مادة المقرر الدراسي

الرياضيات الهندسية-II

Engineering mathematics-II

المستوى الثاني ٢٠٢١/٢٠٢٠

اعداد

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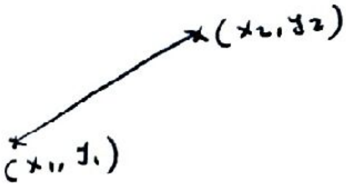
12.2 vectors

- The vector represented by the direct line segment $\overrightarrow{AB}$ has initial point A and terminal point B and it's length is denoted by $|\overrightarrow{AB}|$. Two vector are equal if they have the same length and direction.

- If V is a two-dimentional vector in the plane equal to the vector with initial point at the origin and terminal point (V_1, V_2) then the component form of V is:



$$V = \{V_1, V_2\}$$

$$V_1 = x_2 - x_1 \quad , \quad V_2 = y_2 - y_1$$

- If V is a three-dimensional vector equal to the vector with intial point at the origin and terminal point (V_1, V_2, V_3) then the component form of V is:

$$V = \{V_1, V_2, V_3\}$$

$$V_1 = x_2 - x_1 \quad , \quad V_2 = y_2 - y_1 \quad , \quad V_3 = z_2 - z_1$$

- The magnitude or length of the vector $V = \overrightarrow{PQ}$ is the nonnegative number.

$$|V| = \sqrt{V_1^2 + V_2^2 + V_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- The only vector with length 0 is the Zero vector $0 = \{0, 0\}$ or $0 = \{0, 0, 0\}$. this vector is also the only vector with no specific directi

- vector Algebra operations:

Let $u=\{u_1, u_2, u_3\}$ and $v=\{v_1, v_2, v_3\}$ be vectors :

* Addition : $u+v=\{u_1+v_1, u_2+v_2, u_3+v_3\}$

* scalar multiplication: $ku=\{ku_1, ku_2, ku_3\}$

- Properties of vector operations:

Let u, v, w be vectors and a, b be scalars :

1. $u+v=v+u$.
2. $(u+v)+w=u+(v+w)$
3. $u+0=u$
4. $u+(-u)=0$
5. $0u=0$
6. $1.u=u$
7. $a(bu)=(ab)u$
8. $a(u+v)=au+av$
9. $(a+b)u=au+bu$

- unit vectors:

A vector v of length 1 is called a unit vector. the standard unit vectors are:

$$i=(1,0,0), \quad j=(0,1,0) \quad \text{and} \quad k=(0,0,1)$$

any vector $v=\{v_1, v_2, v_3\}$ can be written as a linear combination of the standard unit vectors as follows:

$$\begin{aligned} v=\{v_1, v_2, v_3\} &= \{v_1,0,0\}+\{0,v_2,0\}+\{0,0,v_3\} \\ &= v_1(1,0,0)+v_2(0,1,0)+v_3(0,0,1) \\ &= v_1 i+v_2 j+v_3 k \end{aligned}$$

* we called the scalar (or number) v_1 the i-component of the vector v, v_2 the j-component and v_3 the k-component. In component form, the vector from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is:

$$\left\{ \overrightarrow{P_1P_2} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k \right\}$$

whenever $v \neq 0$, its length $|v|$ is not zero and

$$\left| \frac{1}{|v|} \cdot v \right| = \frac{1}{|v|} |v| = 1$$

that is $\frac{v}{|v|}$ is a unit vector in the direction of v, called the direction of the nonzero vector. v.

* the equation $v = |v| \cdot \frac{v}{|v|}$ expresses v as its length times its direction.

– Mid point of a line segment:

The midpoint M of the line segment joining point $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is the point:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Exercises 12.2 Page 716

- Let $u = (3,-2)$ and $v(-2,5)$ Find the (a) component form and (b) magnitude (Length) of the vector.

