MECHANICS OF MATERIALS

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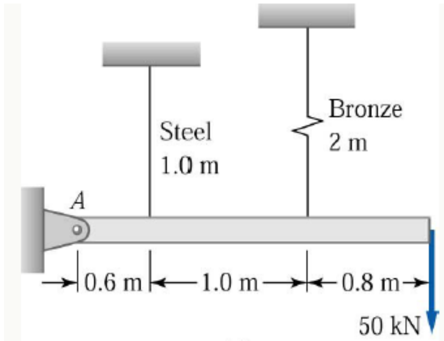
Level YEAR 2

Statically Indeterminate Members:

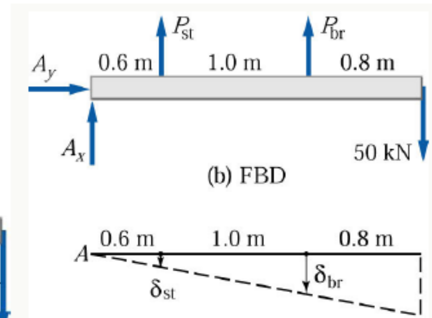
When the reactive forces or the internal resisting forces over a cross section exceed the number of independent equations of equilibrium, the structure is called statically indeterminate. These cases require the use of additional relations that depend on the elastic deformations in the members.

- If the equilibrium equations are sufficient to calculate all the forces (including support reactions) that act on a body, these forces are said to be ***statically determinate***.
- In ***statically determinate problems***, the number of unknown forces is always equal to the number of independent equilibrium equations.
- If the number of unknown forces exceeds the number of independent equilibrium equations, the problem is said to be ***statically indeterminate***.
- A statically indeterminate problem always has geometric restrictions imposed on its deformation. The mathematical expressions of these restrictions known as the ***compatibility equations***, provide us with the additional equations needed to solve the problem.

- Because the source of the compatibility equations is deformation, these equations contain as unknowns either strains or elongations. Use Hooke´s law to express the **deformation** measures in terms of **stresses** or **forces**. The **equations of equilibrium** and **compatibility** can then be solved for the unknown forces (force-displacement equation).
- ***Procedure for Solving Statically Indeterminate Problem***
  - Draw the required free-body diagrams and derive **the equations of equilibrium**.
  - Derive the **compatibility equations**. To visualize the restrictions on deformation, it is often helpful to draw a **sketch** that exaggerates the magnitudes of the deformations.
  - Use **Hooke´s law** to express the deformations (strains) in the compatibility equations in terms of forces (or stresses)
  - Solve the equilibrium and compatibility equations for the unknown forces (force-displacement equation).



(a)



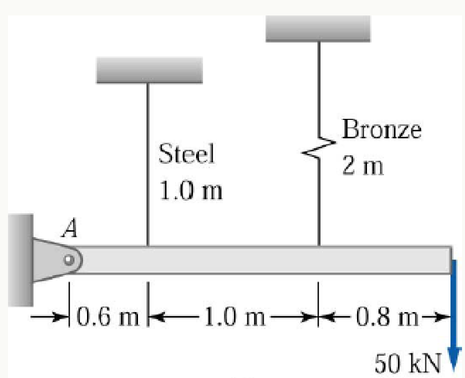
(c)

$$\frac{\delta_{st}}{0.6} = \frac{\delta_{br}}{1.6}$$

Sample problem 1

Figure (a) shows a rigid bar that is supported by a pin at *A* and two rods, one made of steel and the other of bronze. Neglecting the weight of the bar, compute **the stress** in each rod caused by the 50-kN load, the following data :

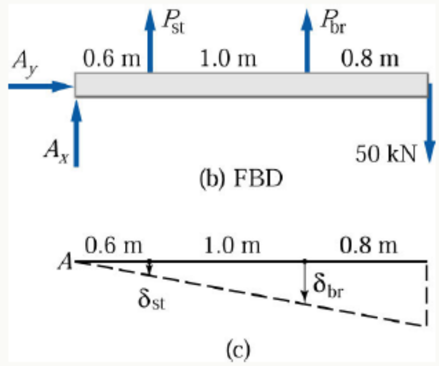
	Steel	Bronze
Area (mm^2)	600	300
E (GPa)	200	83



(a)

Solution

Equilibrium in Fig. (b), contains four unknown forces. Since there are only three independent equilibrium equations, these forces are statically indeterminate. The equilibrium equation that does not involve the pin reactions at A is



$$\Sigma M_A = 0 + \quad 0.6P_{st} + 1.6P_{br} - 2.4(50\times10^3) = 0 \qquad \text{(a)}$$

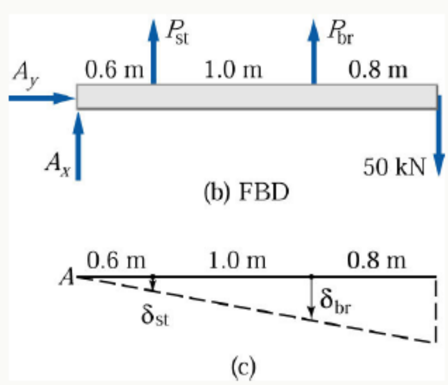
Compatibility The displacement of the bar, consisting of a rigid-body rotation about A, is shown greatly exaggerated in Fig. (c).

