

Soil Mechanics

إن مفهوم ميكانيك التربة يتكون من كلمتين:

Soil -1: يقصد بها التربة حيث إن هنالك عدة مفاهيم للتربة منها:-

❖ Agriculture

وهي طبقة مواد سطحية لها القدرة على إدامة الحياة النباتية سمكها حوالي (0.5m)(Fig)

❖ Geologist

وهي طبقة رقيقة ضمن القشرة الأرضية حيث تمتد فيها جذور النباتات وهي ناتجة عن تفكك الصخور بسبب عوامل التعرية المختلفة سمكها أكبر أو يساوي (10m)(Fig).

❖ Rock

وهو ركام طبيعي صلب و مرصوص يتكون من الحبيبات المترابطة مع بعضها بأواصر دائمية أو غير دائمية.

❖ Soil Engineering (التربة من الناحية الهندسية)

وهي عبارة عن حبيبات ترابية متواجدة في الطبيعة بشكل مفكك أو متماسك حيث تتكون من مواد معدنية، منشأها الصخور وقد تحتوي على مواد عضوية بداخلها ويمكن فصلها عن بعضها بواسطة بعض العمليات الميكانيكية وهي تشكل جزء كبير من القشرة الأرضية يتراوح سمكها من (10-15m).

2- ميكانيك

وهو جزء من علم الفيزياء يدرس تأثير القوى الخارجية و الداخلية المؤثرة على الأجسام.

Mechanics

The part of physical science which treats the action of forces on bodies.

There is Many Types of Mechanics:

1. *static.* ميكانيك السكون (ميكانيك الصلبة)
2. *dynamic.* ميكانيك الحركة
3. *hydraulic.* ميكانيك المياه
4. *pneumatics.* ميكانيك الأجسام الغازية
5. *thermodynamics.* ميكانيك تحول الحرارة إلى طاقة

تعريف ميكانيك التربة (بشكل عام)

Soil Mechanic

Is the branch of mechanics that deals with forces effect on soil and involving the study of soil, its behavior and application as engineering materials.

تعريف ميكانيك التربة (حسب ما عرّفه العالم ترزاكي)

According to Terzaghi (1948):

Soil mechanics is the application of laws of mechanics and hydraulics to engineering problems dealing with sediments and other unconsolidated accumulations of solid particles produced by the mechanical and chemical disintegration of rocks regardless of whether or not they contain an admixture of organic constituent.

تطبيق القوانين الميكانيكية والهيدروليكية لحل ودراسة المشاكل الهندسية المتعلقة بالترسبات والتجمعات للحبيبات الصلبة المتكونة من التعرية الكيماوية والميكانيكية للصخور سواء كانت محتوية على مواد عضوية أم لا.

هناك بعض المصطلحات التي تحتاج الى ايضاح فيما يتعلق بمكانيك التربة:

1. *Geological engineering*
2. *Geotechnical engineering*
3. *Soil engineering*
4. *Soil mechanics*

1. Geotechnical engineering: is a broader term for soil mechanics and it contains:

- *Soil mechanics (soil properties and behavior).*
- *Soil dynamics (dynamics properties of soil ,earth quake engineering and machine foundation.*
- *Foundation engineering (deep and shallow foundation).*
- *Pavement engineering (flexible and rigid pavement).*
- *Rock mechanics (rock stability and tunneling).*
- *Geosynthetics (soil improvement).*

2. Soil engineering contains:-

- Soil mechanics.
- Foundation engineering.
- Soil as a construction materials

Soil engineering study includes:

- Soil as Load supporting material.
- Seepage problem through soil.
- Soil as a Construction material.

a. Load supporting material include:

Swelling.

Stability.

Settlement includes (total sett. and differential sett.).

Bearing capacity include (soft soil, stiff soil). (Fig)

b. Seepage problem

يشمل مشاكل تسرب الماء في السدود كذلك يشمل دراسة خطوط الجريان وخطوط تساوي الجهد (Fig).

c. Construction materials يشمل مايلي:

- أعمال الطرق وسكك الحديد (أي إنشاء الطبقات التحتية للسكك).
- الأعمال الترابية في السدود والسداد .
- في صناعات الطابوق والسيراميك والبوليمرات (Fig) .

تتميز التربة عن المواد الأخرى بما يلي:-

خواص التربة تتغير باستمرار مع تغير المحتوى المائي والظروف البيئية .
تعتبر التربة مادة متغيرة بتغير التركيب والمكان والحجم .

Comparison of soil with other materials

Soil is highly complex material. It differs from conventional structural materials, such as steel and concrete.

1. steel is a manufactured material the properties of which are accurately controlled. The properties of concrete are also controlled to some extent during its preparation.

Soil

is a material which has been subjected to vagaries of nature, without any control. Consequently, soil is a highly heterogeneous and unpredictable material.

أذن التربة مادة غير متجانسة ومتغيرة الخواص

2. The properties of a soil change not only from place to other but also at the place with depth. The properties also change with a change in the environmental, loading and drainage conditions. The properties of a soil depend not only on its type but also on the conditions under which it exists.

تتغير خواص الترب تبعاً لتغير:

- المكان ٢- العمق ٣- الظروف البيئية ٤- وجود الاحمال عليها
- ظروف تصريف المياه ٥-

3. The main engineering properties of steel and concrete are modulus of elasticity and tensile and compressive strength. Most of the design work can be done if these properties are known or determined. however, the engineering properties of soils depend upon a number of factors and it is not possible to characterize them by two or three parameters. Elaborate testing is required to determine the characteristics of the soil before design can be done.

الخواص الهندسية للتربة تعتمد على عدد كبير من العوامل ولا يمكن حصرها بعاملين او ثلاثة مثلاً (معامل المرونة وقوة الشد والانضغاط) كما في حالة الكونكريت والحديد

4. *Because of huge quantities of soil involved, it is not economically feasible to transport the soils from other places like steel or concrete. Soils are generally used in the conditions in which they exist.*

بسبب كميات التربة الكبيرة، فلا يمكن نقل التربة الجيدة الى المواقع ذات التربة الاقل جودة، لذلك يتم تصميم المنشأ بحيث يلائم ظروف التربة الموجودة.

5. *Whereas steel and concrete can be inspected before use, soils for foundations are at great depth and not open to inspection. The sample of the soil taken from the bore holes are generally disturbed and do not represent the true in-situ conditions.*

الترب (تحت الاسس المراد انشائها) تكون على اعماق كبيرة ولا يمكن معاينتها مباشرة، بل يتم اخذ نماذج من الحفر الاختبارية (وهذه النماذج تكون مشوشة الى حد ما ولا تمثل الظروف الحقلية تماما)

Limitation of soil engineering

Soil engineering is not an exact science. Because of the nature and the variability of soils, sweeping *assumptions* are made in the derivation of equations. The solution obtained in most cases are for an idealized, hypothetical material, which may not truly represent the actual soil. *A good engineering judgment is required for the interpretation of the results.* In fact, each problem in soil engineering is a unique problem because the soils at two places are seldom identical.

هندسة التربة لا يكون علماً دقيقاً بسبب خواص التربة المتغيرة، بل يشتمل على فرضيات كثيرة عند اشتقاق المعادلات، لذلك تحتاج النتائج الى قرار هندسي

صائب يعتمد على الخبرة.

The following limitations must be kept in mind when tackling problems related with soils.

1. *As the soil does not possess a linear or unique stress-strain relationship, the solution of the theory of elasticity cannot be directly applied.*

العلاقة بين الاجهاد-الانفعال لا تكون خطية لذلك لا يمكن تطبيق نظرية المرونة بشكل مباشر

2. *The behavior and the strength of soils depend upon pressure, drainage, environment and many other factors.*

These changes must be considered when dealing with soils.

يجب اخذ ظروف الضغط، التصريف، الظروف البيئية وغيرها من العوامل لكونها تؤثر على خواص ومقاومة التربة.

3. *As the soil at every location is different, the results and experience from one project to the other should be transferred with caution.*

خواص التربة تتغير من موقع إلى آخر عليه يجب الانتباه عند نقل الخبرة من مشروع لآخر

4. *Since the soils are very sensitive to disturbance, the results of tests conducted on soil samples should be interpreted carefully.*

نتائج الفحوصات المختبرية تكون حساسة جداً لأي تشوه في النماذج

5. The most of soil is underground and cannot be inspected. Adequate soil exploration should be done to determine the profile of soil strata.

يجب اجراء تحريات كافية للموقع لأعطاء صورة واضحة عن طبقات التربة الموجودة

6. The methods of construction may have to be modified as the work progresses and the properties of the soil begin to unfold. Occasional observations have to be made during and even after the completion of work to check whether the assumptions made were correct.

يجب تغيير اساليب الانشاء في الموقع، عند استمرار العمل وعندما تتبين خواص التربة بشكل اوضح

7. It may not be much use to apply highly mathematical, rigorous solution to a material like soil whose properties cannot be determined to the same accuracy.

استعمال حلول رياضية لا يعطي نتائج ذات دقة عالية للتربة لأن خواصها متغيرة

8. The soil is a particulate material in which the particles are relatively free to move with respect to one another. The behavior of the soil changes as the particles get shifted.

خواص التربة تتغير بتغير موقع جزيئاتها بالنسبة لبعضها.

9. The soil is a multiphase system, consisting of solid, water and air phases. The behavior of a soil depends upon the relative proportion of the three phases

التربة مادة متعددة الطور، تتكون من المادة الصلبة والماء والهواء، وسلوك التربة يعتمد على نسب هذه الاطوار

10. Soil mechanics is relatively new science. It is essential to keep abreast the latest developments in the field.

ميكانيك التربة علم جديد نسبياً، لذلك من المهم الاطلاع على آخر التطورات في هذا المجال

بعض العلماء الذين ساهموا في تطور ميكانيك التربة.

- Vauban (1687) وضع عدة قوانين تطبيقية لتصميم بعض المنشآت .
- Coulomb (1773) طور نظرية ضغط التربة الجانبي التي وضعت عام 1691.
- Rankine (1856) استخدم نظرية المرونة في معالجة الجدران الساندة.
- Darcy (1856) وضع ما يسمى ألان بقانون دارسي والمتعلق بجريان الماء.
- Stock (1856) وضع قانون ستوك لسرعة سقوط المادة الصلبة في مادة سائلة.
- Boussinesq (1885) وضع قوانين لتوزيع الاجهادات داخل التربة.
- Collin (1846) وضع قوانين ميكانيكية الزحف الأرضي .
- Briggs (1907) درس رطوبة التربة في القرن التاسع عشر.
- Atterberg وضع حدود قوام التربة.

- Karl Terzaghi (1925) درس ميكانيك التربة.
- Karl Terzaghi (1948) وضع أسس هندسة التربة.
- العالم ترزاكي هو أول من وضع الأسس لعلم هندسة الأسس والتربة
- ووضع ثلاث شروط أساسية يجب إتباعها لبناء هذا العلم وهذه الشروط هي :
- الملاحظة .
- النظرية والتحليل .
- الجزء العملي والتطبيقي .
- Proctor (1933) وجد عملية الرص.
- Gasagrande (1940) قام بتصنيف التربة.
- Taylor (1948)
- Lambe (1960)
- Bishop and Peck
- Smith and Ziankiewicz study geotechnical engineering by computer
- Fredlund D G 1992 وضع الاسس العلمية لموضوع ميكانيك التربة غير المشبعة

(Fig)



Role of Geotechnical Engineers and Specialists

- دراسة الخصائص الفيزيائية، الدالة والهندسية للتربة physical index.
- تصنيف التربة. soil classification.
- دراسة ملائمة التربة كمادة للإنشاء .
- دراسة وتقويم رد فعل التربة للتغيرات في الأحمال والرطوبة .
- تأثير المياه على خصائص التربة وجريان الماء خلال التربة .
- إيجاد طرق معالجة وحلول لأي مشكلة هندسية تتعلق بميكانيكية التربة

مفردات المهنة

□ Introduction to soil engineering

- Soil formation
- Composition & structure of soil
- Cohesive & cohesionless soil
- Clay mineralogy & effect of water on its consistency
- Soil phases conditions relationship

□ Physico-mechanical properties of soil

- soil texture
- Index properties
- Classification

□ Introduction to soil exploration & sampling

(Fig)

□ Hydraulic properties of soil

- Capillarity
- Permeability
- Flow of water in soils (two dimensional flow)

□ Soil improvement

- Problematic soil
- compaction
- The main soil stabilization methods

□ Stresses within soil mass (total & effective)

□ Hydro-mechanical properties of soil

- Compressibility behaviors
- Consolidation
- Settlement analysis

❑ **Shear strength of soil**

- Tests to determine shear strength of parameters
- Coulomb equation
- Mohr theory of failure & Mohr diagram for stress
- Shear strength of cohesive soil
- Shear strength of cohesionless soil

❑ **Earth pressure of soils**

❑ **stability of slopes**

References

1. Soil mechanics by Lamb & Whitman
2. Soil engineering by Spangler & Handy
3. Soil mechanics by Smith
4. Physical & geotechnical properties of soil by Bowles
5. Principles of soil mechanics by AL-Asho in Arabic
6. soil mechanics & foundation engineering by Sing & Brakash
7. fundamentals of soil mechanics "by MITCHELL, J.K

Some Suggestions

Attend the lectures.

