightarrow$ unstable with this loading case

Improper Constraints

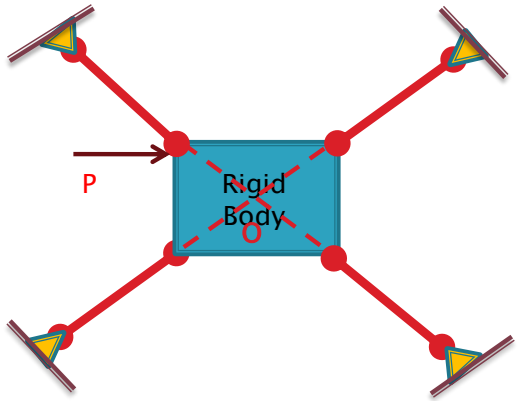
This can occur if all the support reactions are concurrent at a point.



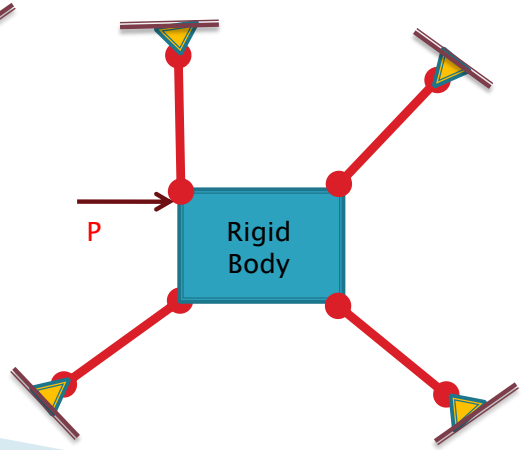
$P \times d \neq 0$



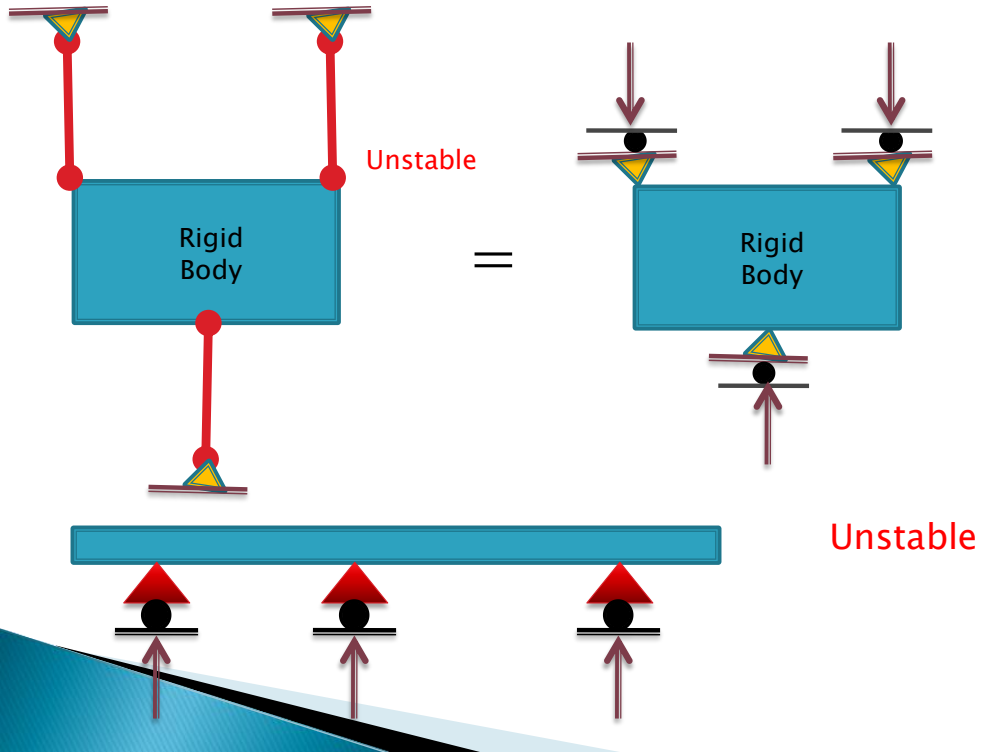
concurrent reactions

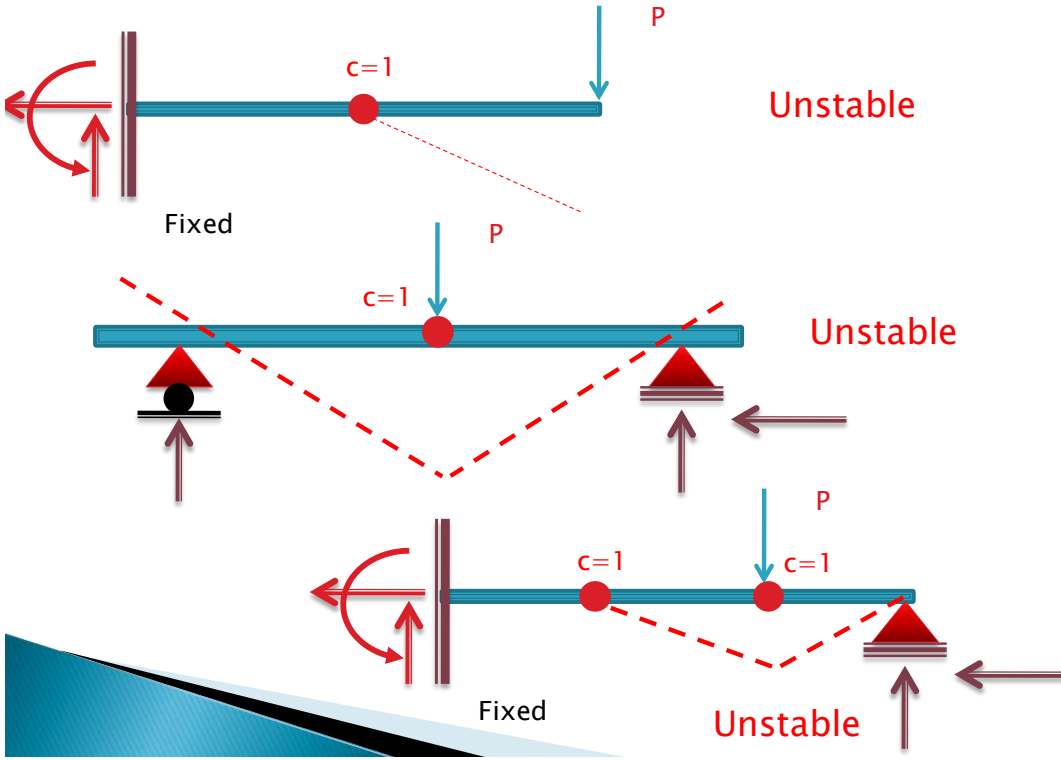


Unstable



stable





THANKS FOR ATTENTION

جامعة الموصل – كلية الهندسة
قسم الهندسة المدنية
2020 -2021

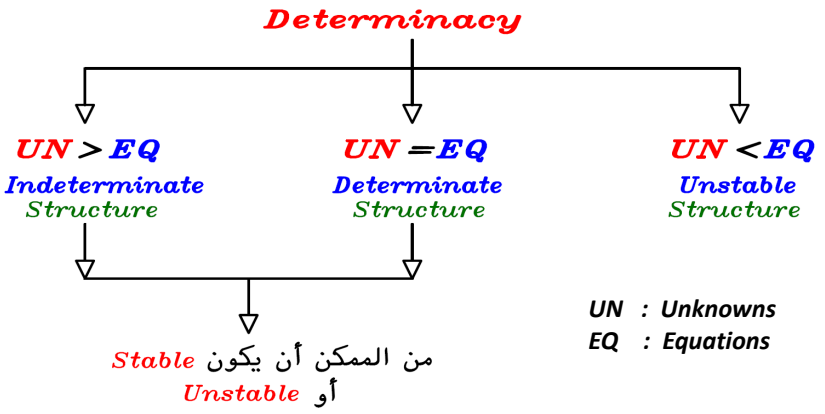
University of Mosul
College of Engineering
Department of Civil Engineering
Dr. Oday Asal Salih
Dr. Jasim Ali Abdulla

Theory of Structures
3rd Class 2020 - 2021

Determinacy and Stability