From similar triangles, we see that the elongations of the supporting rods must satisfy the compatibility condition

$$\frac{\delta_{st}}{0.6} = \frac{\delta_{br}}{1.6} \qquad \text{(b)}$$

Hooke's law When we substitute $\delta = PL/(EA)$ into Eq. (b), the compatibility equation becomes

$$\frac{1}{0.6}\left(\frac{PL}{EA}\right)_{st} = \frac{1}{1.6}\left(\frac{PL}{EA}\right)_{br}$$



Using the given data, we obtain

$$\frac{1}{0.6}\frac{P_{st}(1.0)}{(200)(600)} = \frac{1}{1.6}\frac{P_{br}(2)}{(83)(300)}$$

$$P_{st} = 3.614P_{br} \qquad \text{(c)}$$

Note that the we did not convert the areas from mm^2 to m^2, and we omitted the factor 10^9 from the moduli of elasticity. Since these conversion factors appear on both sides of the equation, they would cancel out.

Solving Eqs. (a) and (c), we obtain

$$P_{st} = 115.08 \times 10^3 \text{ N} \qquad P_{br} = 31.84 \times 10^3 \text{ N}$$

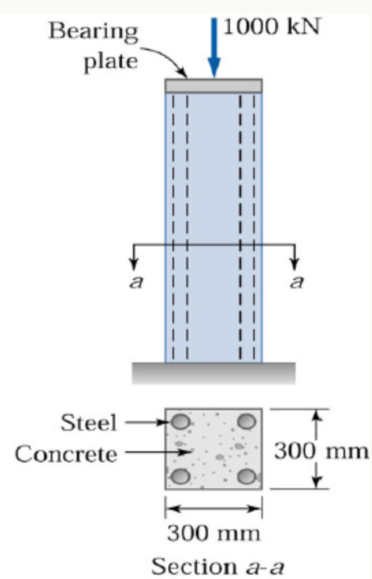
The stresses are

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{115.08\times10^3}{600\times10^{-6}} = 191.8\times10^6\,Pa = 191.8MPa \quad \textit{Answer}$$

$$\sigma_{st} = \frac{P_{br}}{A_{br}} = \frac{31.84\times10^3}{300\times10^{-6}} = 106.1\times10^6\,Pa = 106.1MPa \quad \textit{Answer}$$

Sample Problem **2**

The concrete post in Fig. (a) is reinforced axially with four symmetrically placed steel bars, each of cross-sectional area 900 mm^2. Compute **the stress in each material** when the 1000-kN axial load is applied. The moduli of elasticity are 200 GPa for steel and 14 GPa for .concrete

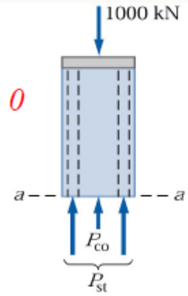


(a)

If each steel bars *was consider*

Solution

$\Sigma F=0 \pm\uparrow \quad 4P_{st}+P_{co}-1.0x10^6 = 0$



(b) FBD

Equilibrium

$\Sigma F=0 \pm\uparrow \quad P_{st}+P_{co}-1.0x10^6 =0$

$\sigma_{st}A_{st}+\sigma_{co}A_{co}= 1.0x10^6 N$ (a)

the problem is statically indeterminate.

Compatibility The changes in lengths of the steel rods and concrete must be equal ; that is, $\delta_{st}=\delta_{co}$, the compatibility equation, written in term of strains, is $\varepsilon_{st}=\varepsilon_{co}$ (b)

Hooke's law (force-displacement equation) From Hooke´s law, Eq. (b) becomes

$$\delta=\frac{\sigma L}{E}=\frac{PL}{EA} \qquad \frac{\sigma_{st}}{E_{co}}=\frac{\sigma_{co}}{E_{co}} \qquad (c)$$

Equations (a) and (c) can now be solved for the stresses. From Eq. (c) we obtain

$$\sigma_{st}=\frac{E_{st}}{E_{co}}\sigma_{co}=\frac{200}{14}\sigma_{co}=14.286\sigma_{co} \qquad (d)$$

Substituting the cross-sectional areas

$A_{st} = 4\ (900x10^{-6}) = 3.6x10^{-3}\ m^2$

$A_{co} = 0.3^2 - 3.6x10^{-3} = 86.4x10^{-3}\ m^2$

and Eq. (d) into Eq. (a) yields

If each steel bars *was consider*

$\Sigma F=0 \pm\uparrow$

$4P_{st}+P_{co}-1.0x10^6=0$

Than $\sigma_{st}=P_{st}/A_{st}$

$(14.286\,\sigma_{co})(3.6x10^{-3})+\sigma_{co}(8.64x10^{-3}) = 1.0x10^6$

Solving for the stress in concrete, we get

$\sigma_{co} = 7.255x10^6\ Pa = 7.255 MPa$ *Answer*

From Eq. (d), the stress in steel is

$\sigma_{st} = 14.286\ (7.255) = 103.6\ MPa$ *Answer*

Sample Problem 3

Let the allowable stresses in the post described in Problem 1 be σ_{st} =120 MPa and σ_{co} = 6 MPa. Compute the maximum safe . axial load P and may be applied