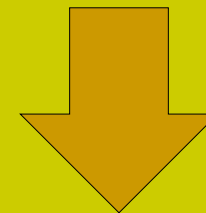
It takes longer to understand from the lecture notes

Develop a good feel for the subject.

It is practical, interesting and makes lot of sense.

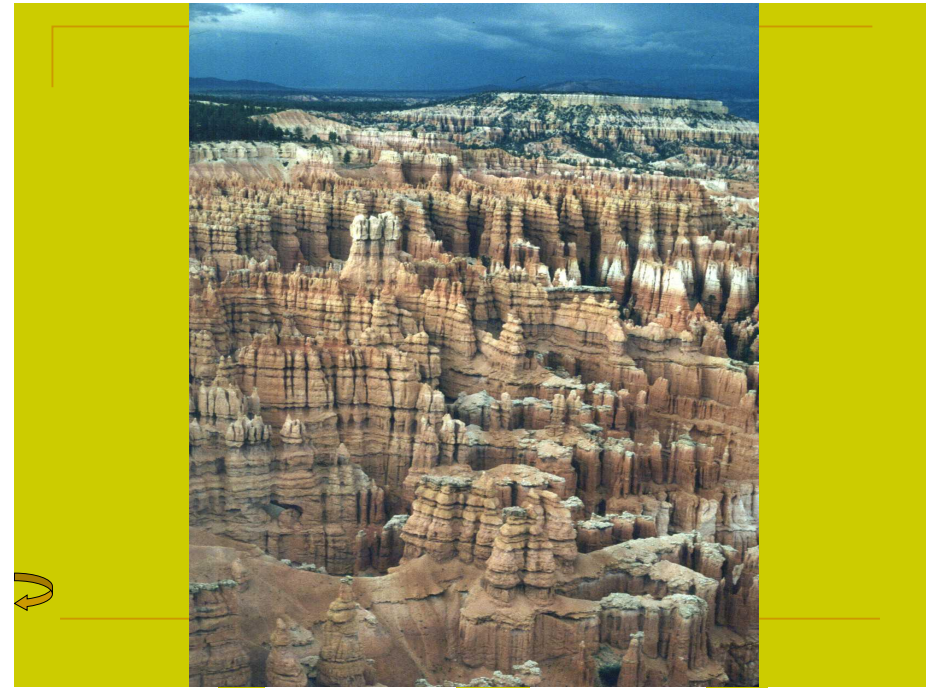
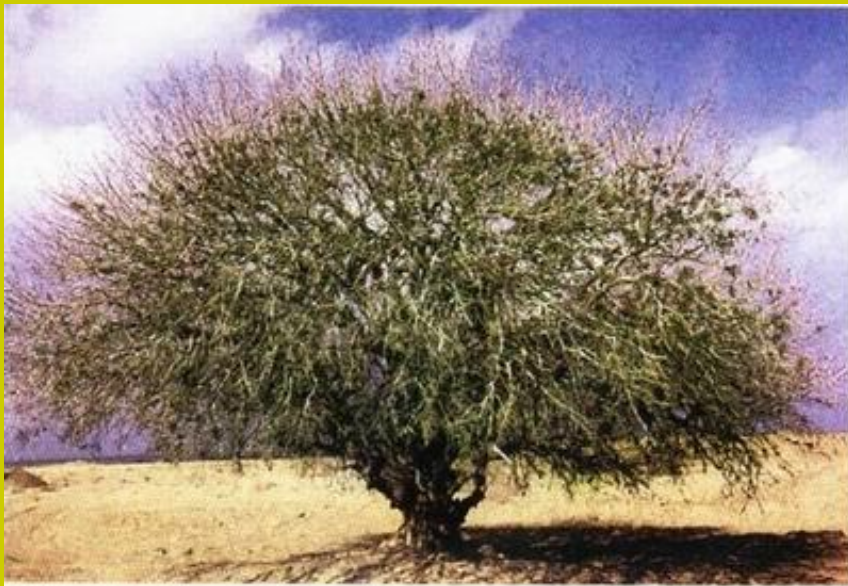
Some Suggestions

Work in groups.



Exams

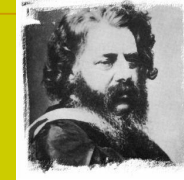
My mama always said, "Exam is like a box of chocolates; you never know what you are gonna get"



Karl Terzaghi
1883-1963



C.A. Coulomb
1736-1806



W.J.M. Rankine
1820-1872



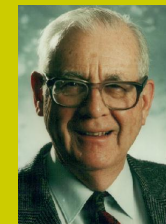
A. Casagrande
1902-1981



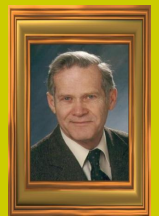
L. Bjerrum
1918-1973



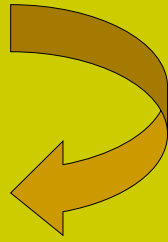
A.W. Skempton
1914-



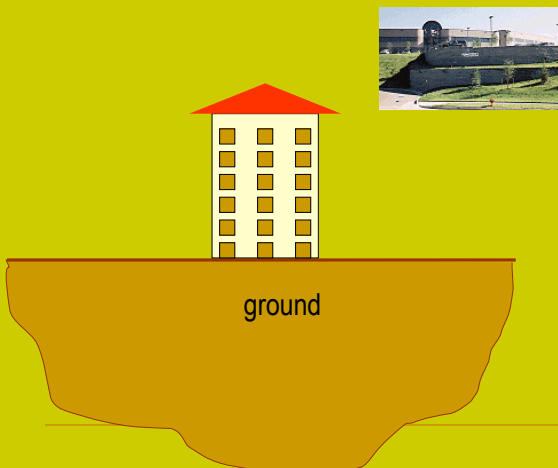
G.F. Sowers
1921-1996



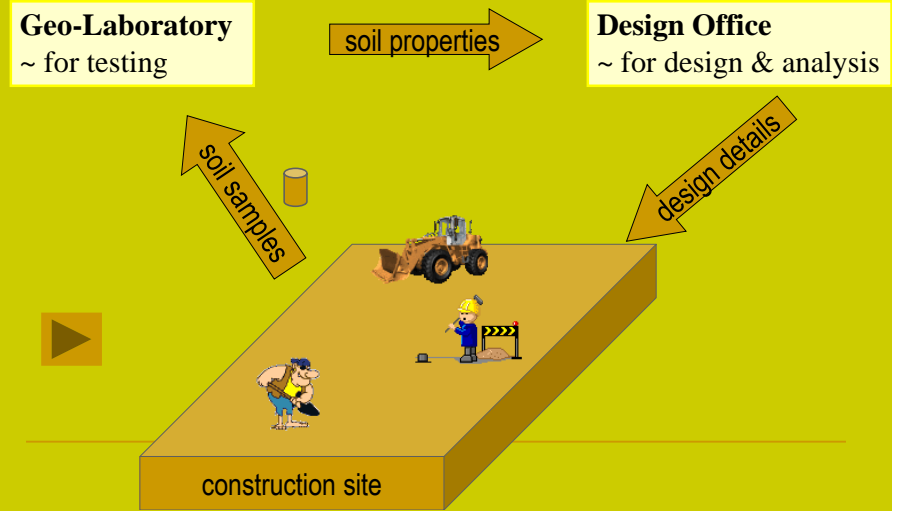
G.A. Leonards
1921-1997



Introduction to Geotechnical Engineering



Typical Geotechnical Project



Soil Exploration



Falling Truck Mounted Rig



CME750 All-Terrain Rig

Soil Exploration



MoDOT Track Mounted Rig



Layne Track Rig

Soil Exploration



Wire line Rig for Kaolin Mines, Macon, GA

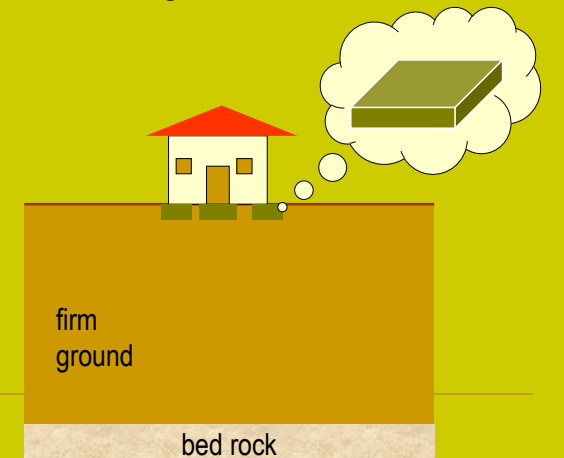


Water Boring from Barge for Bridge Crossing



Shallow Foundations

- ~ for transferring building loads to underlying ground
- ~ mostly for firm soils or light loads



Shallow Foundations



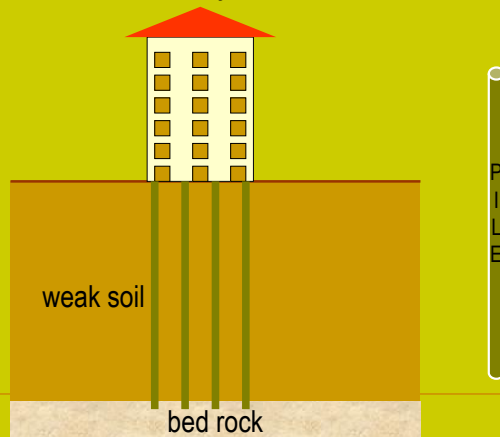
Deep Foundations



Driven timber piles, Pacific Highway

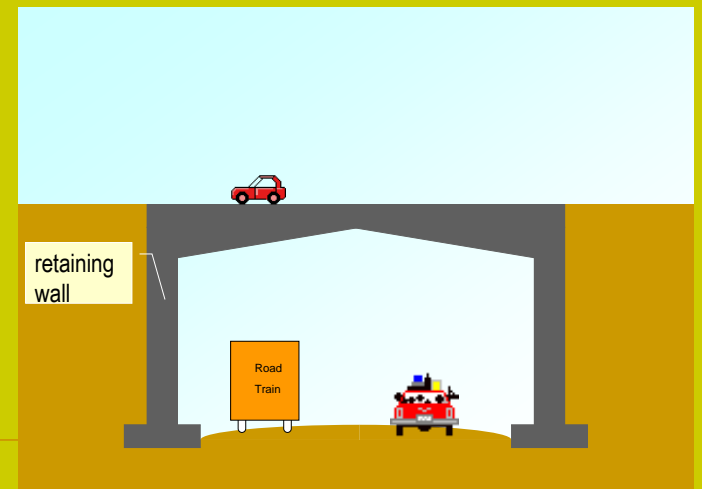
Deep Foundations

- ~ for transferring building loads to underlying ground
- ~ mostly for weak soils or heavy loads



Retaining Walls

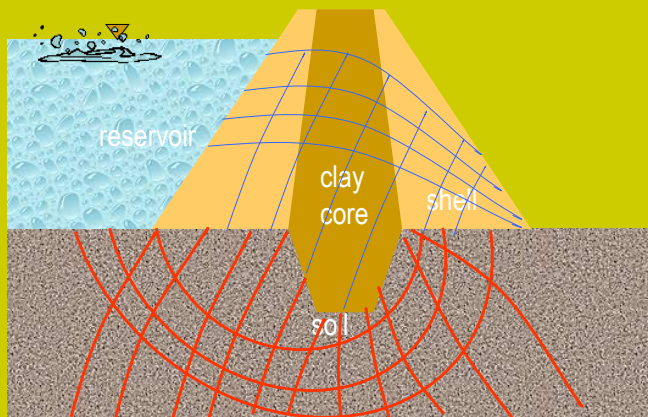
- ~ for retaining soils from spreading laterally



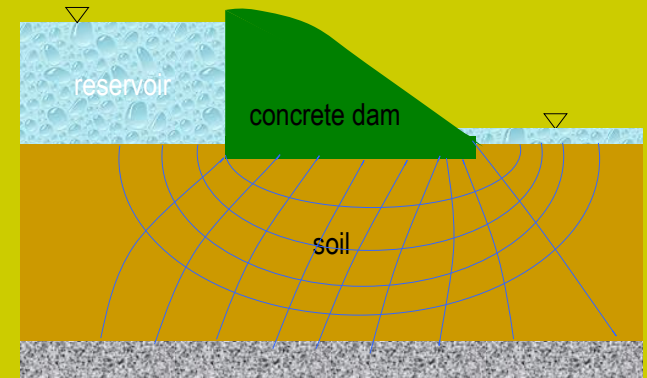


Earth Dams

~ for impounding water



Concrete Dams



Concrete Dams



Three Gorges Dam, Hong Kong

Concrete Dams



Earthworks

~ preparing the ground prior to construction



Roadwork, Pacific Highway



Construction hazard

~ an unwelcome visitor at an earthwork site.