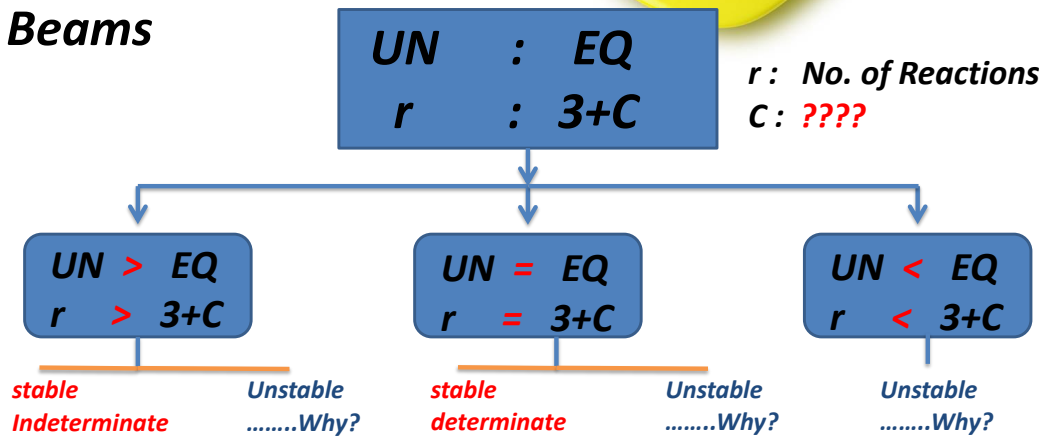
"Trusses and Frames"

(Determinacy and Stability)



(Determinacy and Stability)

Beams



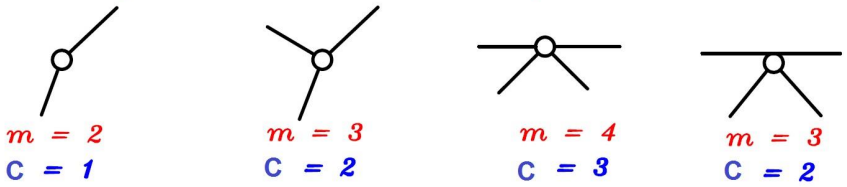
(Determinacy and Stability)

المعادلات التى تعطيها الIntermediate hinges فى حالة وجودها

$$C = m - 1$$

C : No. of equations given by hinge.

m : No. of members connected to hinge.



(Determinacy and Stability)

Frames

UN : EQ

r+3b : 3j+C

r : No. of Reactions

C : ????

b : No. of members

j : No. of Joints

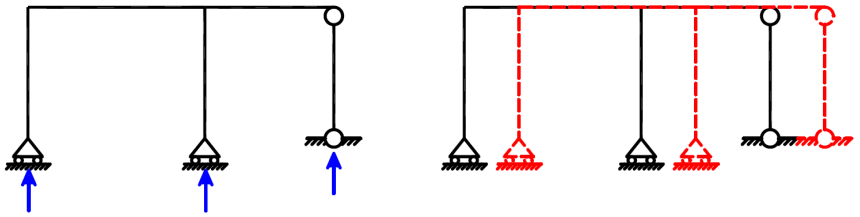
UN > EQ	UN = EQ	UN < EQ
r+3b > 3j+C	r+3b= 3j+C	r+3b < 3j+C
stable Indeterminate	stable determinate	Unstable .....Why?
Unstable .....Why?	Unstable .....Why?	