Solution

substituting the allowable stresses into the equilibrium equation— see Eq. (a) in Sample Problem 2.6. This approach is **incorrect** because it ignores the compatibility condition, the equal strains of , the two materials, $\delta_{st} = \delta_{co}$, From Eq. (d) in Sample problem 2.6

$\sigma_{st} = 14.286\,\sigma_{co}$ (the concrete were broked rather than steel)

Therefore, if the concrete were stressed to its limit of 6 Mpa. The corresponding stress in the steel would be

$$\sigma_{st} = 14.286(6) = 85.72 \text{ MPa}$$

which is below the allowable stress of 120 MPa.

The maximum safe axial load is thus found by substituting σ_{co} = 6 MPa and σ_{st} = 85.72 MPa (rather than σ_{st} = 120 MPa) into the equilibrium equation :

$$P = \sigma_{st} A_{st} + \sigma_{co} A_{co}$$

$$= (85.72 \times 10^6)(3.6 \times 10^{-3}) + (6 \times 10^6)(86.4 \times 10^{-3})$$

$$= 827 \times 10^3 \text{ N} = 827 \text{ kN}$$

Answer

ADDITIONAL PROBLEMS

Problem 1

A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use Eco = 14 GPa and Est = 200 GPa.

Solution 234

$$\delta_{co} = \delta_{st} = \delta$$

$$\left(\frac{PL}{AE}\right)_{co} = \left(\frac{PL}{AE}\right)_{st}$$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{co} L}{14000} = \frac{\sigma_{st} L}{200\,000}$$

$$100\sigma_{co} = 7\sigma_{st}$$

When $\sigma_{st} = 120$ MPa

$$100\sigma_{co} = 7(120)$$

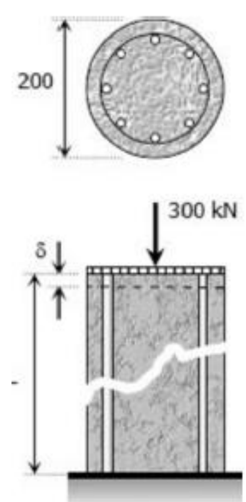
$$\sigma_{co} = 8.4 \text{ MPa} > 6 \text{ MPa} \quad \text{(not ok!)}$$

When $\sigma_{co} = 6$ MPa

$$100(6) = 7\sigma_{st}$$

$$\sigma_{st} = 85.71 \text{ MPa} < 120 \text{ MPa} \quad \text{(ok!)}$$

Use $\sigma_{co} = 6$ MPa and $\sigma_{st} = 85.71$ MPa



$$\Sigma F_V = 0$$
$$P_{st} + P_{co} = 300$$
$$\sigma_{st} A_{st} + \sigma_{co} A_{co} = 300$$
$$85.71Ast + 6\,[\,\tfrac{1}{4}\pi(200)^2 - A_{st}\,] = 300(1000)$$
$$79.71A_{st} + 60\,000\pi = 300\,000$$
$$A_{st} = 1398.9\ \text{mm}^2 \;\rightarrow\; \textit{answer}$$

MECHANICS OF MATERIALS

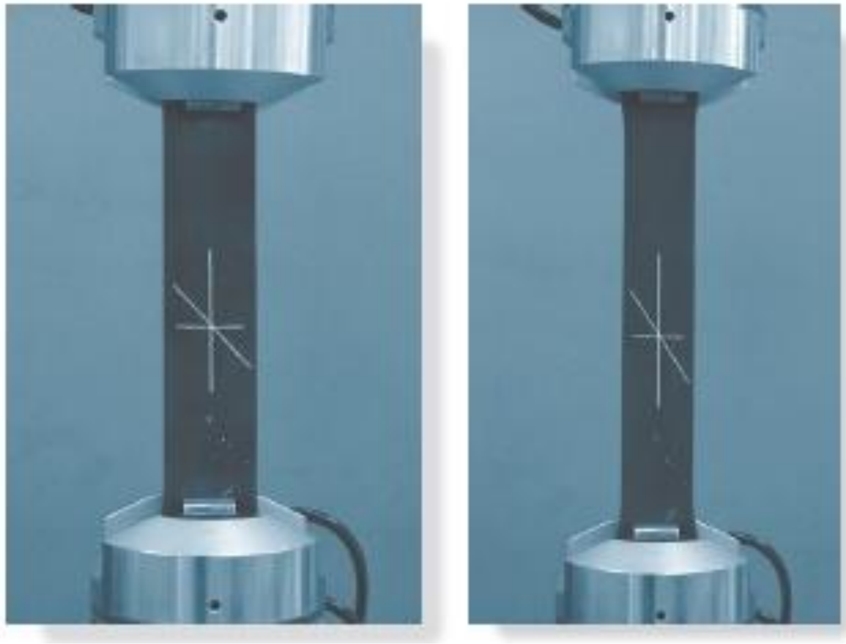
THANKS

Normal Strain and Stress

Normal Strain and Stress, Stress strain diagram, Hooke's Law

Strain

- When a body is subjected to load, it will deform and can be detected through the changes in length and the changes of angles between them.
- The deformation is measured through experiment and it is called as strain.
- The important of strain: it will be related to stress in the later chapter