What does it have to do with Geo?#!

A dead Anaconda python (courtesy: J. Brunskill)

Geofabrics

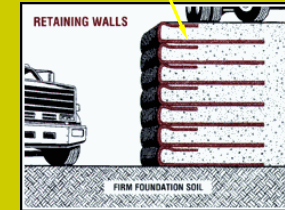
~ used for reinforcement, separation, filtration and drainage in roads, retaining walls, embankments...



Geofabrics used on Pacific Highway

Reinforced Earth Walls

~ using geofabrics to strengthen the soil



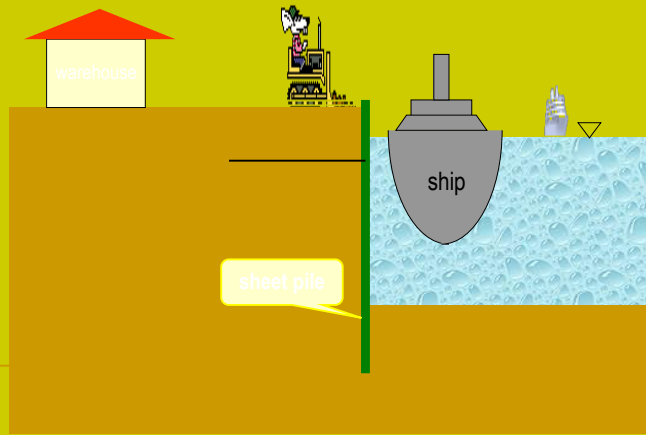
Soil Nailing

~ steel rods placed into holes drilled into the walls and grouted



Sheet Piles

~ sheets of interlocking steel or timber driven into the ground, forming a continuous sheet



Sheet Piles

~ resist lateral earth pressures
~ used in excavations, waterfront structures, ..



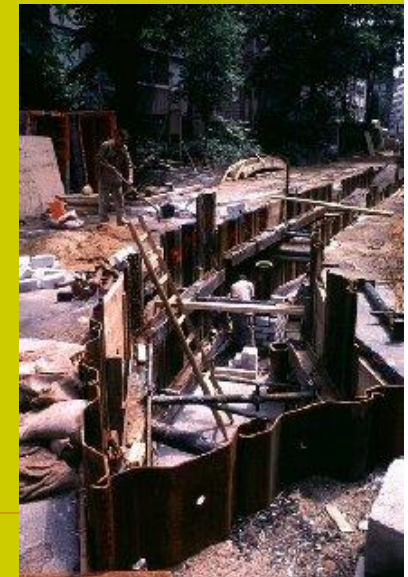
Sheet Piles

~ used in temporary works



Sheet Piles

~ interlocking sections



Cofferdam

~ sheet pile walls enclosing an area, to prevent water seeping in

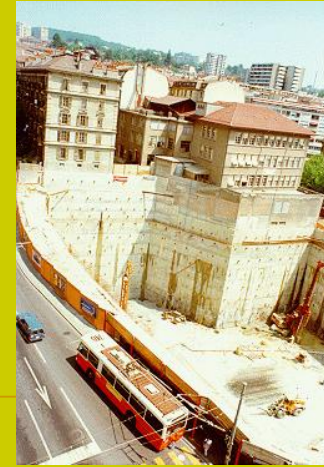


Landslides



Shoring

propping and supporting the exposed walls to resist lateral earth pressures



Tunneling



Blasting



For ore recovery in mines



Ground Improvement



Impact Roller to Compact the Ground

Ground Improvement



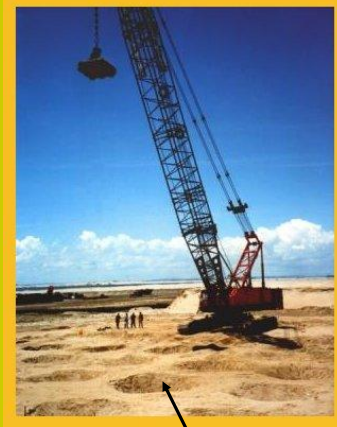
Sheep foot Roller to Compact Clay Soils

Ground Improvement



Smooth-wheeled Roller

Ground Improvement

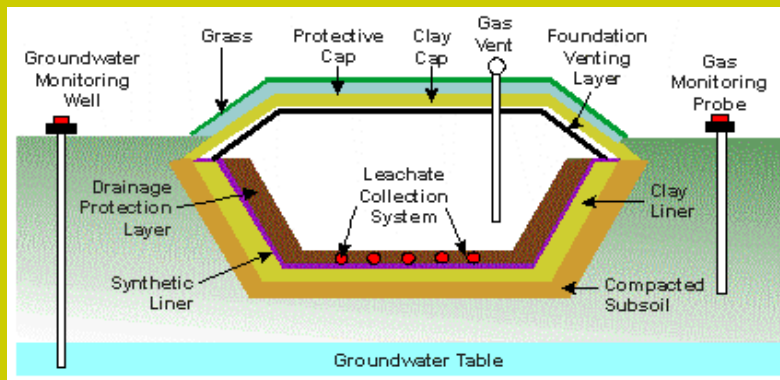


Big weights dropped from 25 m, compacting the ground.



Craters formed in compaction

Environmental Geomechanics



Waste Disposal in Landfills

Soil Engineering: is the application of the principles of soil mechanics to practical problems.

Geotechnical Engineering: is defined as the sub discipline of civil engineering that involves natural materials found close to the surface of the earth. It includes the application of soil mechanics and rock mechanics to the design of foundations, retaining structures, and earth structures.

Lateral Earth Pressure

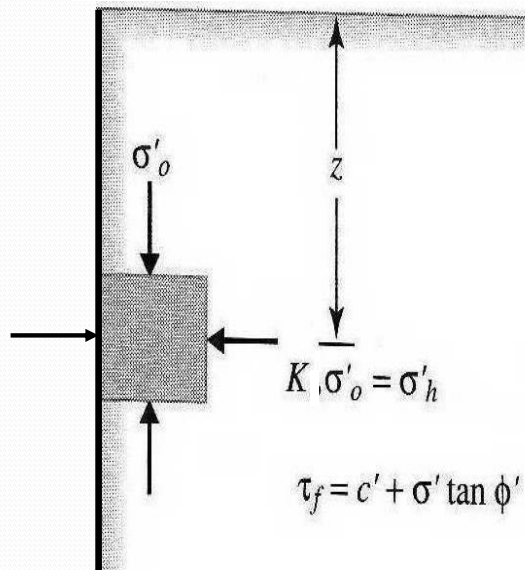
Lateral Earth Pressure

- Retaining structures such as retaining walls, basement walls, bulkheads are support slopes of earth masses.
- To design or construction these structures required to know the lateral forces that acts between the retaining structures and the soil masses being retained.
- The lateral forces are caused by lateral earth pressure.
- Various theories of earth pressure are considered.

At rest, active and Passive Pressures

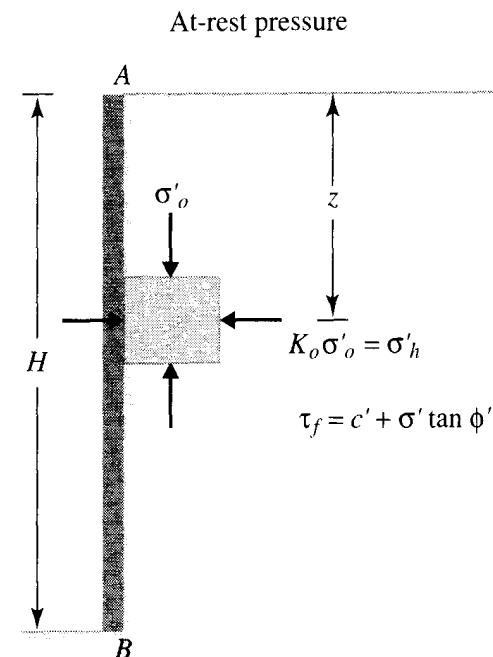
- A soil element located at a depth z is subjected:
- A vertical effective pressure $= \sigma'_o$
- A horizontal effective pressure $= \sigma'_h$
- Ratio of σ'_h to σ'_o is K :

$$K = \frac{\sigma'_h}{\sigma'_o}$$



Case-1:

- If the wall AB is static
- Soil mass in state of a static equilibrium.
- σ'_h = at rest earth pressure.
- $K = K_o = \frac{\sigma'_h}{\sigma'_o}$
- K_o = at rest earth pressure coefficient.

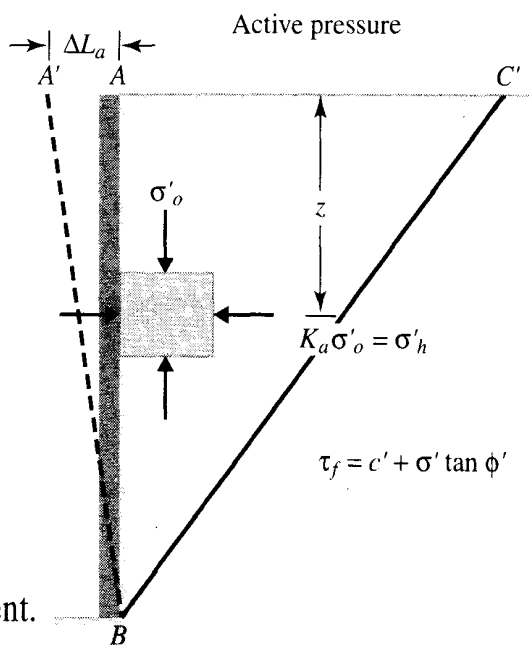


Case-2:

- If the frictionless wall rotates sufficiently about its bottom to a position A'B.
- The soil in ABC reach plastic equilibrium and fail down the plane BC'.

$$K = K_a = \frac{\sigma'_h}{\sigma'_o} = \frac{\sigma'_a}{\sigma'_o}$$

K_a = active earth pressure coefficient.



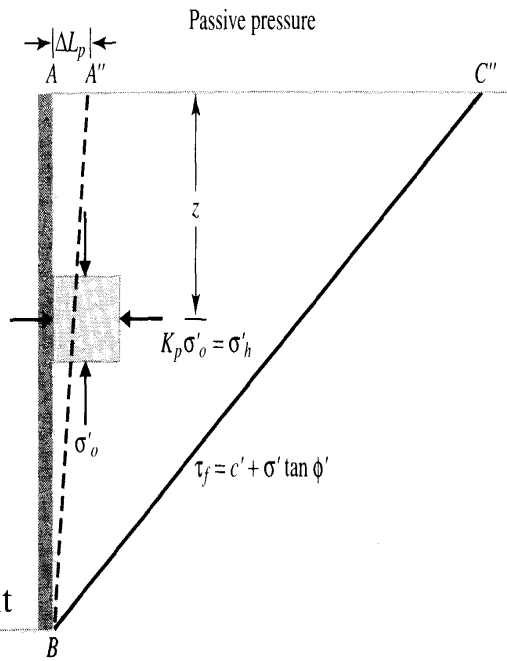
Case-3:

- If the frictionless wall rotates sufficiently about its bottom to a position A''B.
- The soil in ABC'' reach plastic equilibrium and fail down the plane BC''.