⑥ $-2u + 5v$

a: $-2u = -2(3,-2) = (-2\times3, -2\times-2) = (-6,4)$

$5v = (5\times-2, 5\times5) = (-10, 25)$

$-2u + 5v = (-6,4) + (-10,25)$

$-2u + 5v = (-6 + (-10), 4 + 25) = \underline{(-16, 29)}$

b: Length of the vector $= \sqrt{(-16)^2 + (29)^2} = \sqrt{1097} = \underline{33.12}$

⑧ $-\frac{5}{13}u + \frac{12}{13}v$

a: $\frac{-5}{13}u = \frac{-5}{13}(3,-2) = \left(\frac{-5}{13}\times3, \frac{-5}{13}\times-2\right) = \left(\frac{-15}{13}, \frac{10}{13}\right)$

$\frac{12}{13}v = \frac{12}{13}(-2,5) = \left(-2\times\frac{12}{13}, 5\times\frac{12}{13}\right) = \left(\frac{-24}{13}, \frac{60}{13}\right)$

$\therefore \frac{-5}{13}u + \frac{12}{13}v = \left(\frac{-15}{13}, \frac{10}{13}\right) + \left(\frac{-24}{13}, \frac{60}{13}\right)$

$= \left(\frac{-15}{13} + \frac{-24}{13}, \frac{10}{13} + \frac{60}{13}\right)$

$= \left(\frac{-39}{13}, \frac{70}{13}\right)$

b: Length of vector $= \sqrt{\left(\frac{-39}{13}\right)^2 + \left(\frac{70}{13}\right)^2} = \sqrt{37.994} = \underline{6.16}$

- Find the component form of the vector:

(10) The vector $\overrightarrow{OP}$ where $\underline{O}$ is the origin and $\underline{P}$ is the midpoint of segment $\underline{\underline{RS}}$, where $R=(2,-1)$ and $S=(-4,3)$.

Sol.

$$P=\left(\frac{2+(-4)}{2}, \frac{-1+3}{2}\right) \Longrightarrow P=(-1,1)$$

$\therefore O=(0,0)$, $P=(-1,1)$

$\therefore$ The vector $\overrightarrow{OP}=(-1-0, 1-0)=(-1,1)$

(12) The sum of $\overrightarrow{AB}$ and $\overrightarrow{CD}$, where $A(1,-1)$, $B=(2,0)$, $C=(-1,3)$, and $D=(-2,2)$

$$\overrightarrow{AB}=(2-1, 0-(-1)) \Longrightarrow \overrightarrow{AB}=(1,1)$$

$$\overrightarrow{CD}=(-2-(-1), 2-3) \Longrightarrow \overrightarrow{CD}=(-1,-1)$$

$$\therefore \overrightarrow{AB}+\overrightarrow{CD}=(1+(-1), 1+(-1))=\underline{\underline{(0,0)}}$$

(13) The unit vector that makes an angle $\theta = 2\pi/3$ with the positive x-axis.

$\therefore$ the unit vector $=\left(\cos\frac{2\pi}{3}, \sin\frac{2\pi}{3}\right)=\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(15) The unit vector obtained by rotating the vector $(0,1)$ $120°$ counterclockwise about the origin.

Sol.

This is the unit vector $(0,1)$ make an angle of $120° + 90° = 210°$ with the positive x-axis.

$\therefore$ the unit vector $= (\cos 210, \sin 210) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

- Express each vector in the form $v = v_1 i + v_2 j + v_3 k$.

(19) $\overrightarrow{AB}$ if A is the point $(-7,-8,1)$ and B is the point $(-10,8,1)$

$\overline{AB} = (-10-(-7))i + (8-(-8))j + (1-1)k$

$\therefore \overline{AB} = -3i + 16j$

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(21) $5u - v$ if $u = (1,1,-1)$, $v = (2,0,3)$

$5u - v = (5\times 1, 5\times 1, 5\times -1) - (2,0,3)$

$= (5, 5, -5) - (2,0,3)$

$= (5-2, 5-0, -5-3)$

$= (3, 5, -8)$

$5u - v = 3i + 5j - 8k$

- copy vectors u, v, and w head to tail as needed to sketch the indicated vector.