Normal Strain

Normal strain is detected by the changes in length.

$$\varepsilon = \frac{l' - l}{l}$$

$$= \frac{\Delta l}{l}$$

ε (epsilon)

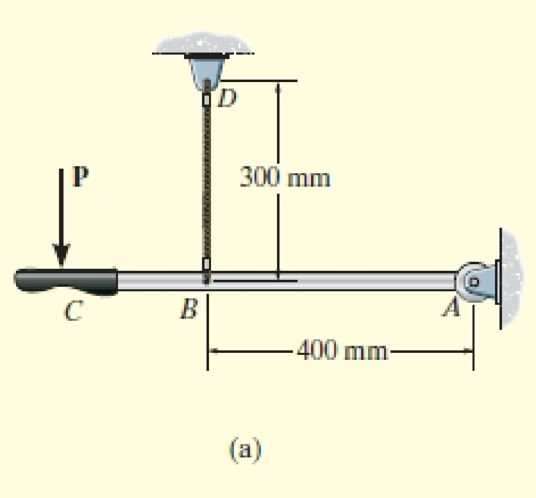
l’: length after deformed

l: original length.

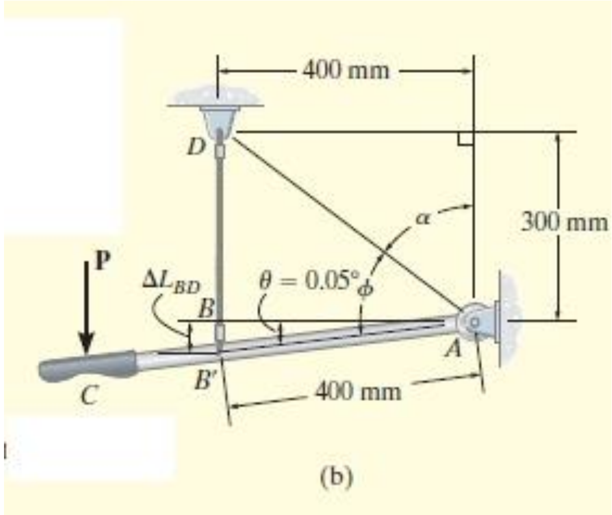
Note ε:

- dimensionless
- very small (normally is μm (=10^{-6} m))
- $480(10)^{-6}$ m/m = 480 μm/m = 480 “micros” = 0.0480 %

Example 1



(a)



(b)

Fig. 2–5

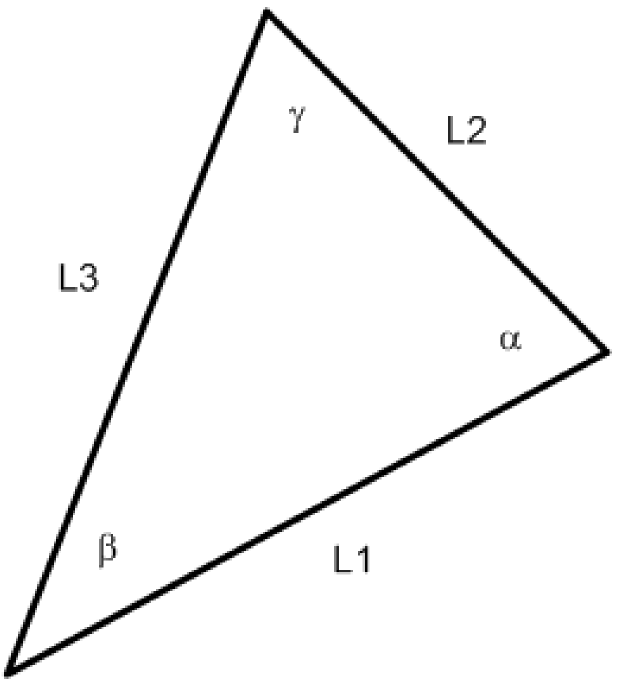
When load P is applied, the RIGID lever arm rotates by 0.05°. Calculate the normal strain of wire BD

Foundation: ΔL/L
Knowledge required: geometrical equation
Rigid: no deformation on the lever

Geometry: The mathematics

Sine and Cosine Rule

$$L1 = \sqrt{L2^2 + L3^2 - 2(L2)(L3)\cos(\gamma)}$$



Example 1

When force P is applied to the rigid lever arm

L_{BD} after deformed is DB'
Cosine rule can be applied here

$$L_{BD'} = \sqrt{L_{AB'}{}^2 + L_{AD}{}^2 - 2(L_{AB'})(L_{AD})\cos(\phi + 0.05)}$$

