(Mechanics of Materials II)
Chapter 5: Stress in Beams

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Chapter Learning Outcome

1. Define bending moment

2. Derive bending moment formulae

3. Calculate the stress in a beam due to bending

4. Find a location of neutral axis

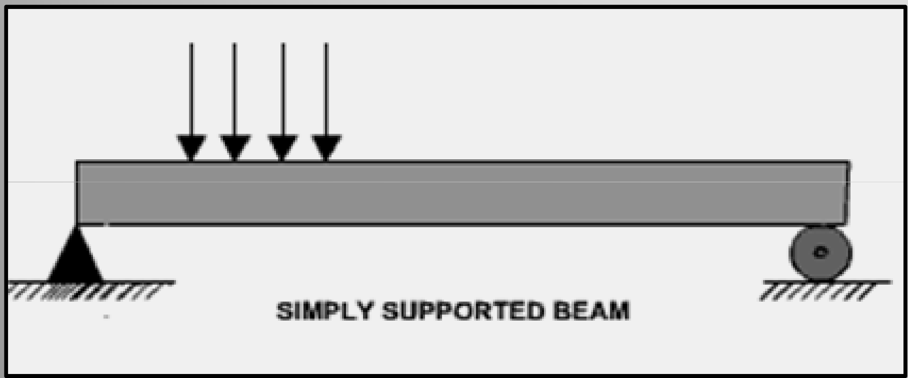
Introduction

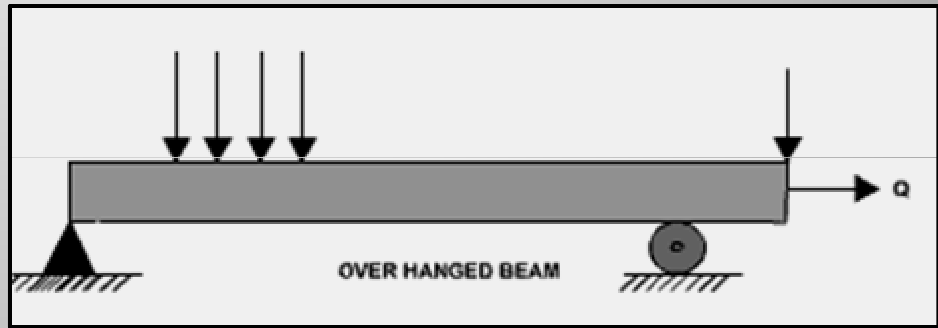
- **Beam**, or **flexural member**, is frequently encountered in structures and machines.
  - member that subjected to loads applied transverse (sideways) to the long dimension, that causing the member to bend.
- Transverse loading causes bending and it is very severe form of stressing a structure.
- The **beat beam** goes into ***tension (stretched)*** on one side and ***compression*** on the other.

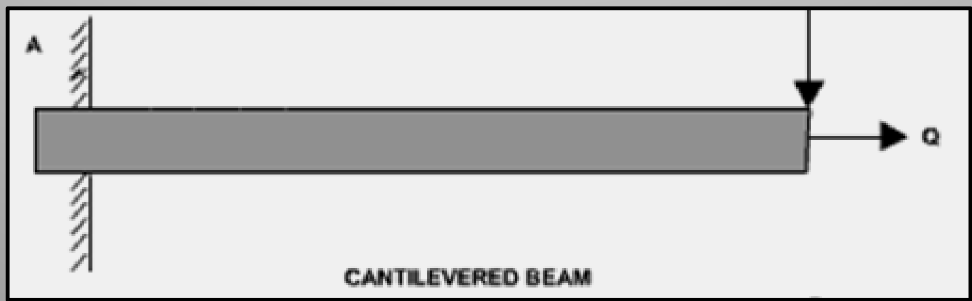
Example: a simply-supported beam loaded at its Mid-point will deform into the exaggerated bent shape shown in figure

Classify Types of Beams

- **Beams** - classified on the basis of supports or reactions such as *pins, rollers, or smooth surfaces* at the ends .