(Determinacy and Stability)

Frames - Unstable

1—All reactions are parallel

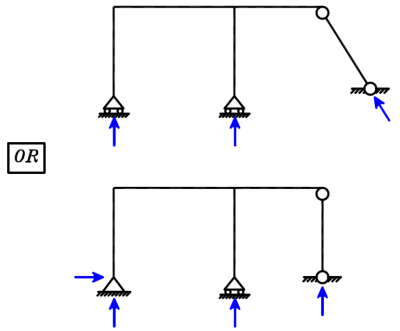
لو كانت كل الReactions متوازية فان الStructure يكون Unstable لانه سوف يتحرك فى الاتجاه العمودى على هذه الReactions.



(Determinacy and Stability)

Frames - Unstable --Stable

و لجعل هذا المنشأ Stable اما أن نغير اتجاه احد الReactions أو أن نضيف Reaction فى اتجاه اخر كالتالى .



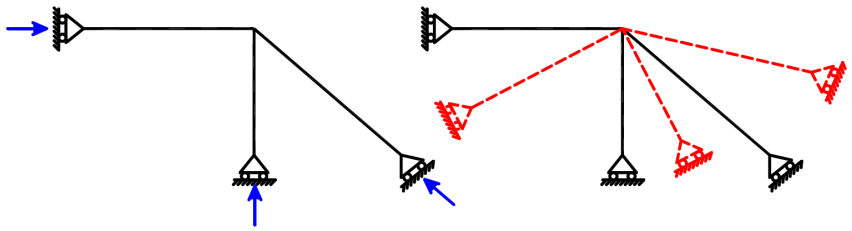
(Determinacy and Stability)

Frames - Unstable

2–All reactions are intersecting at a point

لو كانت كل الReactions متقاطعة فى نقطة قان الStructure يكون Unstable

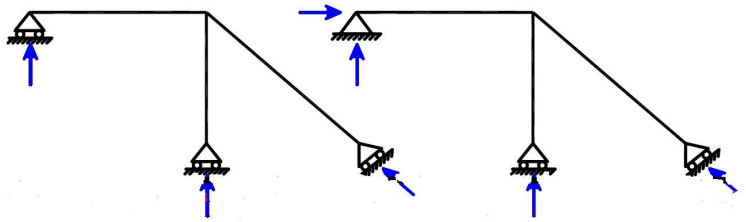
لانه سوف يدور حول هذه النقطة .



(Determinacy and Stability)

Frames - Unstable --Stable

و لجعل هذا المنشأ Stable اما أن نغير اتجاه احد الReactions حتى لاتكون متلاقية أو نضيف Reaction بحيث لا يمر بالنقطة .

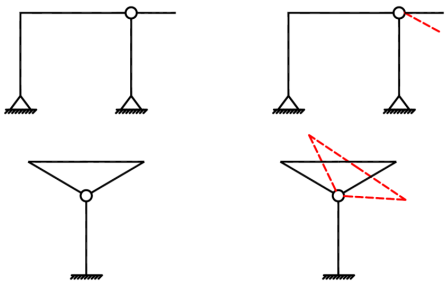


(Determinacy and Stability)

Frames - Unstable

3–Geometrical Unstability

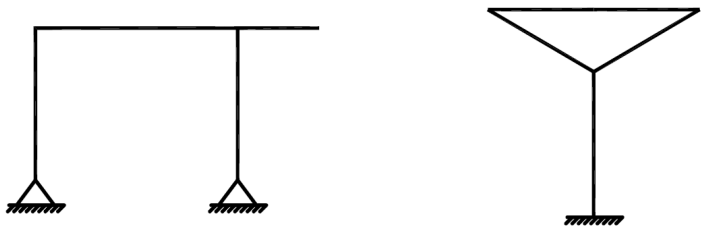
تحدث عندما يكون جزء من المنشأ متصل بـ Intermediate hinge من ناحية Free من الناحية الاخرى حيث أن هذا الجزء عند التأثير عليه باى قوة سوف دور حول الـ Intermediate hinge .



(Determinacy and Stability)

Frames - Unstable --Stable

و لجعل هذا المنشأ متزن نزيل الـ Intermediate hinge .



(Determinacy and Stability)

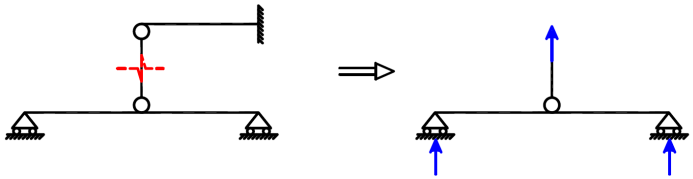
Frames - Unstable

4–Part of structure is Unstable

فى حالة وجود جزء من الـ Structure غير متزن يكون الـ Structure كله غير متزن لانه لو انهار جزء من الـ Structure فهذا معناه أن المنشأ كله أصبح غير آمن .

و يحدث هذا فى احد الحالات التالية :

a–Reactions are parallel for a part of the structure .