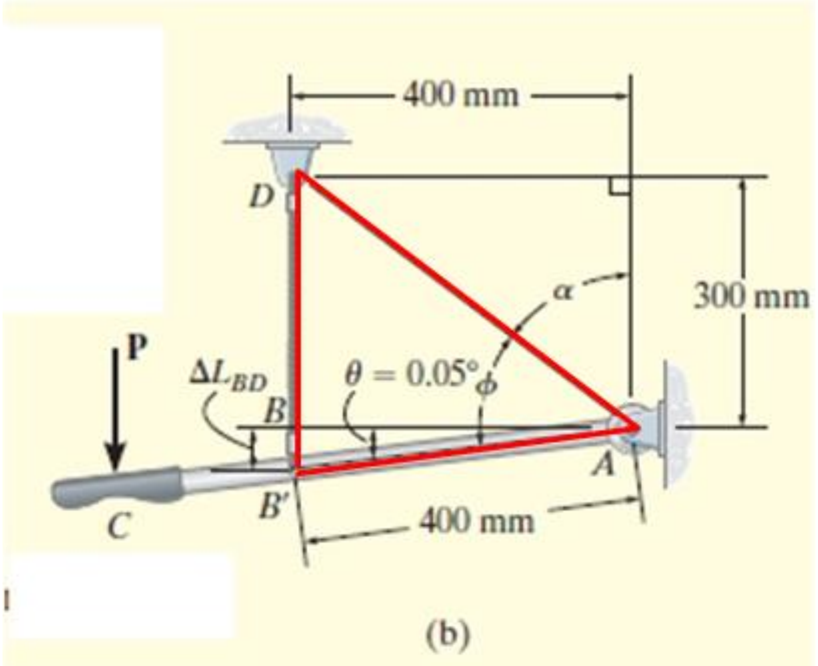


Fig. 2–5

Strain:

$$\varepsilon_{BD} = \frac{L_{DB'} - L_{DB}}{L_{DB}}$$

Example 1

When force P is applied to the rigid lever arm

L_{BD} after deformed is DB'
Cosine rule can be applied here

$$L_{BD'} = \sqrt{L_{AB'}^{\ 2} + L_{AD}^{\ 2} - 2(L_{AB'})(L_{AD})\cos(\phi + 0.05)}$$

$$L_{BD'} = 300.3491\, mm$$

Strain:

$$\varepsilon_{BD} = \frac{L_{DB'} - L_{DB}}{L_{DB}}$$

$$\varepsilon_{BD} = 0.00116\, mm/mm$$

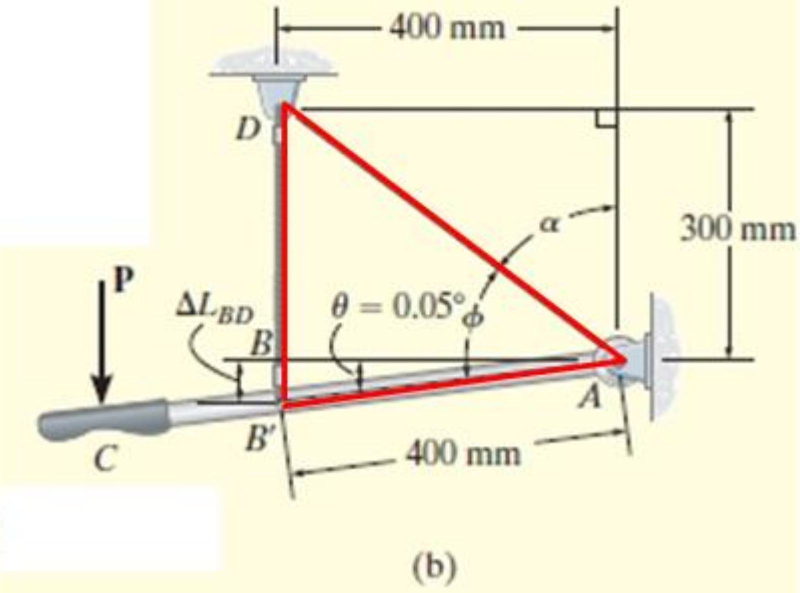
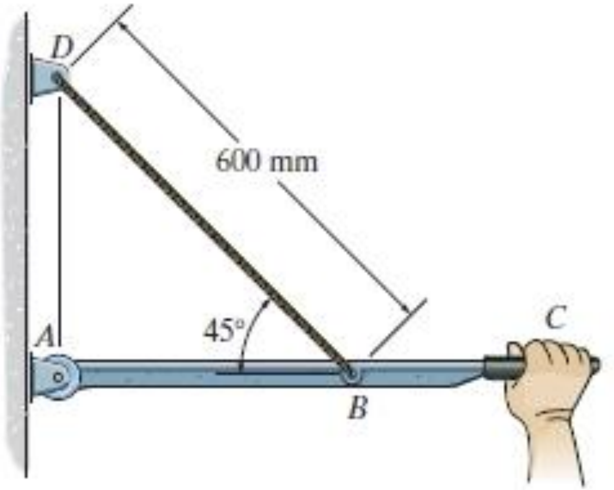