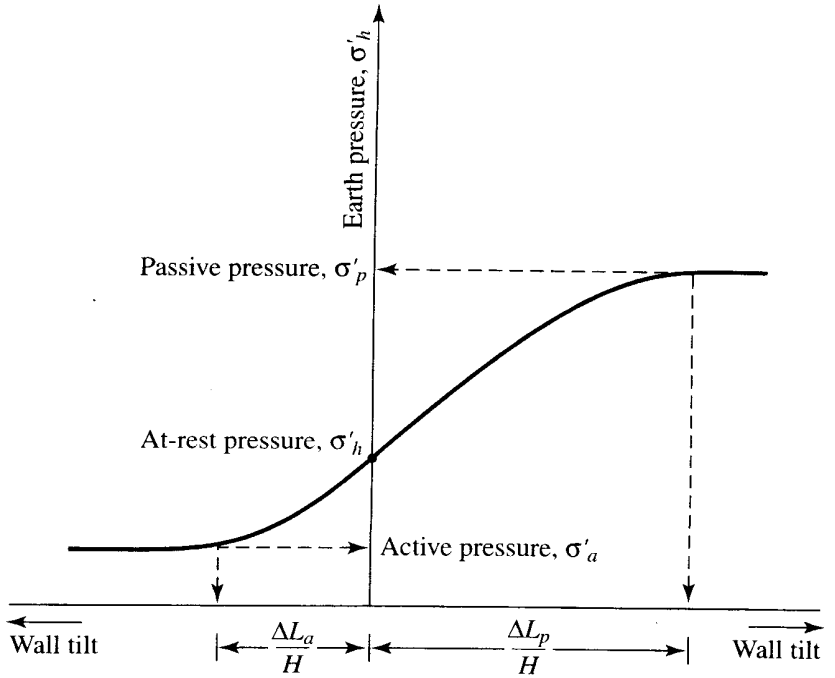
$$\sigma'_h = \sigma'_p$$

$$K = K_p = \frac{\sigma'_h}{\sigma'_o} = \frac{\sigma'_p}{\sigma'_o}$$

K_p = passive earth pressure coefficient



- Nature of variation of the lateral earth pressure with the wall tilt.



AT-REST LATERAL EARTH PRESSURE

- A wall AB retaining a dry soil with a unit weight γ .
- The wall is static.
- At a depth z ,

Vertical effective stress = $\sigma'_o = \gamma z$

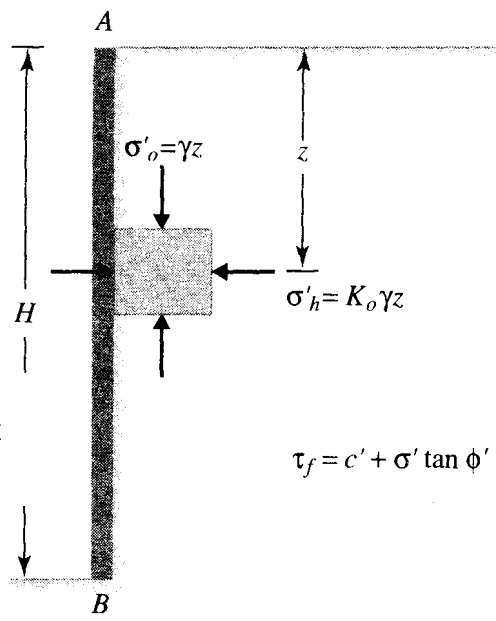
Horizontal effective stress = $\sigma'_h = K_o \gamma z$

$$K_o = \frac{\sigma'_h}{\sigma'_o} = \text{at-rest earth pressure coefficient}$$

For coarse-grained soils,

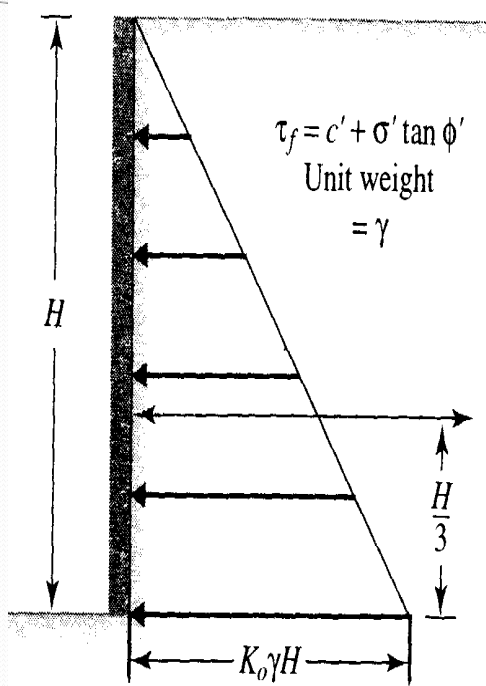
$$K_o = 1 - \sin \phi'$$

where ϕ' = drained friction angle.



- Distribution of lateral earth pressure at rest on a wall of height H retaining a dry soil having unit weight of γ
- The total force per unit length of the wall, P_o is:

$$P_o = \frac{1}{2} K_o \gamma H^2$$



For fine-grained, normally consolidated soils,

$$K_o = 0.44 + 0.42 \left[\frac{PI (\%)}{100} \right]$$

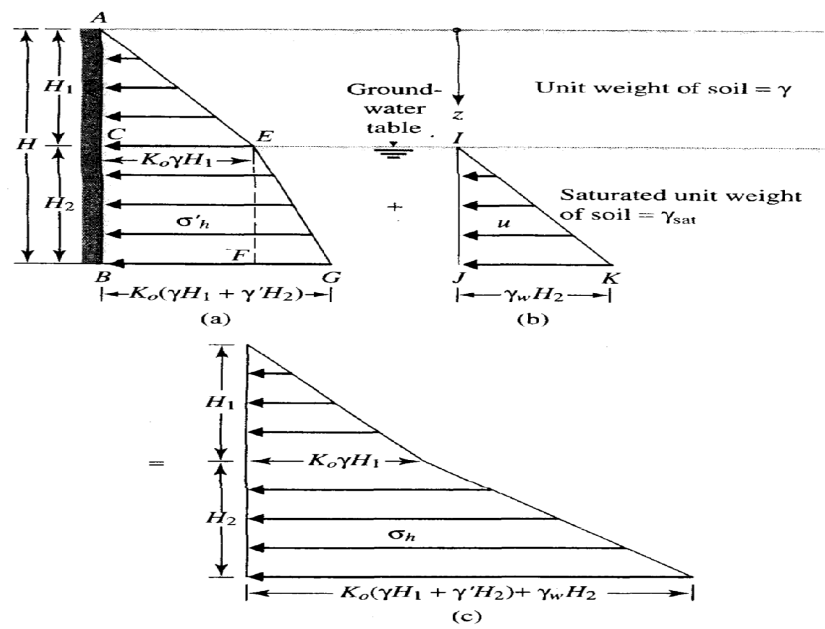
For overconsolidated clays,

$$K_o(\text{overconsolidated}) = K_o(\text{normally consolidated}) \sqrt{OCR}$$

where OCR = overconsolidation ratio.

$$OCR = \frac{\text{Preconsolidation pressure, } \sigma'_c}{\text{Present effective overburden pressure, } \sigma'_c}$$

EARTH PRESSURE AT REST FOR PARTIALLY SUBMERGED SOIL



For $z \leq H_1$, the lateral earth pressure at rest effective vertical pressure = σ'_o

$$\sigma'_o = \gamma H_1 + \gamma'(z - H_1)$$

where $\gamma' = \gamma_{sat} - \gamma_w =$

Effective lateral earth pressure at rest is:

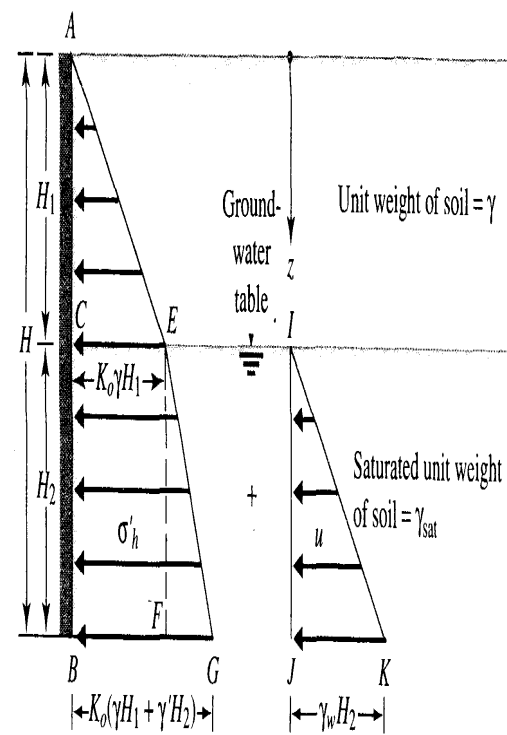
$$\sigma'_h = K_o \sigma'_o = K_o [\gamma H_1 + \gamma'(z - H_1)]$$

$$u = \gamma_w (z - H_1)$$

for $z \geq H_1$ (i.e., below the groundwater table)

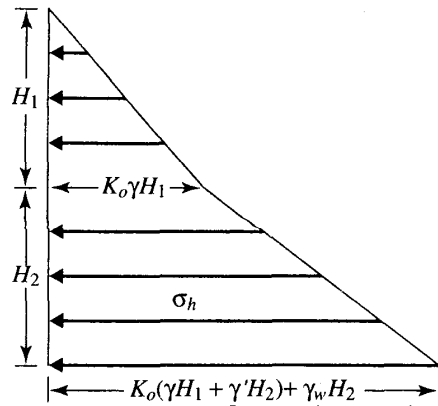
$$\sigma_h = \sigma'_h + u$$

$$= K_o [\gamma H_1 + \gamma'(z - H_1)] + \gamma_w (z - H_1)$$



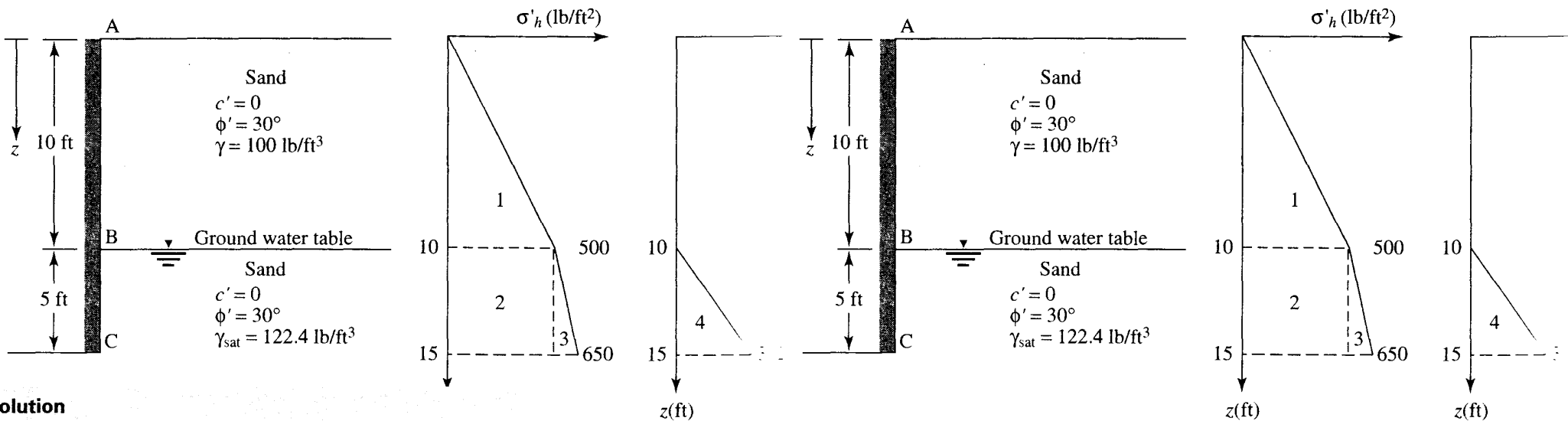
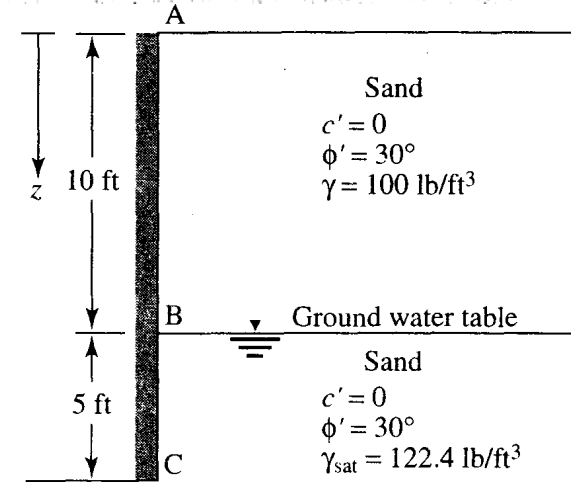
Example 12.1

Figure 12.6a shows a 15-ft-high retaining wall. The wall is restrained from yielding. Calculate the lateral force P_o per unit length of the wall. Also, determine the location of the resultant force.