(Determinacy and Stability)

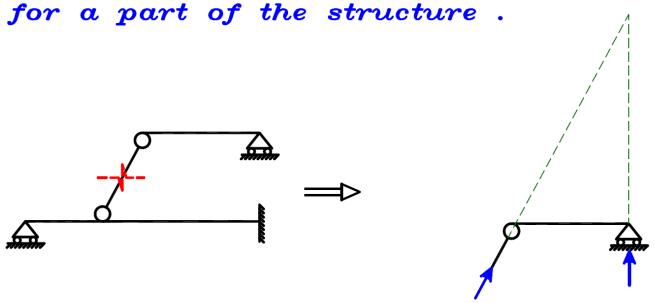
Frames - Unstable --Stable

و لجعل هذا المنشأ Stable اما أن نغير اتجاه احد الـ Reactions أو أن نضيف Reaction فى اتجاه اخر كالتالى .

(Determinacy and Stability)

Frames - Unstable

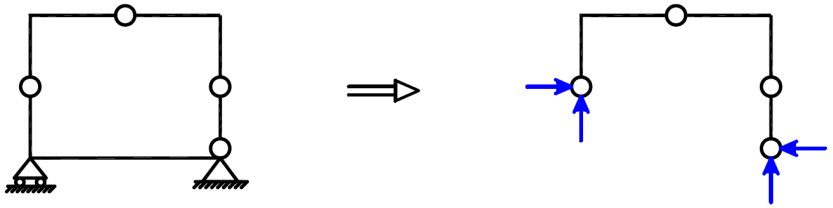
b–Reactions are intersecting at one point for a part of the structure .



(Determinacy and Stability)

Frames - Unstable

C–Case of more than three intermediate hinges in a closed frame .



(Determinacy and Stability)

Frames - Unstable --Stable

لانه فى هذه الحالة نجد أن الجزء المفصول به عدد ال $UN = 4$ بينما عدد ال EQ يكون كالتالى (٣ معادلات و هم $\Sigma M = 0$ $\Sigma Y = 0$ $\Sigma X = 0$ بالاضافة الى معادلات ال $I.H.$ فنجد أن 2 $I.H.$ بهما 2 $members$ فكل واحدة تعطى معادلة و 2 $I.H.$ بهما $member$ فلا يعطيا معادلات و بالتالى تكون ال $EQ = 5$) حيث أن هذا الجزء $UN < EQ$ يكون $Unstable$ و بالتالى يكون المنشأ كله $Unstable$.

و لكى يكون المنشأ $Stable$ نزيل احد ال $I.H$.

(Determinacy and Stability)

Degree of indeterminacy

فى أى من المنشآت السابقة لتحديد ال $Degree\ of\ indeterminacy$ تكون

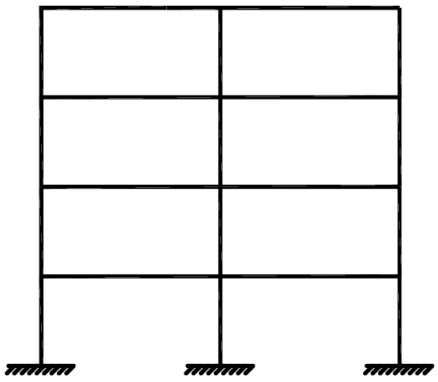
$$Degree\ of\ indeterminacy = UN - EQ$$

$UN = 4$ & $EQ = 2 + 1 = 3$

$UN > EQ$ ----- $Indeterminate\ structure$

$UN - EQ = 1$ $\Longrightarrow$ $Once\ statically\ indeterminate$

(Determinacy and Stability)



UN : EQ

$r+3b$: $3j+C$

$UN = r+3b = 9+3(20) = 69$

$EQ = 3j+C = 3(15)+0 = 45$

$UN > EQ$

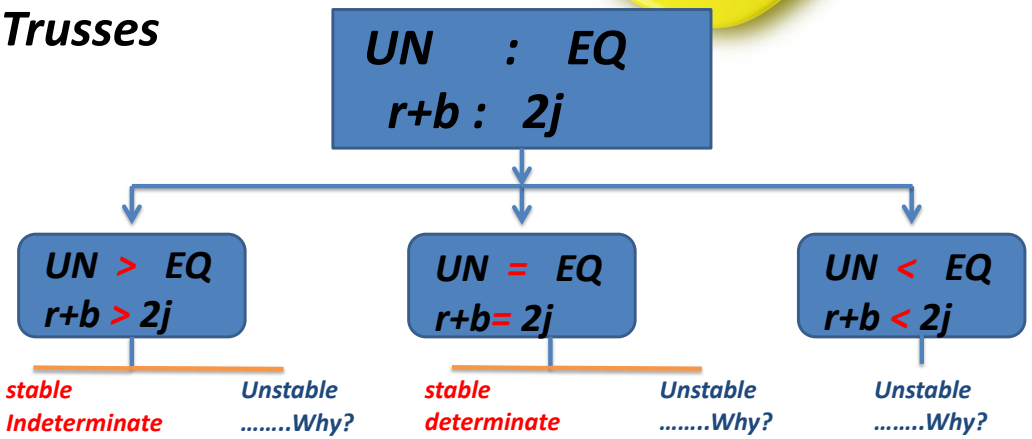
Indeteminate

$UN - EQ = 24$

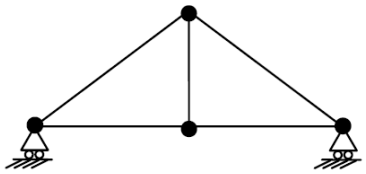
Stable & Indeterminate to the **24**

(Determinacy and Stability)

Trusses



(Determinacy and Stability)