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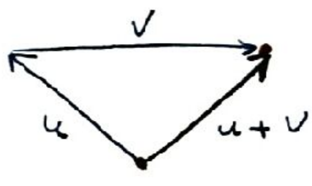
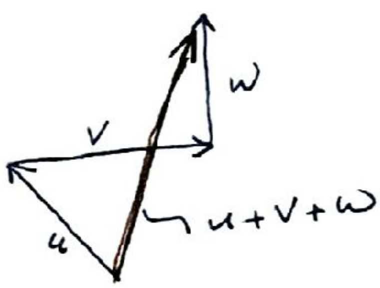
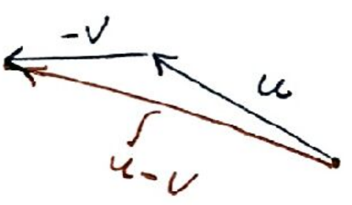
a. $u + v$

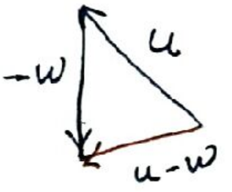


b. $u + v + w$



c. $u - v$





d. $u - w$

- Express each vector as a product of its Length and direction :

(26) $9i - 2j + 6k$

* Length $= |9i - 2j + 6k| = \sqrt{9^2 + (-2)^2 + 6^2} = 11$

* the direction is $\frac{V}{|V|} = \boxed{\frac{9}{11} i - \frac{2}{11} j + \frac{6}{11} k}$

* Length * direction $= 11\left(\frac{9}{11} i - \frac{2}{11} j + \frac{6}{11} k\right)$

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(29) $\frac{1}{\sqrt{6}} i - \frac{1}{\sqrt{6}} j - \frac{1}{\sqrt{6}} k$

* Length $= \left| \frac{1}{\sqrt{6}} i - \frac{1}{\sqrt{6}} j \cancel{+} - \frac{1}{\sqrt{6}} k \right| = \sqrt{\left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{-1}{\sqrt{6}}\right)^2 + \left(\frac{-1}{\sqrt{6}}\right)^2}$

$\therefore$ Length $= \sqrt{1/2}$

* Direction is $\frac{V}{|V|} = \frac{\frac{1}{\sqrt{6}}}{\sqrt{1/2}} i - \frac{1/\sqrt{6}}{\sqrt{1/2}} j - \frac{1/\sqrt{6}}{\sqrt{1/2}} k =$

$= \frac{1}{\sqrt{3}} i - \frac{1}{\sqrt{3}} j - \frac{1}{\sqrt{3}} k$

$\therefore$ Length * direction $= \sqrt{1/2}\left(\frac{1}{\sqrt{3}} i - \frac{1}{\sqrt{3}} j - \frac{1}{\sqrt{3}} k\right)$

(31) Find the vectors whose Lengths and directions are given:

b- Length $= \sqrt{3}$, direction $= -K$

Length $= |V|$, direction $= \frac{V}{|V|}$

$\therefore$ Length $*$ direction $= |V| * \frac{V}{|V|} = \underline{\underline{V}}$ vectors

$= \sqrt{3} * (-K) = -\sqrt{3}\,K$

C- Length $= 1/2$, direction $= \frac{3}{5}j + \frac{4}{5}K$

$\therefore$ vector $= 1/2 \left(\frac{3}{5} j + \frac{4}{5} K \right)$

$= 1/2 * \frac{3}{5} j + 1/2 * \frac{4}{5} K$

vector $= \frac{3}{10} j + \frac{4}{10} K$

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(34) Find a vector of magnitude $\underline{\underline{3}}$ in the direction opposite to the direction of $V = \frac{1}{2}i - \frac{1}{2}j - \frac{1}{2}K$