The force per unit length of the wall can be found from the sum of the areas of the pressure diagrams in Figures 12.5a and 12.5b and is equal to (Figure 12.5c)

$$P_o = \underbrace{\frac{1}{2}K_o\gamma H_1^2}_{\text{Area ACE}} + \underbrace{K_o\gamma H_1 H_2}_{\text{Area CEFB}} + \underbrace{\frac{1}{2}(K_o\gamma' + \gamma_w)H_2^2}_{\text{Areas EFG and IJK}} \quad (12.15)$$



Solution

$$K_o = 1 - \sin \phi' = 1 - \sin 30 = 0.5$$

$$\text{At } z = 0: \quad \sigma'_o = 0; \quad \sigma'_h = 0; \quad u = 0$$

$$\text{At } z = 10 \text{ ft: } \sigma'_o = (10)(100) = 1000 \text{ lb/ft}^2$$

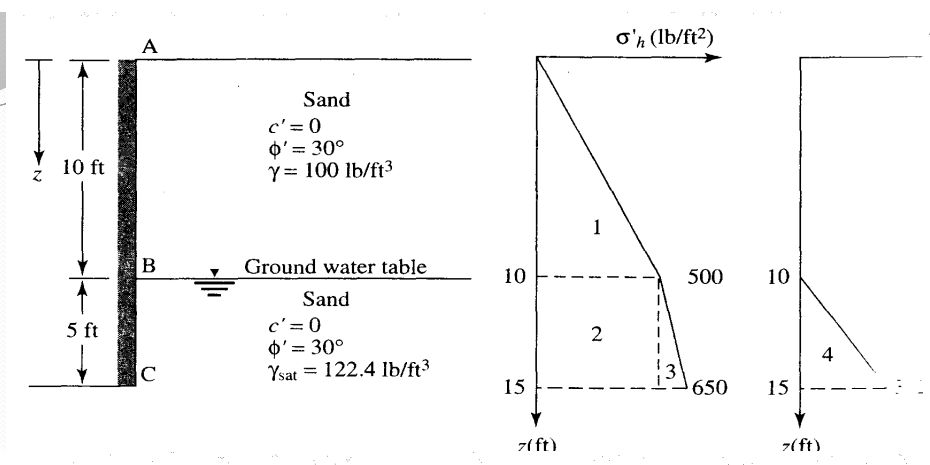
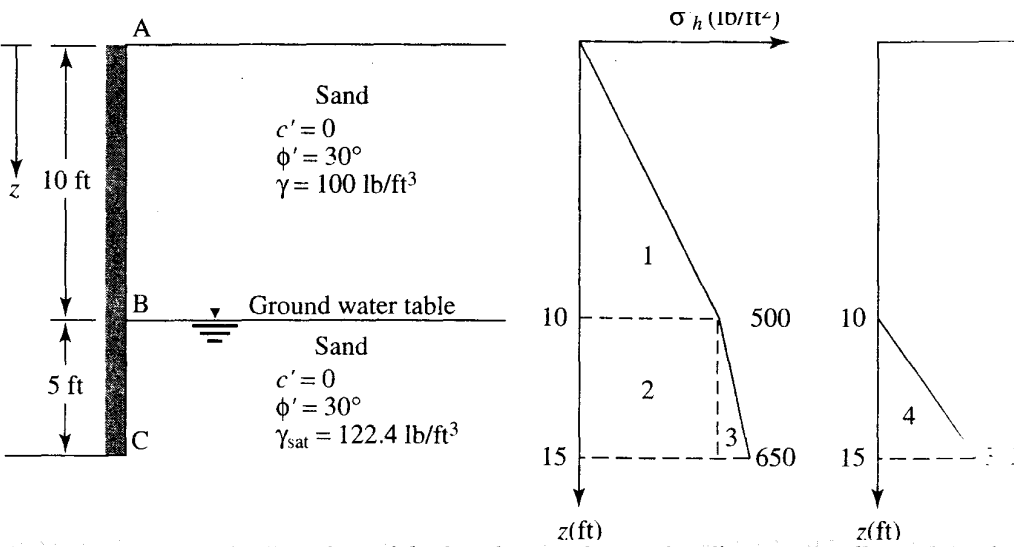
$$\sigma'_h = K_o\sigma'_o = (0.5)(1000) = 500 \text{ lb/ft}^2$$

$$u = 0$$

$$\text{At } z = 15 \text{ ft: } \sigma'_o = (10)(100) + (5)(122.4 - 62.4) = 1300 \text{ lb/ft}^2$$

$$\sigma'_h = K_o\sigma'_o = (0.5)(1300) = 650 \text{ lb/ft}^2$$

$$u = (5)(\gamma_w) = (5)(62.4) = 312 \text{ lb/ft}^2$$



The variations of σ'_h and u with depth are shown in Figures 12.6b and 12.6c.

The location of the resultant, measured from the bottom of the wall, is

$$\bar{z} = \frac{\sum \text{moment of pressure diagram about C}}{P_o}$$

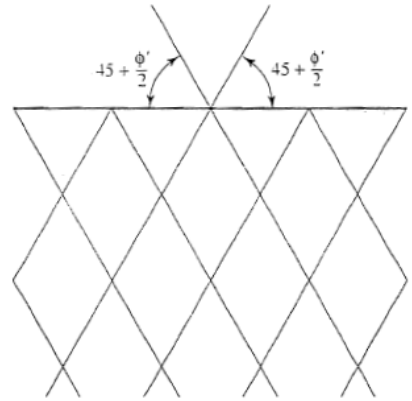
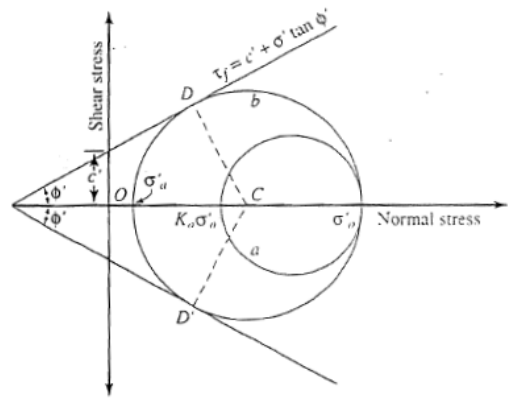
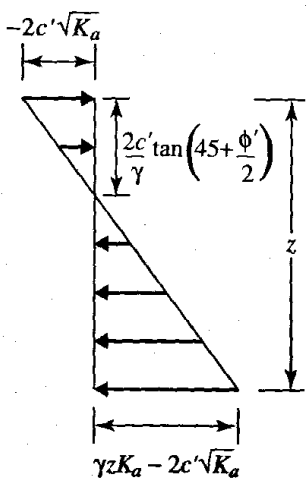
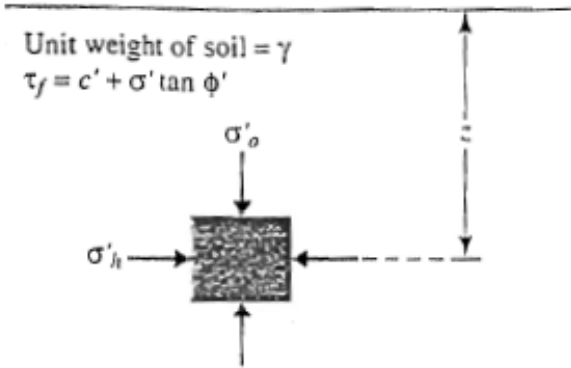
$$\bar{z} = \frac{(2500)\left(5 + \frac{10}{3}\right) + (2500)\left(\frac{5}{2}\right) + (375)\left(\frac{5}{3}\right) + (780)\left(\frac{5}{3}\right)}{6155} = 4.71 \text{ ft}$$

Lateral force $P_o = \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4}$

$$P_o = \left(\frac{1}{2}\right)(10)(500) + (5)(500) + \left(\frac{1}{2}\right)(5)(150) + \left(\frac{1}{2}\right)(5)(312) = 2500 + 2500 + 375 + 780 = 6155 \text{ lb/ft}$$

RANKINE'S LATERAL EARTH PRESSURE

Rankine's Theory of Active Pressure



$$\sin \phi' = \frac{CD}{AC} = \frac{CD}{AO + OC}$$

$$CD = \text{radius of the failure circle} = \frac{\sigma'_o - \sigma'_a}{2}$$

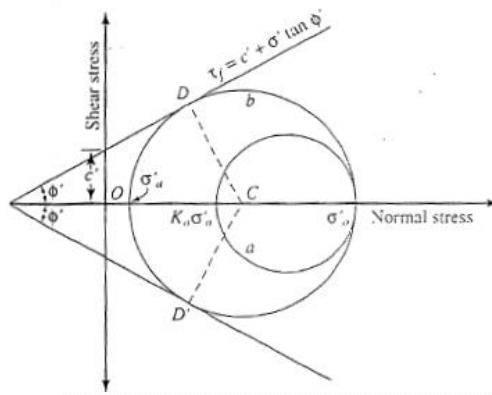
$$AO = c' \cot \phi'$$

$$OC = \frac{\sigma'_o + \sigma'_a}{2}$$

$$\sin \phi' = \frac{\frac{\sigma'_o - \sigma'_a}{2}}{c' \cot \phi' + \frac{\sigma'_o + \sigma'_a}{2}}$$

$$c' \cos \phi' + \frac{\sigma'_o + \sigma'_a}{2} \sin \phi' = \frac{\sigma'_o - \sigma'_a}{2}$$

$$\sigma'_a = \sigma'_o \frac{1 - \sin \phi'}{1 + \sin \phi'} - 2c' \frac{\cos \phi'}{1 + \sin \phi'}$$



σ'_o = vertical effective overburden pressure = γz

$$\frac{1 - \sin \phi'}{1 + \sin \phi'} = \tan^2 \left(45 - \frac{\phi'}{2} \right)$$

$$\frac{\cos \phi'}{1 + \sin \phi'} = \tan \left(45 - \frac{\phi'}{2} \right)$$

Substituting the preceding values into Eq. (12.24), we get

$$\sigma'_a = \gamma z \tan^2 \left(45 - \frac{\phi'}{2} \right) - 2c' \tan \left(45 - \frac{\phi'}{2} \right)$$

The variation of σ'_a with depth is shown in Figure 12.9c. For cohesionless soils, $c' = 0$ and