$r = 2,\ b = 5,\ j = 4$

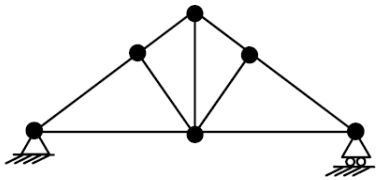
$b + r = 7,\ 2j = 8$

$7 < 8$ (Unstable)

UN < EQ

r+b < 2j

(Determinacy and Stability)



$r = 3,\ b = 9,\ j = 6$

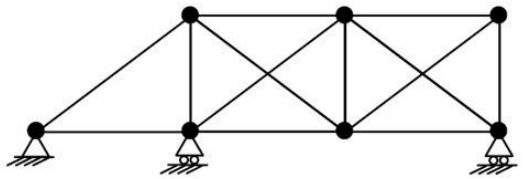
$b + r = 12,\ 2j = 12$

Stable & Determinate

UN = EQ

r+b= 2j

(Determinacy and Stability)



$r = 4,\ b = 13,\ j = 7$

$b + r = 17,\ 2j = 14$

Stable & Indeterminate to the 3rd degree

UN > EQ

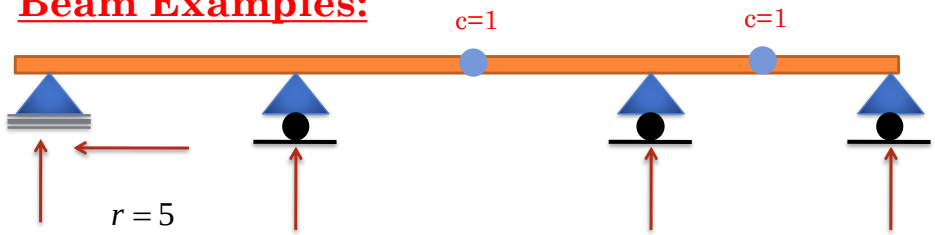
r+b > 2j

Thanks for your Attention

Examples in Stability & Determinacy of Structures

Dr. Jasim Ali Abdullah
Dr. Oday Asal Salih

Beam Examples:

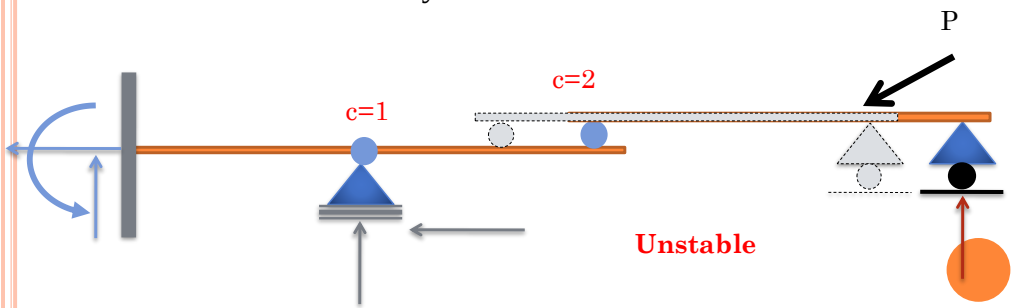


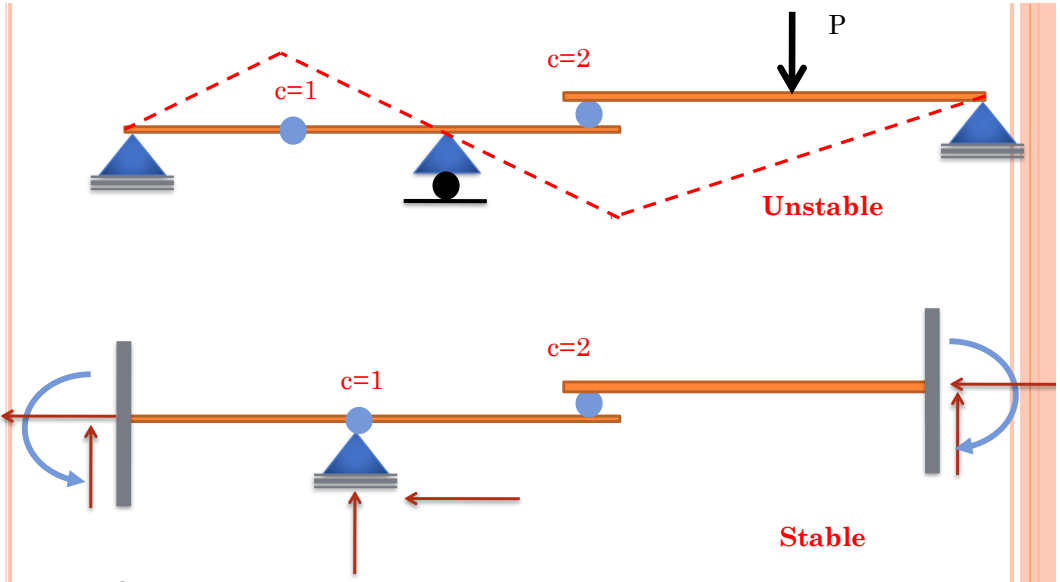
$r = 5$

$c = 2$

$r : 3 + c$

$5 > 3 + 2$ Statically determinate



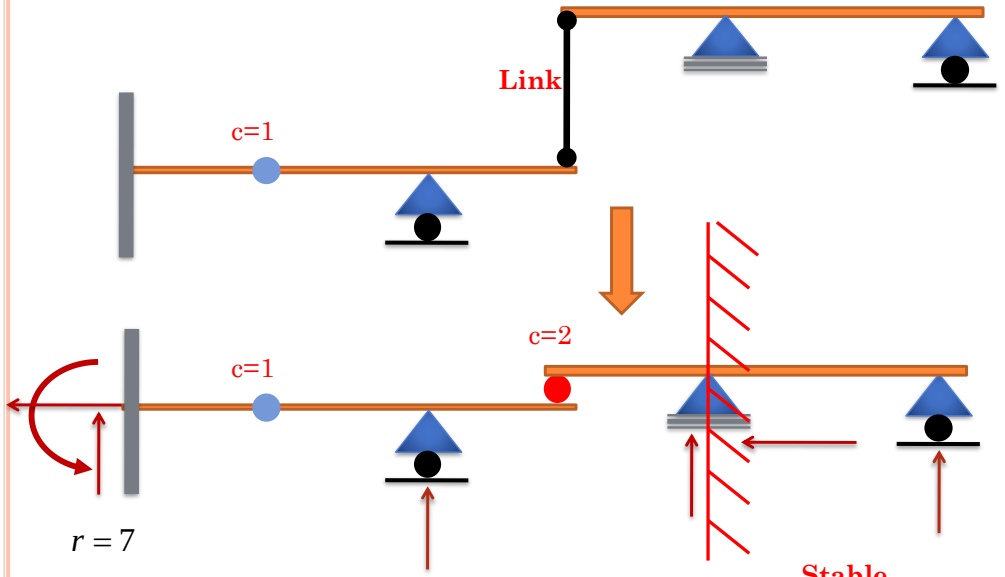


$r = 8$

$c = 3$

$r : 3 + c$

$8 > 3 + 3$ Statically indeterminate of 2^{nd} degree



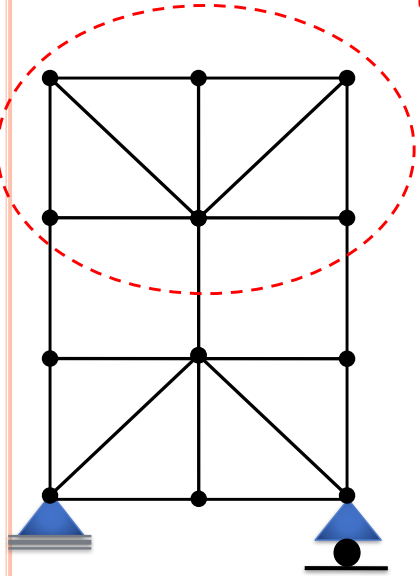
$r = 7$

$c = 3$

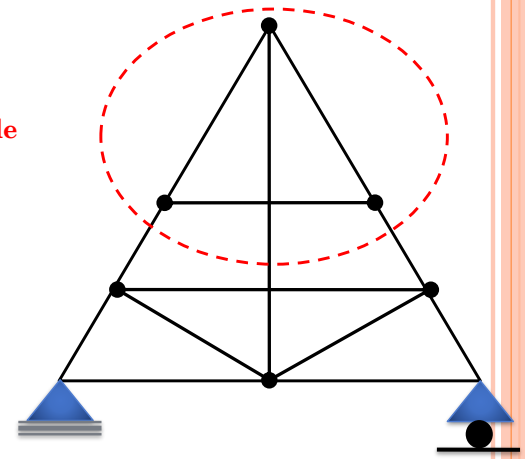
$r : 3 + c$

$7 > 3 + 3$ Statically indeterminate of 1^{st} degree

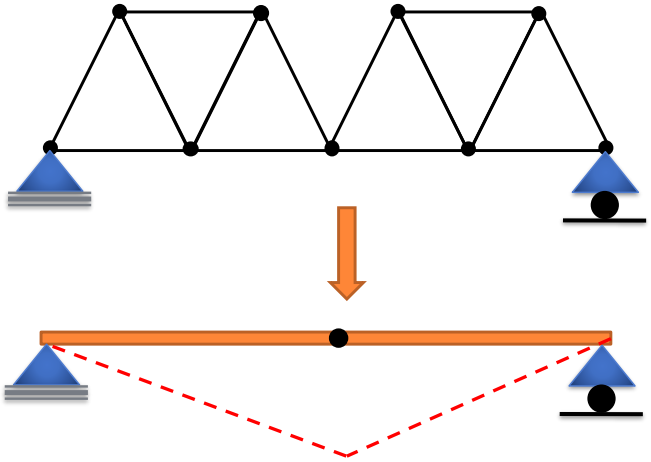
Truss Examples:



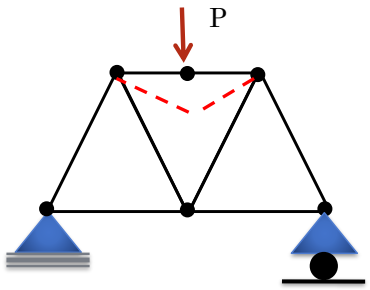
Unstable



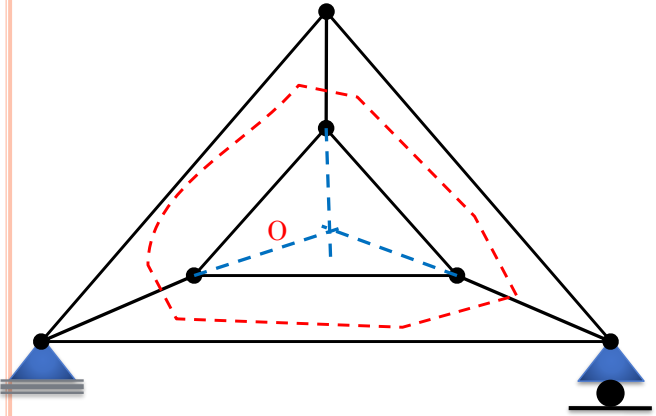
Unstable



Unstable

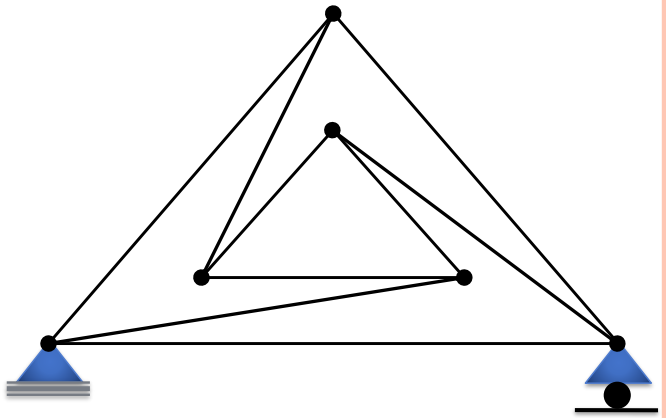


Unstable



Unstable

Stable



$r = 3$

$b = 9$

$j = 6$

$r + b : 2j$

$3 + 9 = 2 \times 6$ Statically determinate

Stable

$r = 3$

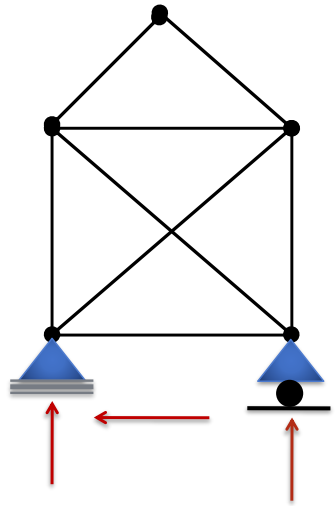
$b = 8$

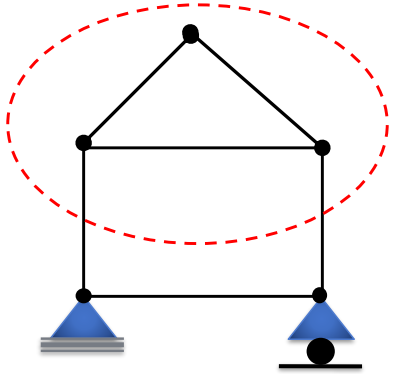
$j = 5$

$r + b : 2j$

$3 + 8 > 2 * 5$ Indeterminate

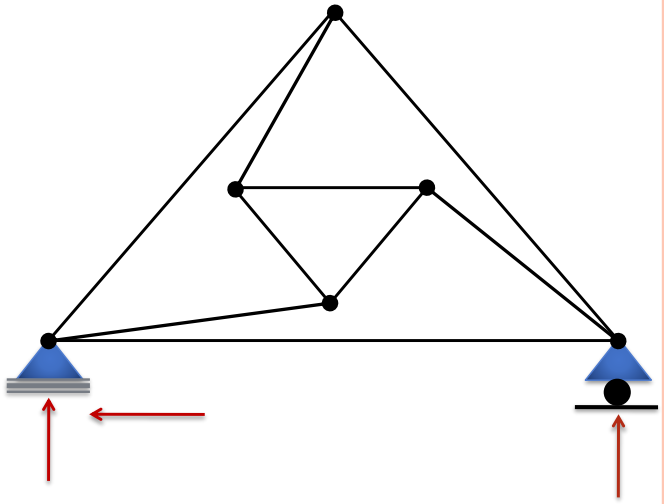
of 1st degree





Unstable

Stable



$r = 3$

$b = 9$

$j = 6$

$r + b : 2j$

$3 + 9 = 2 \times 6$ Statically determinate

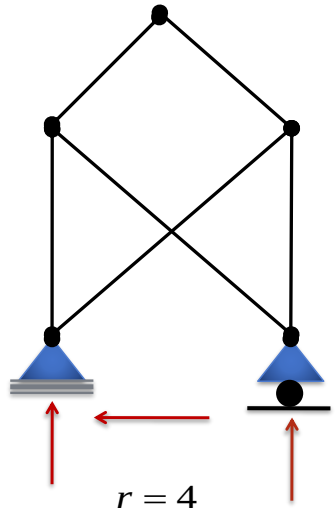
$r = 3$

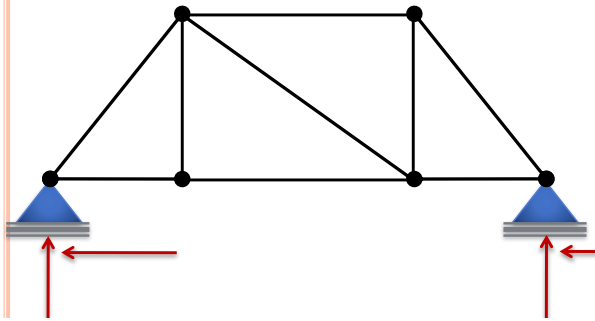
$b = 6$

$j = 5$

$r + b : 2j$

$3 + 6 \langle 2 * 5$ Unstable





$r = 4$

$b = 9$

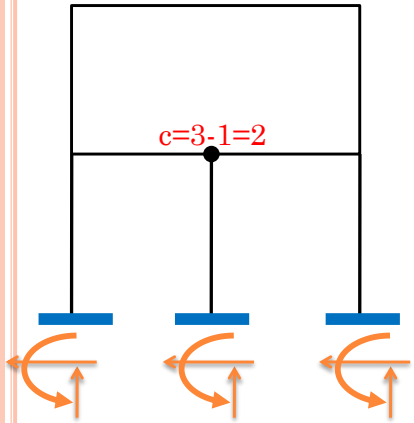
$j = 6$

$r + b : 2j$

$4 + 9 \rangle 2 * 6$ Indetrminte

1st degree

Frame Examples:



Unstable

$r = 9$

$b = 8$

$j = 8$

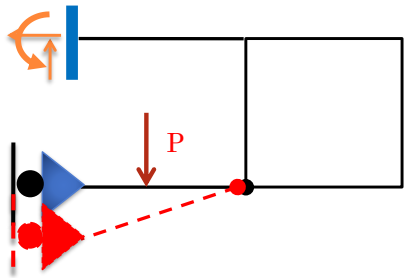
$c = 2$

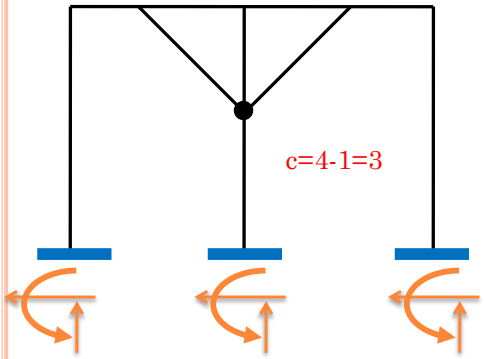
$r + 3b : 3j + c$

$9 + 3 \times 8 > 3 \times 8 + 2$

$33 > 26$

Indeterminate of 7th degree





$r = 9$

$b = 10$

$j = 9$

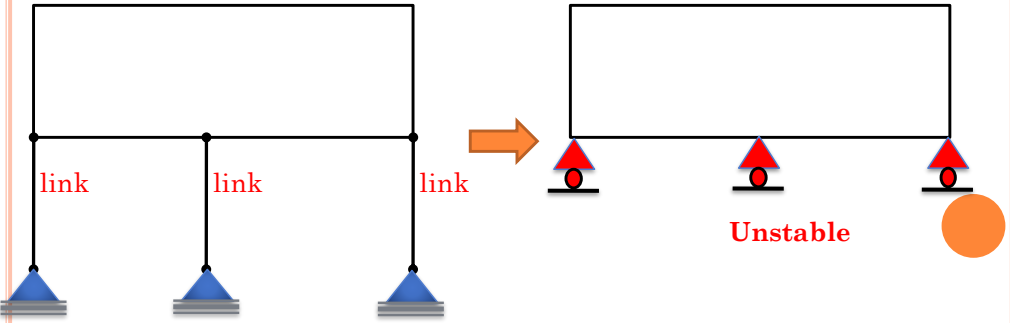
$c = 3$

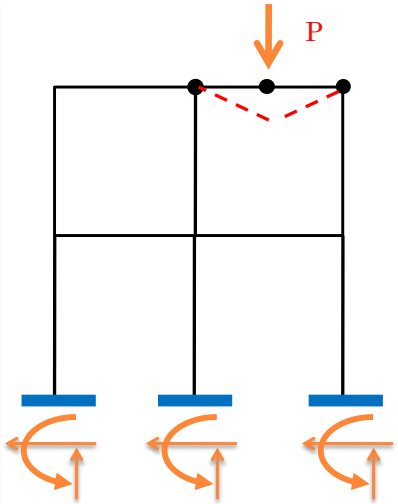
$r + 3b : 3j + c$

$9 + 3 \times 10 > 3 \times 9 + 3$

$39 > 30$

Indeterminate of 9^{th} degree





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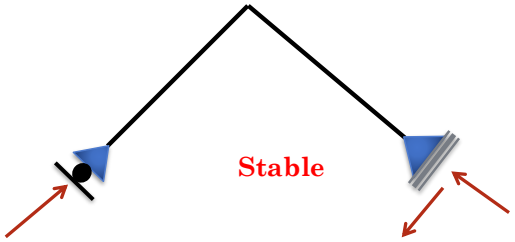


$r = 3$

$b = 2$

$j = 3$

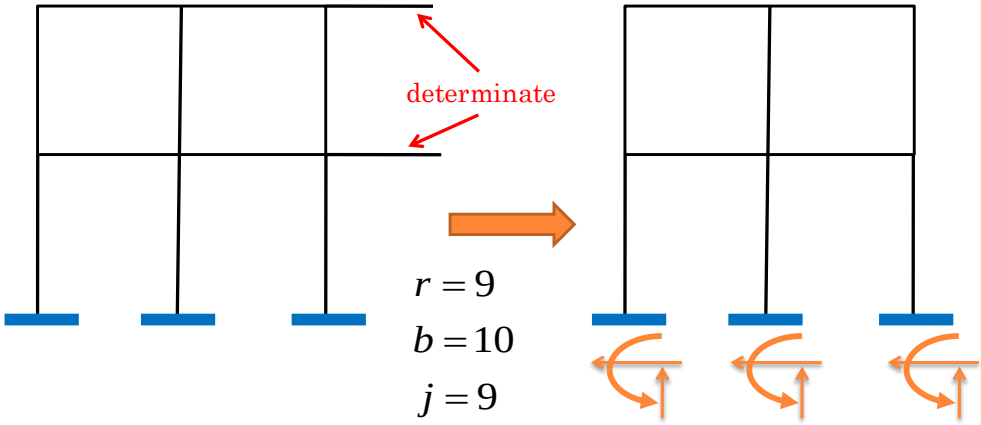
$c = 0$

$r + 3b : 3j + c$

$3 + 3 \times 2 = 3 \times 3 + 0$

$9 = 9$

Determinate



$r = 9$

$b = 10$

$j = 9$

$c = 0$

$r + 3b : 3j + c$

$9 + 3 \times 10 > 3 \times 9 + 0$

$39 > 27$

Indeterminate of 12^{th} degree

THANKS FOR ATTENTION