Length of $V = \sqrt{(\frac{1}{2})^2 + (-\frac{1}{2})^2 + (-1/2)^2} = \frac{\sqrt{3}}{2}$

direction $= \frac{V}{|V|} = \frac{1/2}{\sqrt{3}/2} i - \frac{1/2}{\sqrt{3}/2} j - \frac{1/2}{\sqrt{3}/2} K$

direction $= \frac{1}{\sqrt{3}} i - \frac{1}{\sqrt{3}} j - \frac{1}{\sqrt{3}} K$

$\therefore$ the vector in direction opposite of V = $\ominus$ Length $*$ direction

$= -3 \left(\frac{1}{\sqrt{3}} i - \frac{1}{\sqrt{3}} j - \frac{1}{\sqrt{3}} K \right) = \boxed{-\sqrt{3}\,i + \sqrt{3}\,j + \sqrt{3}\,K}$

- Find
  - a- the direction of $\overrightarrow{P_1P_2}$
  - b- the mid point of line segment P_1P_2.

(35) $P_1(-1,1,5)$, $P_2(2,5,0)$

a- $\overrightarrow{P_1P_2} = (x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)k$

$= (2-(-1))i + (5-1)j + (0-5)k$

$\overrightarrow{P_1P_2} = 3i + 4j - 5k$

$|P_1P_2| = \sqrt{3^2+4^2+(-5)^2} = 5\sqrt{2}$

$\therefore$ Direction $= \dfrac{V}{|V|} = \dfrac{3}{5\sqrt{2}}i + \dfrac{4}{5\sqrt{2}}j - \dfrac{1}{\sqrt{2}}k$

b- midpoint $= \left(\dfrac{2+(-1)}{2}, \dfrac{5+1}{2}, \dfrac{0+5}{2}\right)$

$\therefore$ midpoint $= \left(\dfrac{1}{2}, 3, \dfrac{5}{2}\right)$

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(39) if $\overrightarrow{AB} = i+4j-2k$ and B is the point $(5,1,3)$ find A

$\overrightarrow{AB} = (x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)k$

$\therefore$ $5-x_1 = 1 \Rightarrow x_1 = 4$

$1-y_1 = 4 \Rightarrow y_1 = -3$

$3-z_1 = -2 \Rightarrow z_1 = 5$

$\therefore$ $A = (4,-3,5)$

(42) Let $u = i - 2j$, $v = 2i + 3j$, and $w = i + j$,

write $u = u_1 + u_2$, where u_1 is parallel to v and u_2 is parallel to w.

Sol.

$$u_1 = a(2i + 3j) = 2ai + 3aj$$

$$u_2 = b(i + j) = bi + bj$$

$$u = u_1 + u_2$$

$$i - 2j = (2a+b)i + (3a+b)j$$

$$\therefore\ 2a + b = 1 \Rightarrow b = 1 - 2a \quad \cdots (1)$$

$$3a + b = -2 \Rightarrow 3a + (1 - 2a) = -2 \Rightarrow 3a + 1 - 2a = -2$$

$$a + 1 = -2 \Rightarrow a = -3$$

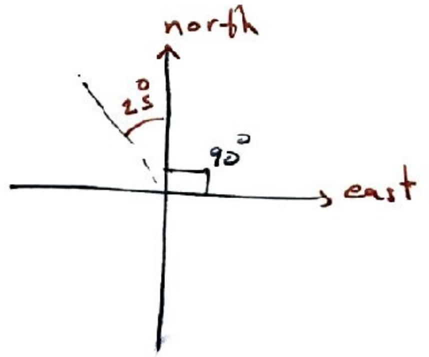
$$\therefore\ b = 1 - (2 \times -3) = 7$$

$$\therefore\ u_1 = ((2 \times -3))i + 3 \times (-3)j$$

$$\boxed{u_1 = -6i - 9j}$$

$$u_2 = bi + bj \Rightarrow \boxed{u_2 = 7i + 7j}$$

(43) An air plane is flying in the Direction 25° west of nourth at 800 Km/h. Find the component form of the velocity of the air plane, assuming that the Positive x-axis represents due east and the positive y-axis represents due north.