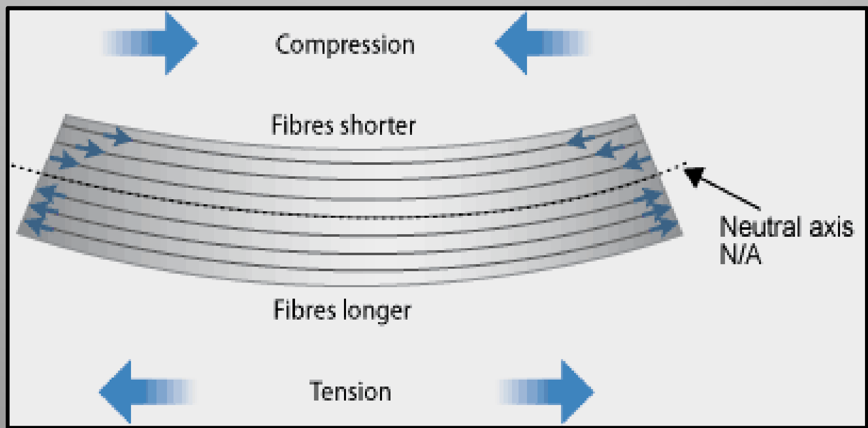






The Bending Theory Neutral

- Neutral Axis (N.A)
  - axis along the length of the beam which remains unstressed when it is bend.
  - It is the region that separates the tensile forces from compressive force
    - i.e. bending stress equal to zero. The position of the neutral axis must pass through the centroid of the section hence this position is important



Derivation of flexural formula

– To establish the bending stress formula several assumptions are used:-

1. The cross section of the beam is plane and must remain plane after bending

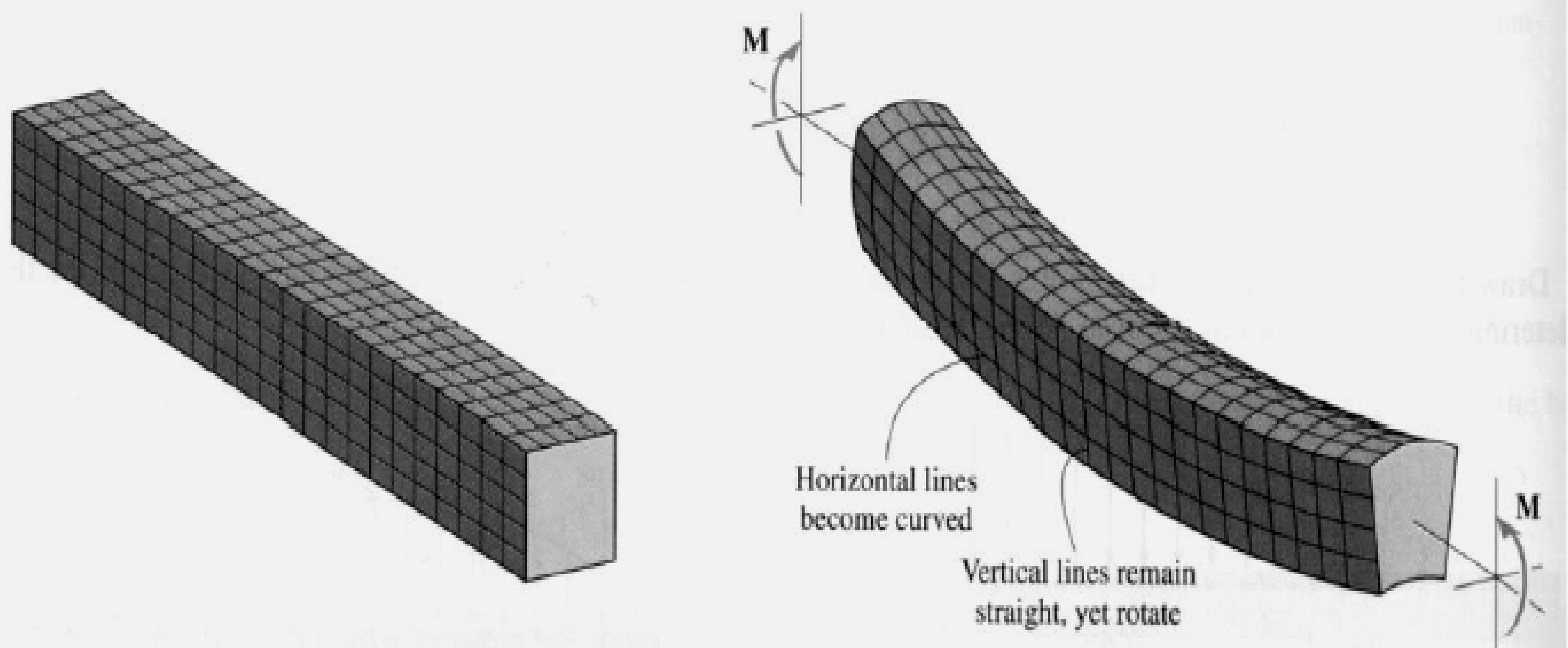
2. The beam's material is homogeneous and obey Hooke's Law