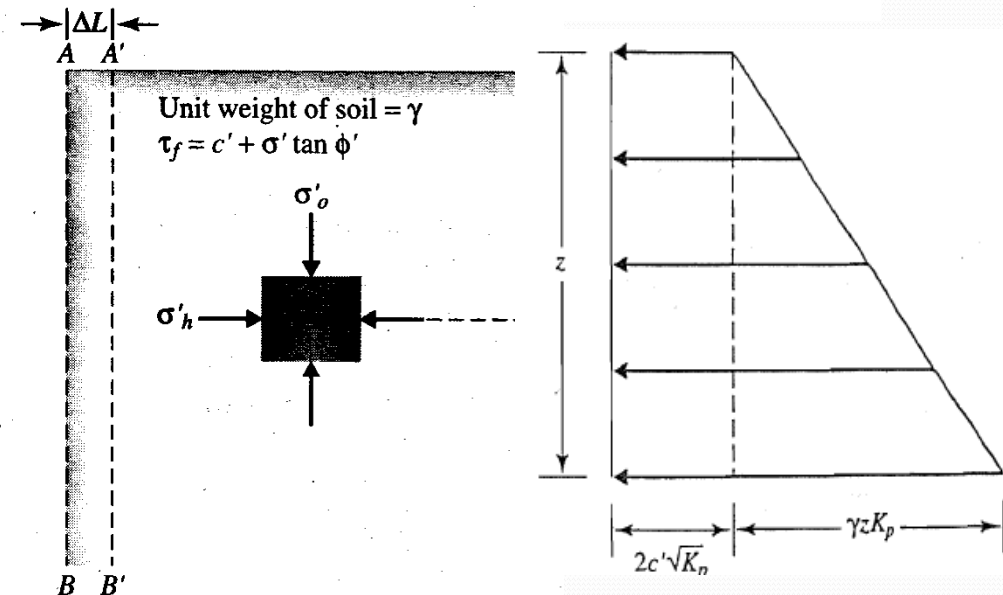
$$\sigma'_a = \sigma'_o \tan^2 \left(45 - \frac{\phi'}{2} \right)$$

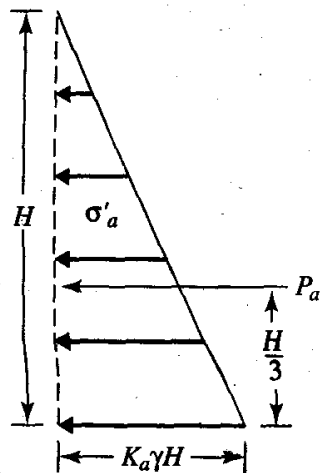
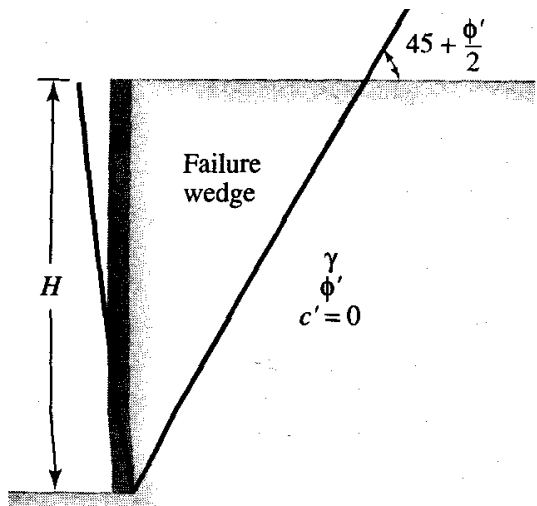
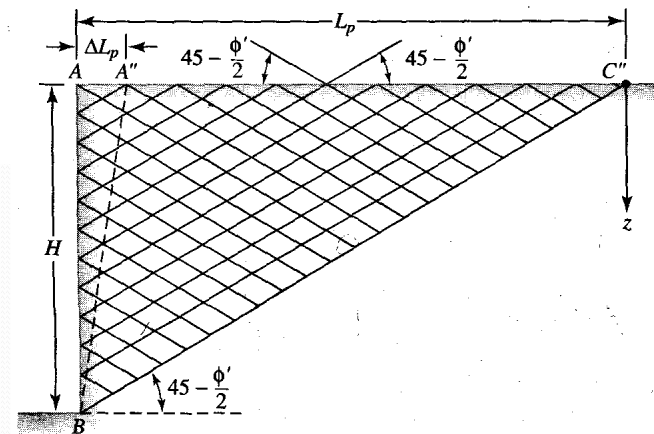
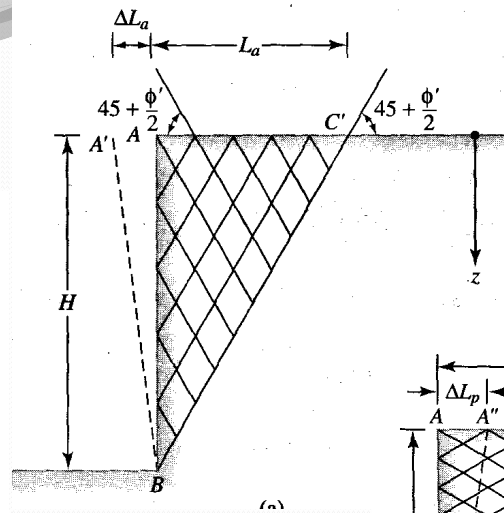
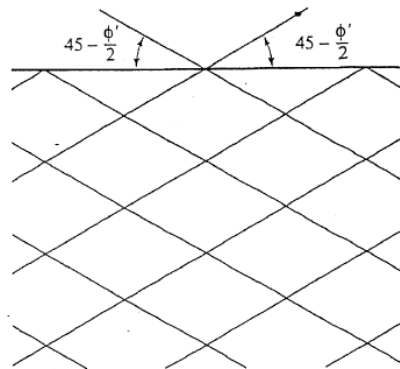
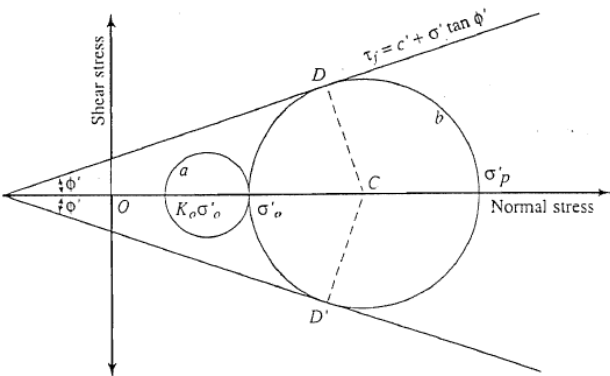
The ratio of σ'_a to σ'_o is called the *coefficient of Rankine's active earth pressure* and is given by

$$K_a = \frac{\sigma'_a}{\sigma'_o} = \tan^2 \left(45 - \frac{\phi'}{2} \right)$$

$$\sigma_a = \gamma z \tan^2 \left(45 - \frac{\phi'}{2} \right) - 2c \tan \left(45 - \frac{\phi'}{2} \right)$$

Theory of Rankine's Passive Pressure

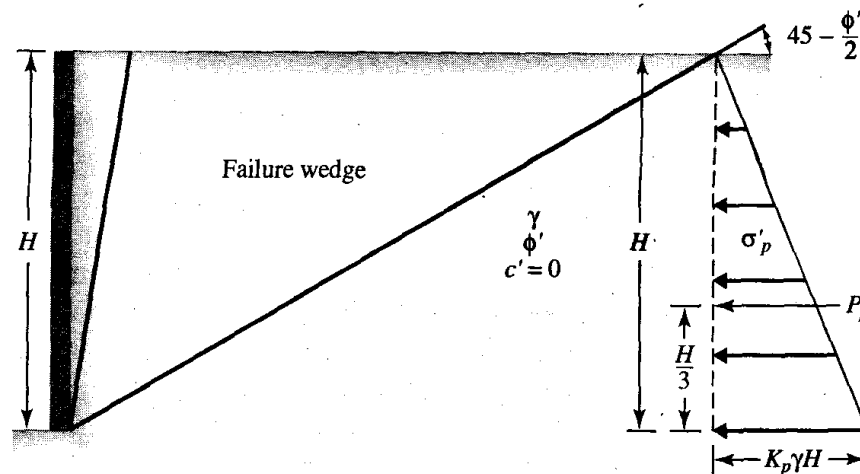




$$\sigma'_a = K_a \gamma z \quad (\text{Note: } c' = 0.)$$

$$\sigma'_a = K_a \gamma H$$

$$P_a = \frac{1}{2} K_a \gamma H^2$$

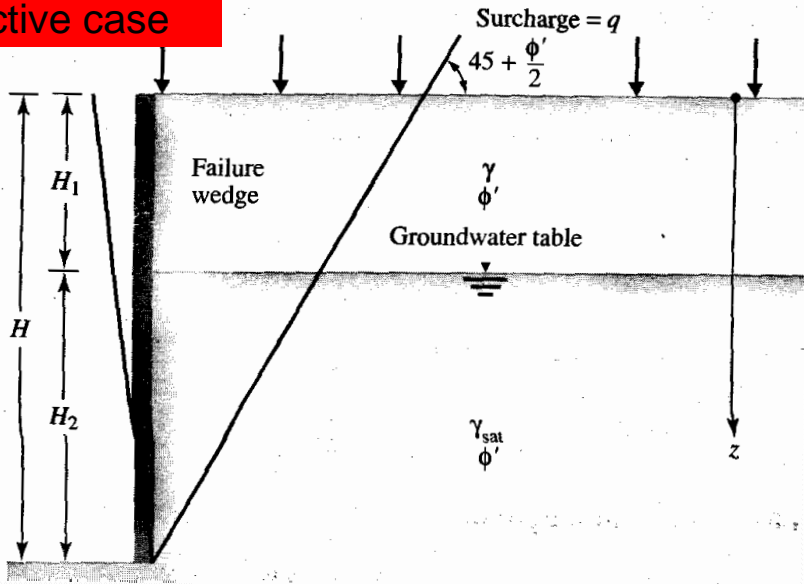


$$\sigma'_p = K_p \gamma H$$

$$P_p = \frac{1}{2} K_p \gamma H^2$$

Backfill—Partially Submerged Cohesionless Soil Supporting a Surcharge

Active case



$$\sigma'_a = K_a \sigma'_o$$

where σ'_o and σ'_a = the effective vertical pressure and lateral pressure, respectively.