$\theta = 25 + 90 = 115°$

component of velocity $= 800(\cos 115, \sin 115)$

$\approx (-338.095, 725.046)$

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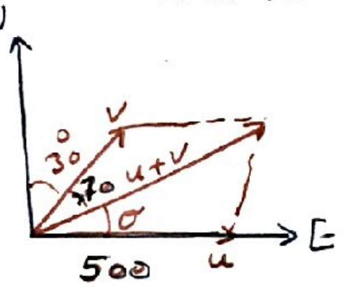
(44) A jet airliner flying due east at 500 mph in still air encounters a 70 mph tail wind blowing in the direction 60° north of east. what speed and direction have in order for the resultant vector to be 500 mph due east?



Let u = velocity of air plane alone

V = " " tail wind

$u = (\cancel{500}\ x, y)$, $V = (70\cos 60, 70\sin 60)$

$V = (35, 35\sqrt{3})$

Let the resultant $= u + V = (500, 0)$

$(x, y) + (35, 35\sqrt{3}) = (500, 0)$

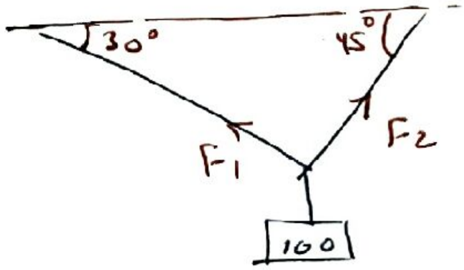
$$\left.\begin{array}{l} x + 35 = 500 \Rightarrow x = 465 \\ y + 35\sqrt{3} = 0 \Rightarrow y = -35\sqrt{3} \end{array}\right\} \Rightarrow u = (465, -35\sqrt{3})$$

$|u| = \sqrt{465^2 + (-35\sqrt{3})^2} \approx 468.9$ mph

$\tan\theta = \frac{-35\sqrt{3}}{465} \Rightarrow \theta = -7.4° \Rightarrow 7.4$ south of east

(45) Consider a 100-N weight suspended by two wires as shown in the figure. Find the magnitude and components of the force vectors F_1 and F_2.

Sol.



$F_1 = (-|F_1|\cos 30, |F_1|\sin 30)$

$F_1 = \left(-\frac{\sqrt{3}}{2}|F_1|, \frac{1}{2}|F_1|\right)$

$F_2 = (|F_2|\cos 45°, |F_2|\sin 45)$

$F_2 = \left(\frac{1}{\sqrt{2}}|F_2|, \frac{1}{\sqrt{2}}|F_2|\right)$

$W = (0, -100)$

$F_1 + F_2 = W$

$\left(\frac{-\sqrt{3}}{2}|F_1|, \frac{1}{2}|F_1|\right) + \left(\frac{1}{\sqrt{2}}|F_2|, \frac{1}{\sqrt{2}}|F_2|\right) = (0, -100)$

$\therefore \quad -\frac{\sqrt{3}}{2}|F_1| + \frac{1}{\sqrt{2}}|F_2| = 0 \Rightarrow |F_2| = \frac{\sqrt{6}}{2}|F_1| \quad \cdots (1)$

$\frac{1}{2}|F_1| + \frac{1}{\sqrt{2}}|F_2| = -100$

$\frac{1}{2}|F_1| + \frac{1}{\sqrt{2}} * \frac{\sqrt{6}}{2}|F_1| = -100 \Rightarrow |F_1| = \frac{200}{1+\sqrt{3}} \approx 73.205N$

$|F_2| = \frac{\sqrt{6}}{2} * \frac{200}{1+\sqrt{3}} = \frac{100\sqrt{6}}{1+\sqrt{3}} \approx 89.658 N$

$\therefore F_1 \approx (-63.397, 36.603)$

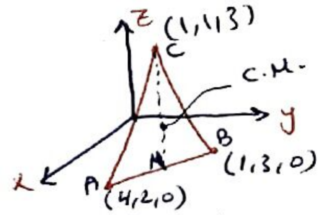
$F_2 \approx (63.397, 63.397)$

(51) suppose that A, B and C are the corner points of the thin traingular plate of constant density shown in figure.

a. Find the vector from C to the midpoint M of side AB

b. Find the vector from C to the point that lies two-third of the way from C to M on the median CM.

c. Find the coordinates of the point in which the medians of ΔABC intersect. This point is the plat's center of mass.