Fig. 2–5

Example 2

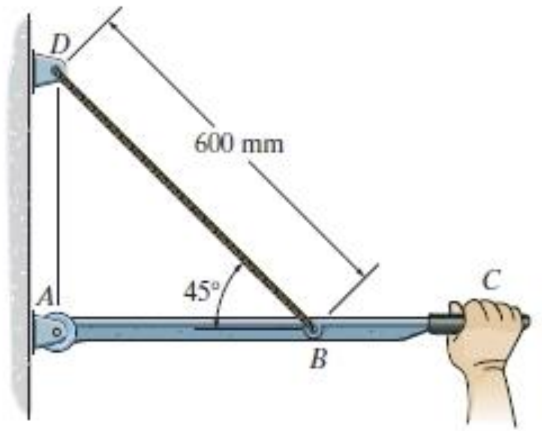
The force applied to the handle of the rigid lever the arm to rotate clockwise through an angle of 3° about pin A. Determine the average normal strain developed in the wire. Originally, the wire is unstretched.



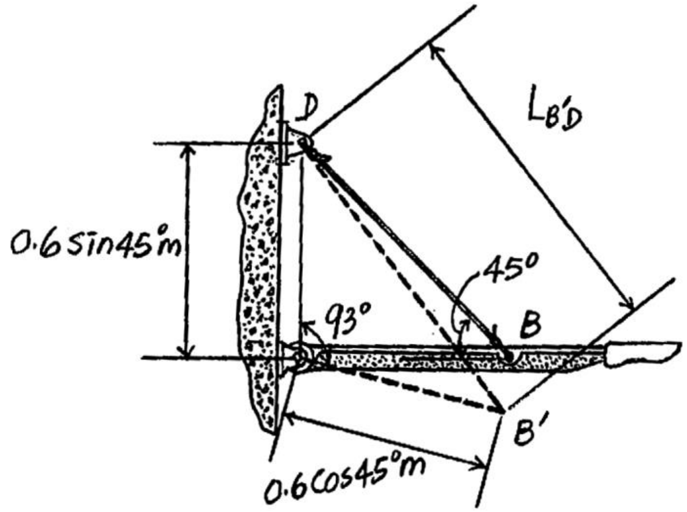
Prob. 2–21

Discuss the approach?

Solution



Prob. 2–21



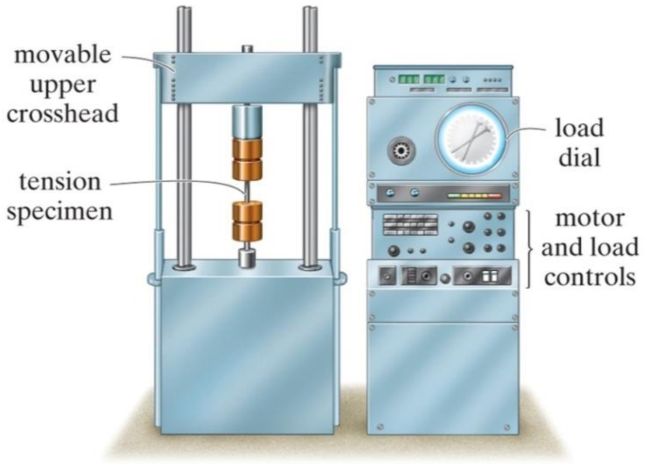
(a)

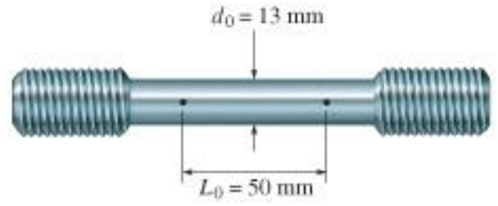
$L_{B'D} = 0.6155$ m

$\varepsilon = 0.0258$ m/m

Simple Tensile Test

- Strength of a material can only be determined by *experiment*
- The test used by engineers is the *tension or compression test*
- This test is used primarily to determine the relationship between the average normal stress and average normal strain in common engineering materials, such as metals, ceramics, polymers and composites