$$\text{At } z = 0, \quad \sigma'_o = \sigma'_a = q$$

$$\sigma'_a = K_a q$$

$$\text{At depth } z = H_1, \quad \sigma'_o = (q + \gamma H_1)$$

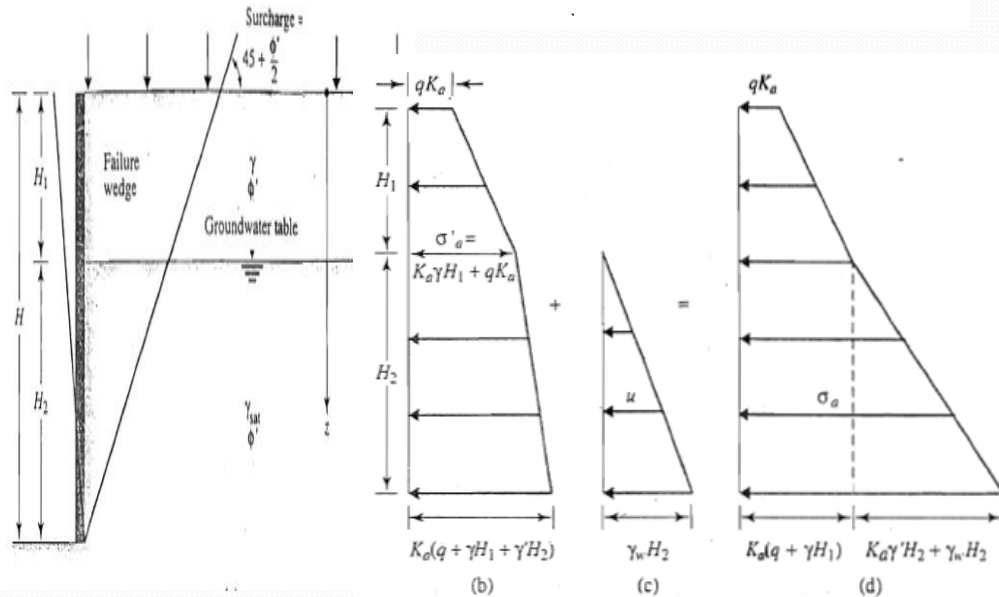
$$\sigma'_a = K_a (q + \gamma H_1)$$

$$\text{At depth } z = H, \quad \sigma'_o = (q + \gamma H_1 + \gamma' H_2)$$

$$\sigma'_a = K_a (q + \gamma H_1 + \gamma' H_2)$$

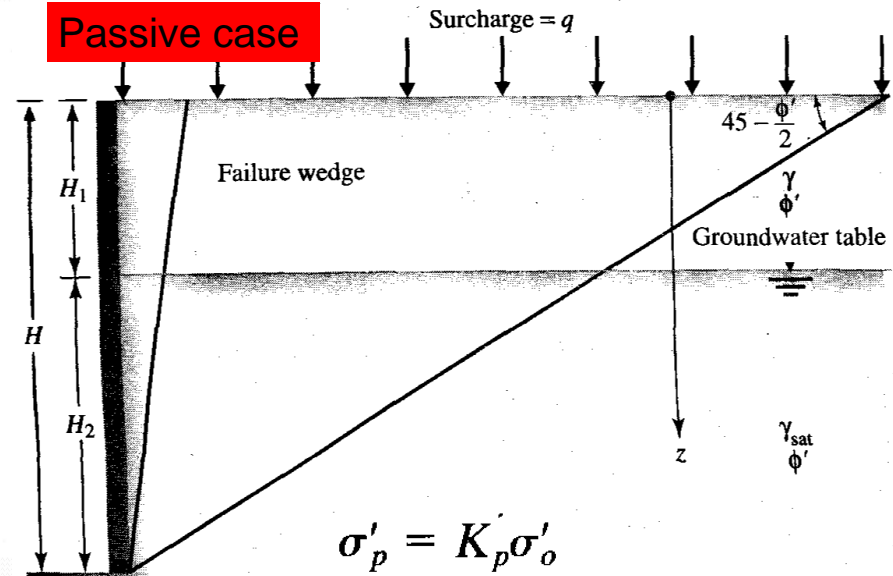
where $\gamma' = \gamma_{\text{sat}} - \gamma_w$.

The variation of σ'_a with depth is shown in Figure



Backfill—Partially Submerged Cohesionless Soil Supporting a Surcharge

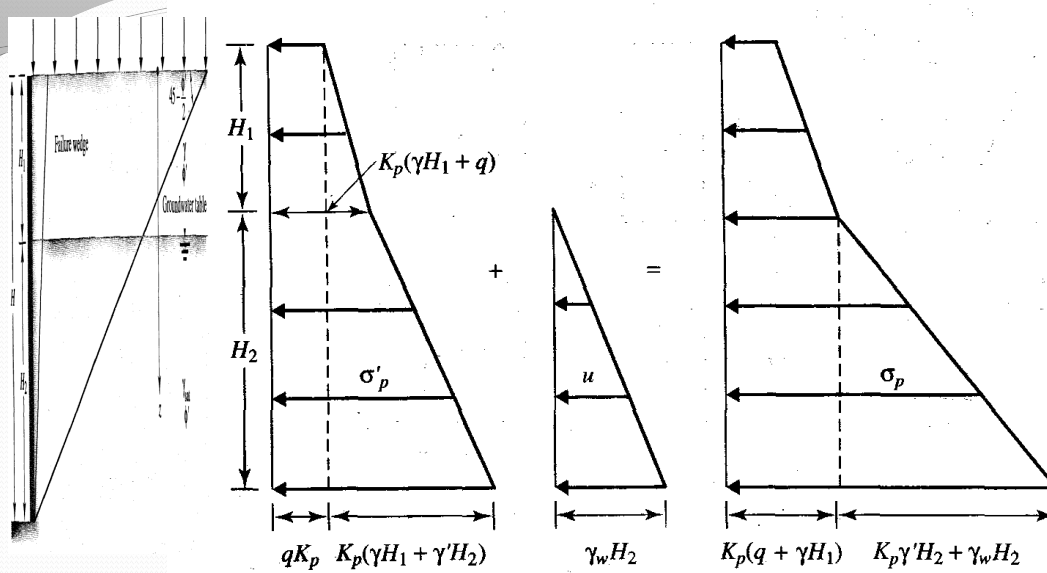
Passive case



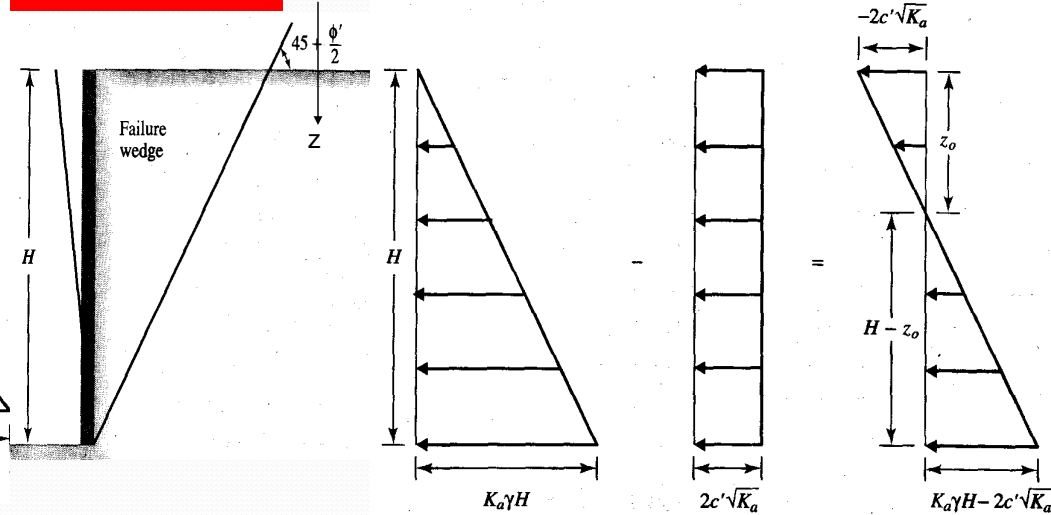
$$\sigma'_p = K_p \sigma'_o$$

$$P_a = K_a q H + \frac{1}{2} K_a \gamma H_1^2 + K_a \gamma H_1 H_2 + \frac{1}{2} (K_a \gamma' + \gamma_w) H_2^2 \quad P_p = K_p q H + \frac{1}{2} K_p \gamma H_1^2 + K_p \gamma H_1 H_2 + \frac{1}{2} (K_p \gamma' + \gamma_w) H_2^2$$

Backfill—Cohesive Soil with Horizontal Backfill



Active case



$$\sigma'_a = K_a \gamma z - 2\sqrt{K_a} c'$$

Depth ' z_o ' at which active earth pressure=0

$$K_a \gamma z_o - 2\sqrt{K_a} c' = 0$$

$$z_o = \frac{2c'}{\gamma \sqrt{K_a}} = \text{Depth of tension cracks}$$

For the undrained condition — that is, $\phi = 0$, $K_a = \tan^2 45 = 1$, and $c = c_u$ (undrained cohesion)

$$z_o = \frac{2c_u}{\gamma}$$

The total active force per unit length of the wall

$$P_a = \frac{1}{2} K_a \gamma H^2 - 2\sqrt{K_a} c' H$$

For the $\phi = 0$ condition, $P_a = \frac{1}{2} \gamma H^2 - 2c_u H$

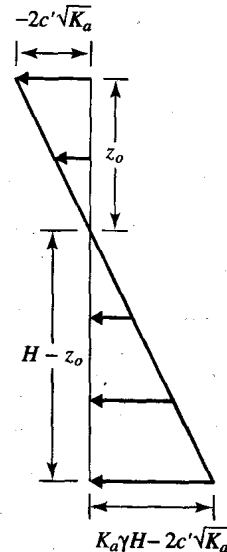
Due to development of tensile cracks, the active pressure between $z = 2c'/(\gamma \sqrt{K_a})$ and H

$$P_a = \frac{1}{2} (K_a \gamma H - 2\sqrt{K_a} c') \left(H - \frac{2c'}{\gamma \sqrt{K_a}} \right)$$

$$= \frac{1}{2} K_a \gamma H^2 - 2\sqrt{K_a} c' H + 2 \frac{c'^2}{\gamma}$$

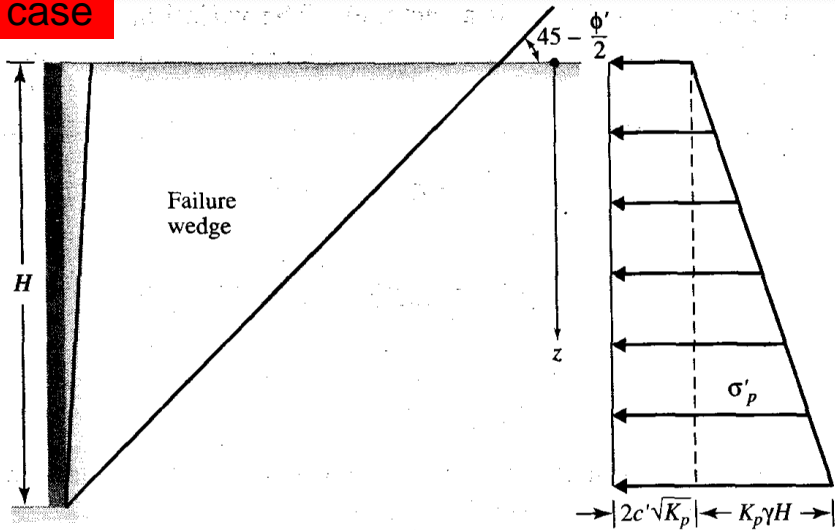
For the $\phi = 0$ condition,

$$P_a = \frac{1}{2} \gamma H^2 - 2c_u H + 2 \frac{c_u^2}{\gamma}$$



Backfill—Cohesive Soil with Horizontal Backfill

Passive case



$$\sigma'_p = K_p \gamma z + 2\sqrt{K_p} c'$$

Rankine's passive pressure

$$\sigma'_p = K_p \gamma z + 2\sqrt{K_p} c'$$

At $z = 0$,

$$\sigma'_p = 2\sqrt{K_p} c'$$

at $z = H$,

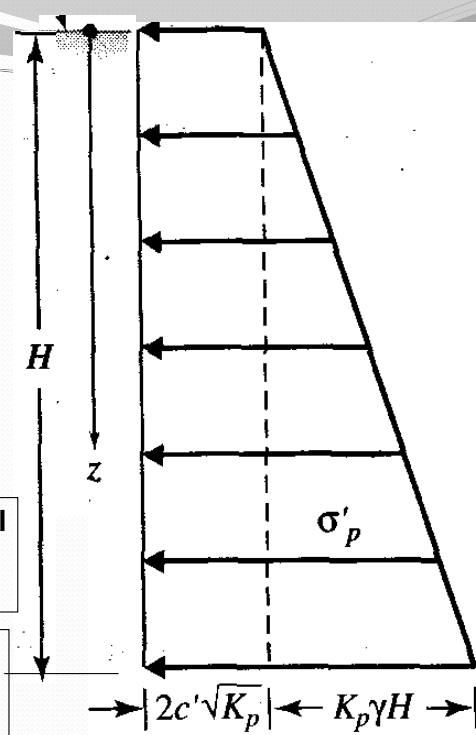
$$\sigma'_p = K_p \gamma H + 2\sqrt{K_p} c'$$

Total passive force/unit length of wall

$$P_p = \frac{1}{2} K_p \gamma H^2 + 2\sqrt{K_p} c' H$$

For the $\phi = 0$ condition, $K_p = 1$

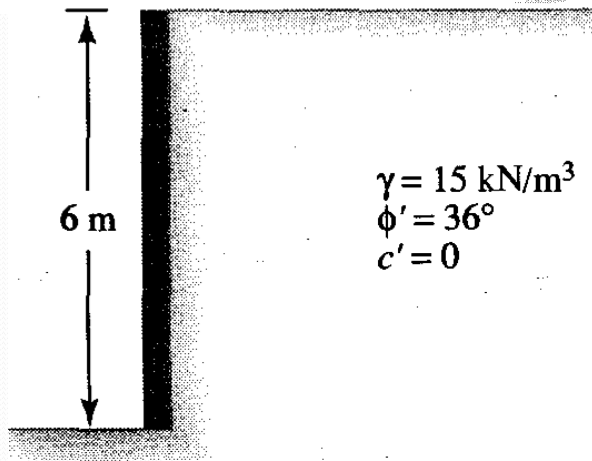
$$P_p = \frac{1}{2} \gamma H^2 + 2c_u H$$