Sol.

a. from figure $A(4,2,0)$, $B(1,3,0)$, $C(1,1,3)$

$$M = \text{midpoint of } AB = \left(\frac{4+1}{2}, \frac{2+3}{2}, \frac{0+0}{2}\right)$$

$$M = \left(\frac{5}{2}, \frac{5}{2}, 0\right)$$

$\therefore$ the vector $\overrightarrow{CM} = \left(\frac{5}{2}-1\right)i + \left(\frac{5}{2}-1\right)j + (0-3)K$

$$\therefore \boxed{\overrightarrow{CM} = \frac{3}{2}i + \frac{3}{2}j - 3K}$$

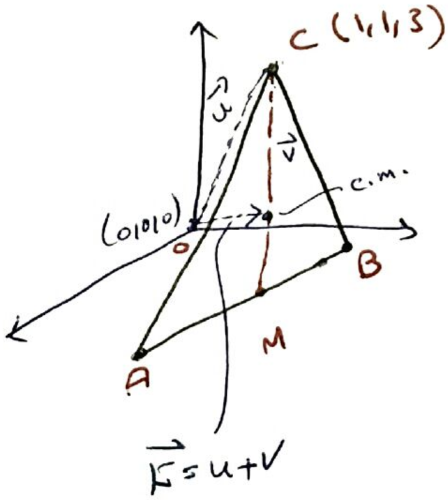
b. the vector $= \frac{2}{3}\overline{CM} = \frac{2}{3}\left(\frac{3}{2}i + \frac{3}{2}j - 3K\right)$

$$\boxed{\vec{V} = i + j - 2K}$$

C:

$\vec{u} = \vec{OC} = (1-0)i + (1-0)j + (3-0)k$

$\vec{OC} = i + j + 3k$



$\vec{v}$ = the vector from part b

$\vec{v} = i + j - 2k$

$\vec{k} = \vec{u} + \vec{v} = (i + j + 3k) + (i + j - 2k)$

$= (1+1)i + (1+1)j + (3-2)k$

$\vec{k} = \vec{u} + \vec{v} = 2i + 2j + k$

vector $\vec{k}$ from origin (0,0,0) To c.m. (x,y,z)

$\therefore$ $x - 0 = 2 \Rightarrow x = 2$

$y - 0 = 2 \Rightarrow y = 2$

$z - 0 = 1 \Rightarrow z = 1$

$\therefore$ the Point (center of mass) = (2,2,1)

12.3 The Dot Product   P. 718

- **Angle between two vectors**: The angle θ between two nonzero vectors $u = \{u_1, u_2, u_3\}$ and $v = (v_1, v_2, v_3)$ is given by

$$\theta = \cos^{-1}\left(\frac{u_1v_1 + u_2v_2 + u_3v_3}{|u|\,|v|}\right) \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{u \cdot v}{|u|\,|v|}\right)$$

- The dot product $u \cdot v$ ("u dot v") of vectors $u = \{u_1, u_2, u_3\}$ and $v = \{v_1, v_2, v_3\}$ is the scalar

$$u \cdot v = u_1v_1 + u_2v_2 + u_3v_3$$

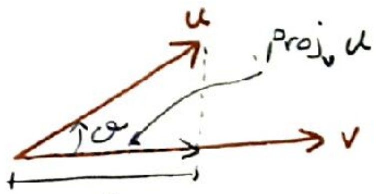
- The vectors u and v are orthogonal if $\boxed{u \cdot v = 0}$