Conventional Stress–Strain Diagram

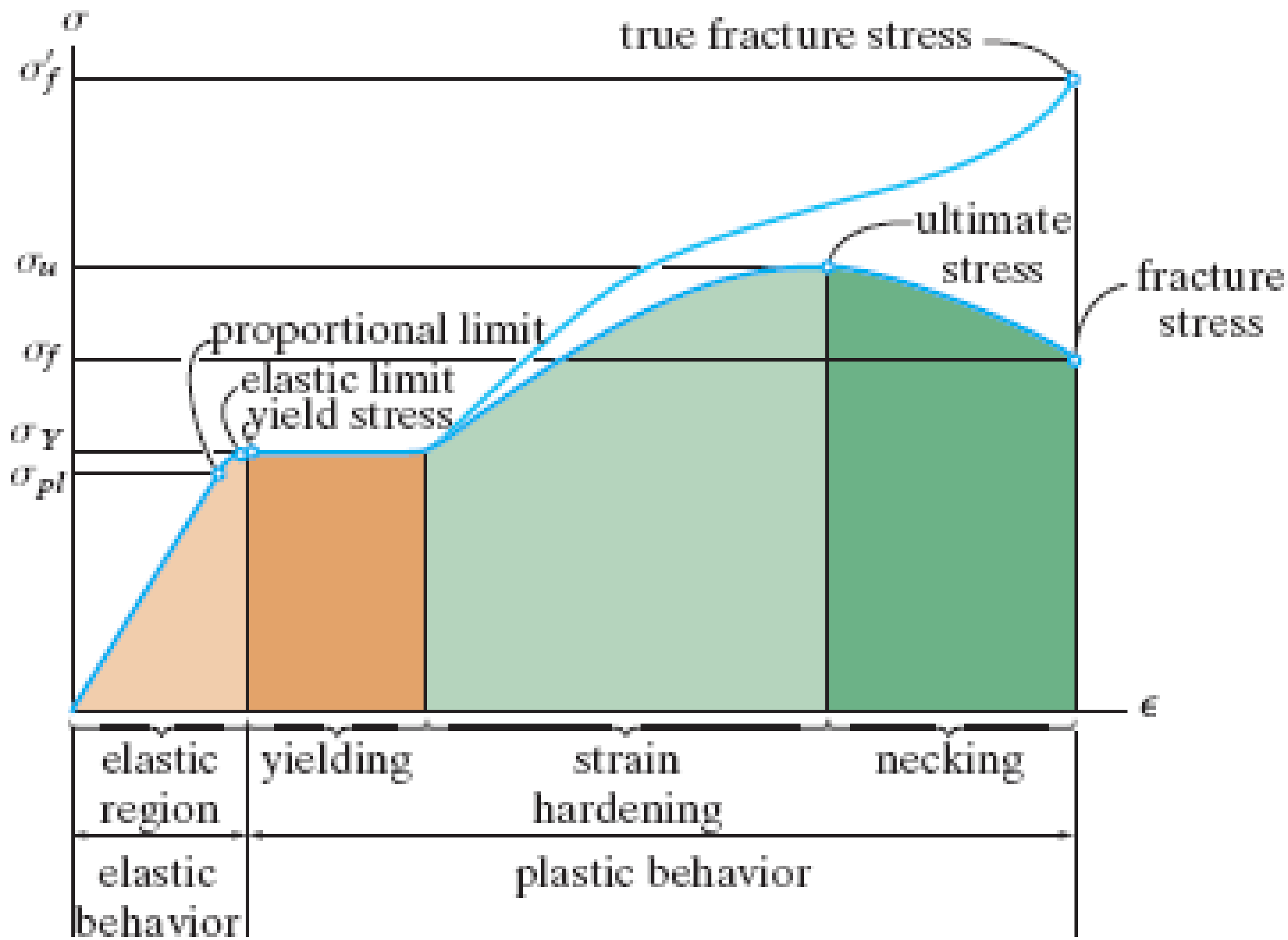
- ***Nominal*** or ***engineering stress*** is obtained by dividing the applied load *P* by the specimen's *original* cross-sectional area.

$$\sigma = \frac{P}{A_0}$$

- ***Nominal*** or ***engineering strain*** is obtained by dividing the change in the specimen's gauge length by the specimen's original gauge length.

$$\varepsilon = \frac{\delta}{L_0}$$

Conventional Stress–Strain Diagram



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Conventional Stress–Strain Diagram

Elastic Behaviour

- A straight line
- Stress is proportional to strain, i.e., linearly elastic
- Upper stress limit, or *proportional limit*; σ_{pl}
- If load is removed upon reaching elastic limit, specimen will return to its original shape

Yielding

- Material deforms permanently; yielding; plastic deformation
- Yield stress, σ_Y
- Once yield point reached, specimen continues to elongate (strain) *without any increase in load*
- Note figure not drawn to scale, otherwise induced strains is 10-40 times larger than in elastic limit
- Material is referred to as being *perfectly plastic*

Hooke's Law

- *Hooke's Law* defines the *linear relationship* between stress and strain within the elastic region.

$$\sigma = E\varepsilon$$

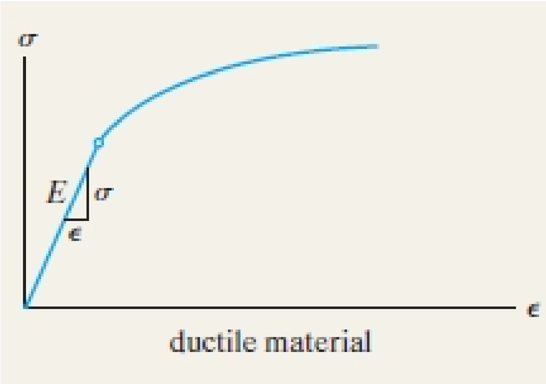
σ = stress

E = modulus of elasticity or *Young's modulus*

ε = strain

- *E* can be used only if a material has *linear–elastic* behaviour.