Example 12.3

An 6 m high retaining wall is shown in Figure 12.17a. Determine

- The Rankine active force per unit length of the wall and the location of the resultant
- The Rankine passive force per unit length of the wall and the location of the resultant



Solution

a. Because $c' = 0$, to determine the active force, we can use

$$\sigma'_a = K_a \sigma'_o = K_a \gamma z$$

$$K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \frac{1 - \sin 36}{1 + \sin 36} = 0.26$$

At $z = 0$, $\sigma'_a = 0$; at $z = 6$ m,

$$\sigma'_a = (0.26)(15)(6) = 23.4 \text{ kN/m}^2$$

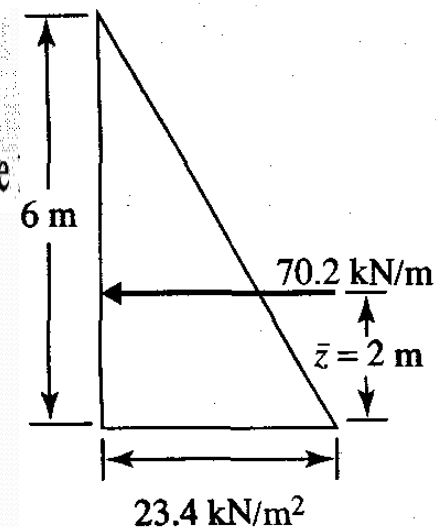
The pressure distribution diagram is shown in Figure

The active force/unit length of the wall is:

$$P_a = \frac{1}{2} K_a \gamma H^2$$

$$P_a = \frac{1}{2} (6)(23.4) = 70.2 \text{ kN/m}$$

$$\bar{z} = \frac{6 \text{ m}}{3} = 2 \text{ m}$$



b. To determine the passive force, we are given that $c' = 0$.

$$\sigma'_a = K_a \sigma'_o = K_a \gamma z$$

$$K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \frac{1 - \sin 36}{1 + \sin 36} = 0.26$$

At $z = 0$, $\sigma'_a = 0$

at $z = 6 \text{ m}$,

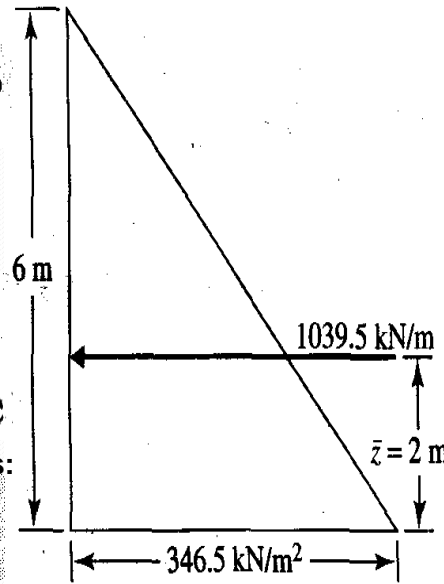
$$\sigma_p = (3.85)(15)(6) = 346.5 \text{ kN/m}^2$$

The pressure distribution diagram is shown in Figure

The passive force /unit length of the wall is:

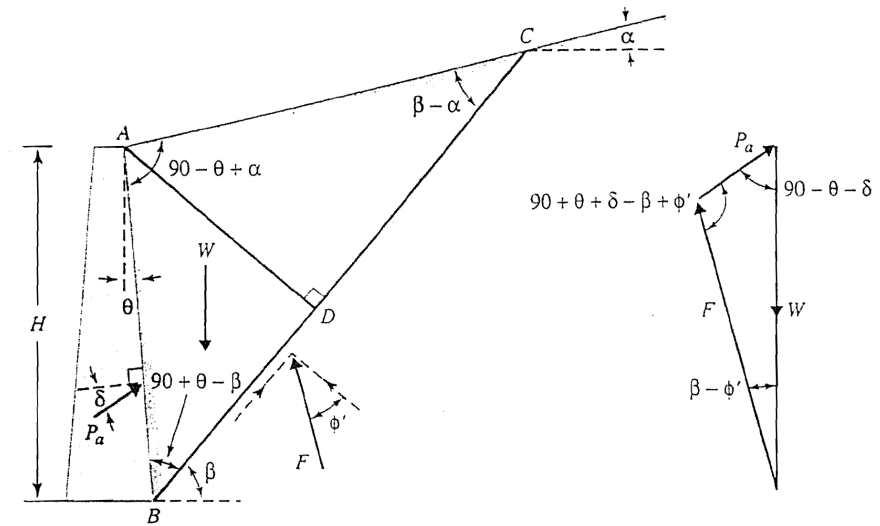
$$P_p = \frac{1}{2}(6)(346.5) = 1039.5 \text{ kN/m}$$

$$\bar{z} = \frac{6 \text{ m}}{3} = 2 \text{ m}$$



COULOMB'S EARTH PRESSURE THEORY

Coulomb's Active Pressure



$$\frac{W}{\sin(90 + \theta + \delta - \beta + \phi')} = \frac{P_a}{\sin(\beta - \phi')}$$

or

$$P_a = \frac{\sin(\beta - \phi')}{\sin(90 + \theta + \delta - \beta + \phi')} W$$

The preceding equation can be written in the form

$$P_a = \frac{1}{2} \gamma H^2 \left[\frac{\cos(\theta - \beta) \cos(\theta - \alpha) \sin(\beta - \phi')}{\cos^2 \theta \sin(\beta - \alpha) \sin(90 + \theta + \delta - \beta + \phi')} \right]$$

$$P_a = \frac{1}{2} K_a \gamma H^2$$

where K_a is Coulomb's active earth pressure coefficient and is given by

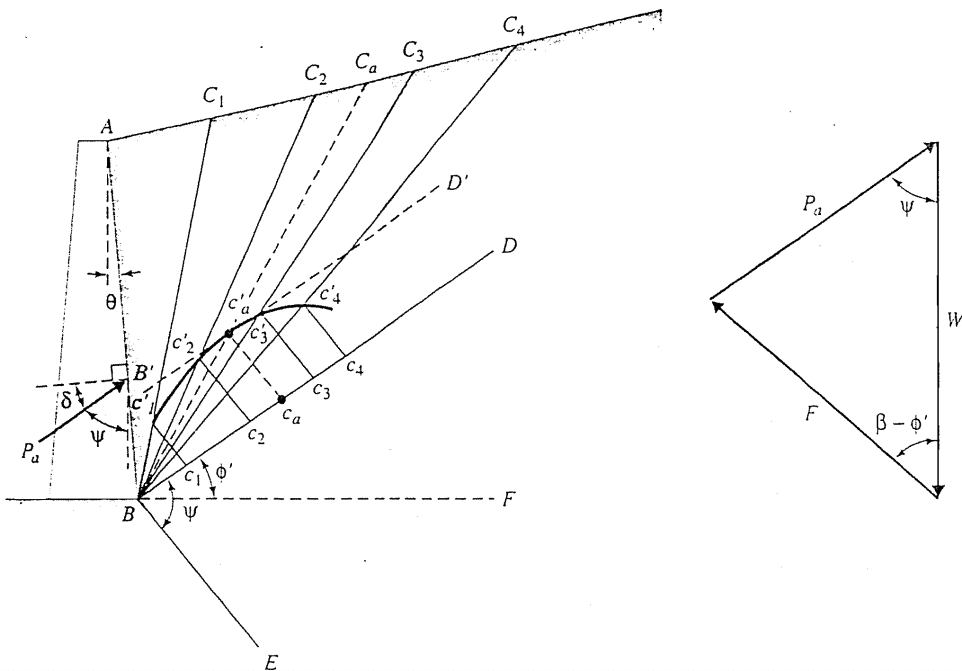
$$K_a = \frac{\cos^2(\phi' - \theta)}{\cos^2 \theta \cos(\delta + \theta) \left[1 + \sqrt{\frac{\sin(\delta + \phi') \sin(\phi' - \alpha)}{\cos(\delta + \theta) \cos(\theta - \alpha)}} \right]^2}$$

Graphic Solution for Coulomb's Active Earth Pressure

1. Draw the features of the retaining wall and the backfill to a convenient scale.
2. Determine the value of ψ (degrees) = $90 - \theta - \delta$, where θ = the inclination of the back face of the retaining wall with the vertical, and δ = angle of wall friction.
3. Draw a line BD that makes an angle ϕ' with the horizontal.
4. Draw a line BE that makes an angle ψ with line BD .
5. To consider some trial failure wedges, draw lines $BC_1, BC_2, BC_3, \dots, BC_n$.
6. Find the areas of $ABC_1, ABC_2, ABC_3, \dots, ABC_n$.
7. Determine the weight of soil, W , per unit length of the retaining wall in each of the trial failure wedges as follows:
 $W_1 = (\text{Area of } ABC_1) \times (\gamma) \times (1)$
 $W_2 = (\text{Area of } ABC_2) \times (\gamma) \times (1)$
 $W_3 = (\text{Area of } ABC_3) \times (\gamma) \times (1)$
 \vdots
 $W_n = (\text{Area of } ABC_n) \times (\gamma) \times (1)$

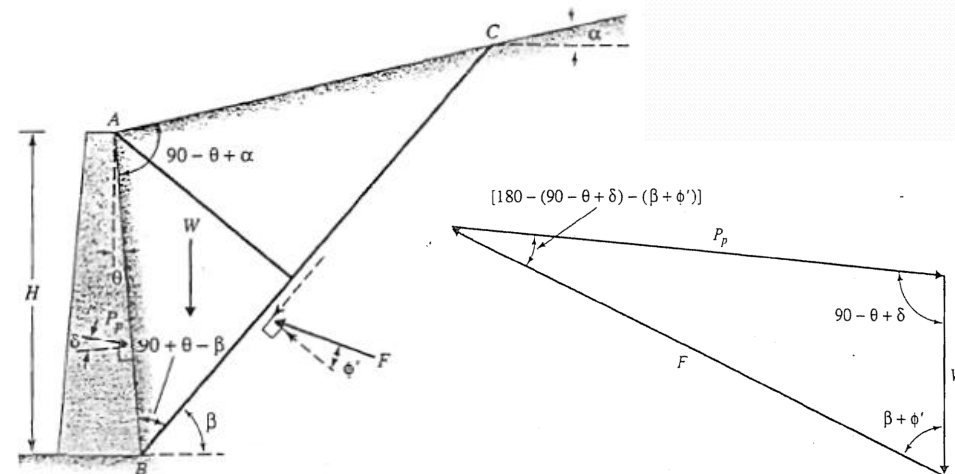
8. Adopt a convenient load scale and plot the weights $W_1, W_2, W_3, \dots, W_n$ determined from step 7 on line BD . (Note: $Bc_1 = W_1, Bc_2 = W_2, Bc_3 = W_3, \dots, Bc_n = W_n$.)
9. Draw $c_1c'_1, c_2c'_2, c_3c'_3, \dots, c_nc'_n$ parallel to the line BE . (Note: $c'_1, c'_2, c'_3, \dots, c'_n$ are located on lines $BC_1, BC_2, BC_3, \dots, BC_n$, respectively.)
10. Draw a smooth curve through points $c'_1, c'_2, c'_3, \dots, c'_n$. This curve is called the *Culmann line*.
11. Draw a tangent $B'D'$ to the smooth curve drawn in step 10. $B'D'$ is parallel to line BD . Let c'_a be the point of tangency.
12. Draw a line $c_a c'_a$ parallel to the line BE .
13. Determine the active force per unit length of wall as

$$P_a = (\text{Length of } c_a c'_a) \times (\text{Load scale})$$
14. Draw a line $Bc'_a c_a$. ABC_a is the desired failure wedge.



Coulomb's Passive Pressure

$$P_p = \frac{1}{2} K_p \gamma H^2$$



$$K_p = \frac{\cos^2(\phi' + \theta)}{\cos^2 \theta \cos(\delta - \theta) \left[1 - \sqrt{\frac{\sin(\phi' - \delta) \sin(\phi' + \alpha)}{\cos(\delta - \theta) \cos(\alpha - \theta)}} \right]^2}$$

$$K_p = \frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2 \left(45 + \frac{\phi'}{2} \right)$$