* Properties of the Dot Product:

If $u, v,$ and w are any vectors and c is a scalar, then:

1. $u \cdot v = v \cdot u$
2. $(cu) \cdot v = u \cdot (cv) = c(u \cdot v)$
3. $u \cdot (v + w) = u \cdot v + u \cdot w$
4. $u \cdot u = |u|^2$
5. $0 \cdot u = 0$

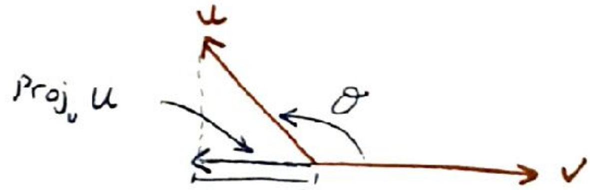
* The vector Projection of $\underline{\underline{u}}$ on to $\underline{\underline{V}}$ is the vector:



Length $= |u| \cos\theta$

$\boxed{\text{if } \cos\theta \geq 0}$

$$\boxed{\operatorname{Proj}_v u = \left(\frac{u \cdot v}{|v|^2}\right) v}$$



Length $= -|u| \cos\theta$

$\boxed{\text{if } \cos\theta < 0}$

* The scalar component of $\underline{\underline{u}}$ in the direction of $\underline{\underline{V}}$ is the scalar:

$$\boxed{|u| \cos\theta = \frac{u \cdot v}{|v|} = u \cdot \frac{v}{|v|}}$$

The <u>work</u> done by a constant force $\underline{\underline{F}}$ acting through a displacement $\underline{\underline{D}} = \overrightarrow{PQ}$ is

$$\boxed{W = F \cdot D}$$

Exercises 12.3 P. 724

- For the following vectors find:

a. $v \cdot u$, $|v|$, $|u|$, b. the cosine of the angle betwеen v and u

c. the scalar component of u in the direction of $\underline{v}$.

d. the vector $Proj_v \cdot u$.

(3) $v = 10i + 11j - 2K$, $u = 3j + 4K$

a. $v \cdot u = 10 \times 0 + 11 * 3 - 2 \times 4 = \underline{\underline{25}}$

$|v| = \sqrt{(10)^2 + (11)^2 + (-2)^2} = \underline{\underline{15}}$

$|u| = \sqrt{(3)^2 + (4)^2} = \underline{\underline{5}}$

b. $\cos\theta = \dfrac{u \cdot v}{|u||v|} = \dfrac{25}{15 \times 5} = \dfrac{1}{3}$

c. $|u| \cos\theta = \dfrac{u \cdot v}{|v|} = \dfrac{25}{15} = \dfrac{5}{3}$

d. $Proj_v u = \left(\dfrac{u \cdot v}{|v|^2}\right) v = \dfrac{25}{(15)^2} * (10i + 11j - 2K)$

$v = \dfrac{1}{9}(10i + 11j - 2K)$

- Find the angles between the vectors to the nearest hundredth of a radian.

(10) $u = 2i - 2j + k$, $v = 3i + 4k$

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$$

$u \cdot v = 2 \times 3 + (-2) * (0) + (1 * 4) = \underline{\underline{10}}$

$|u| = \sqrt{(2)^2 + (-2)^2 + (1)^2} = \sqrt{9} = 3$

$|v| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

$\therefore \theta = \cos^{-1}\left(\frac{10}{3 * 5}\right) \approx 0.84$ rad.

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(12) $u = i + \sqrt{2}j - \sqrt{2}k$, $v = -i + j + k$

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right) =$$

$u \cdot v = (1 * -1) + (\sqrt{2} * 1) + (-\sqrt{2} * 1) = \underline{\underline{-1}}$

$|u| = \sqrt{(1)^2 + (\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{5}$

$|v| = \sqrt{(-1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

$\therefore \theta = \cos^{-1}\left(\frac{-1}{\sqrt{3} * \sqrt{5}}\right) \approx 1.83$ rad.