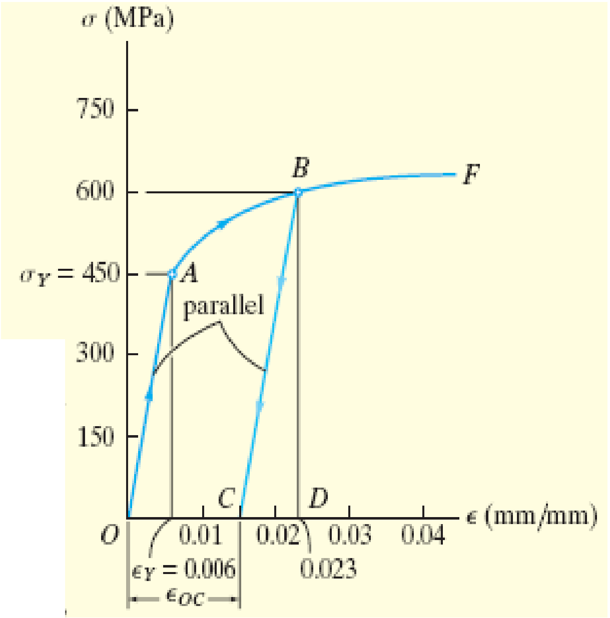
E can be derived from stress and strain graph.
What is it?



Example

- The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is shown. When material is stressed to 600 MPa, find the permanent strain that remains in the specimen when load is released. Also, compute the modulus of resilience both before and after the load application.

- Approach to the problem:
  - Parallel to elastic line
  - Both slope is equal
  - Distance CD can be calculated based on the slope
  - Permanent strain: 0.023 – distance CD



Solution

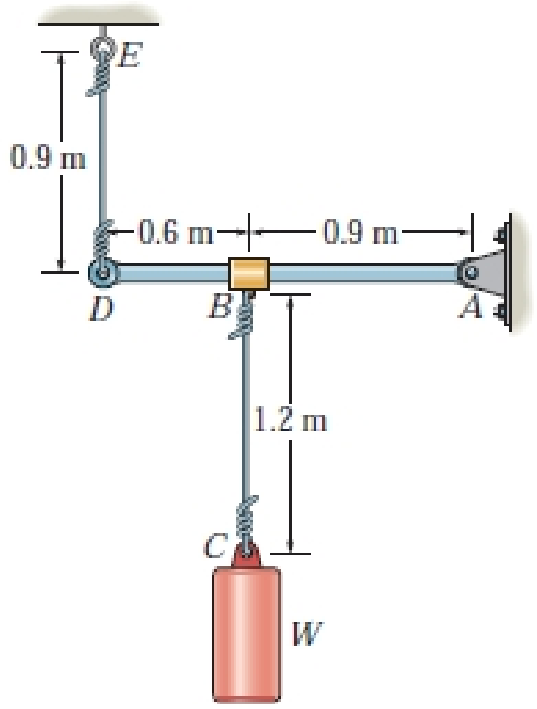
- When the specimen is subjected to the load, the strain is approximately 0.023 mm/mm.
- The slope of line *OA* is the modulus of elasticity,
- From triangle *CBD,*

$$E = \frac{450}{0.006} = 75.0 \text{ GPa}$$

$$E = \frac{BD}{CD} = \frac{600\left(10^6\right)}{CD} = 75.0\left(10^9\right) \Rightarrow CD = 0.008 \text{ mm/mm}$$

Example

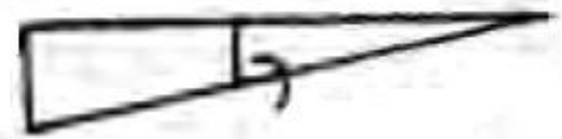
The bar DA is rigid and is originally held in the horizontal position when the weight W is supported from C. If the weight causes B to be displaced downward 0.625mm, determine the strain in wires DE and BC. Also if the wires are made of A-36 steel and have a cross-sectional area of 1.25 mm^2, determine the weight W.



Prob. 3–42

Discuss the approach????

1) Calculate the displacement of D



$$\frac{\delta_D}{1.5} = \frac{\delta_B}{0.9}$$

$$\delta_D = 0.625(\frac{1.5}{0.9})$$

$$\delta_D = 1.0417mm$$

2) Based on displacement on D, calculate the strain and normal stress

$$\varepsilon_D = \frac{\delta_D}{L_D} = \frac{1.0417}{900} = 1.157(10)^{-3} mm/mm$$

** strain in mm/mm, stress and E in MPa, F in N and length in mm*

$$\sigma_D = E\varepsilon = 200(10)^3 1.157(10)^{-3} = 231.4MPa$$

3) Based on normal stress at wire DE, calculate the T of wire D

$$\sigma_D = 231.4MPa = \frac{T_{ED}}{A}$$

$$T_{ED} = \sigma_D A = 289.3N$$

4) Calculate W, based on FBD of bar DA

$$\sum M_A = 0$$

$$-T_{DE}(1.5) + W(0.9) = 0$$

$$W = 482.2N$$

5) Calculate normal stress of wire CB and strain of wire CB

$$\sigma_{BC} = \frac{T_{BC}}{A} = \frac{482.2}{1.25} = 385.7MPa$$

Strain can not be calculated as normal stress goes beyond yield stress (Sy = 250 MPa), elastic property is no more applied. Therefore it requires the stress and strain curve to predict the strain