3. The material must be free from any resistance force and from impurities, holes, or grooves.

4. The bending moment of elasticity in tension must be the same for compression.

5. The beam has constant cross section.

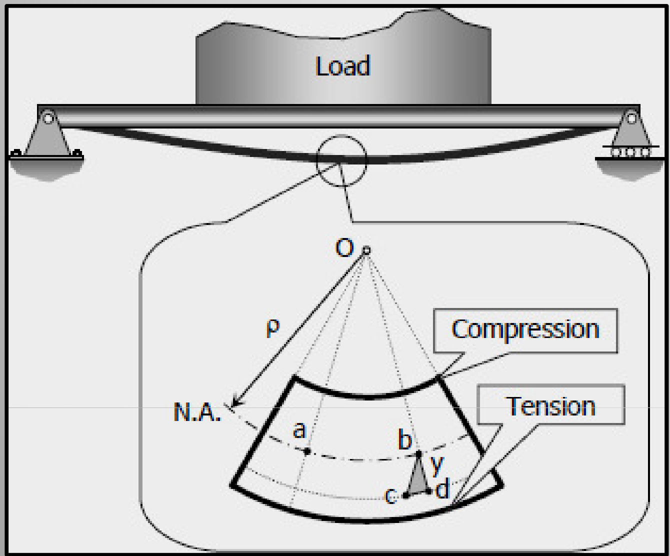
6. The beam is subjected to pure bending

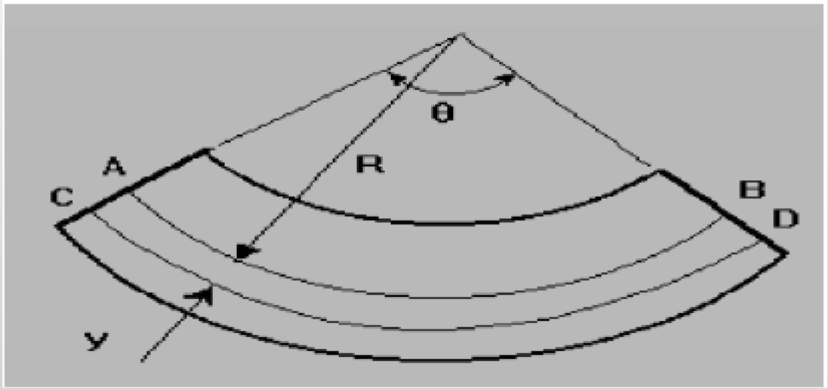


Before deformation

After deformation

- Due to the action of load beam will bend. Consider a beam that bent into an arc of a circle through angle Θ radians. AB is on the neutral axis (will be the same length before and after bending). R is the radius of neutral axis.





- Beams in whatever shape will basically form a curve of x-y graph. One should be noted that the radius of curvature at any point on the graph is the radius of a circle

- The length of AB; $AB = R\theta$
- Consider a layer of material with distance y from the N.A. The radius of this layer is R + y. The length of this layer which denoted by the line CD is;

$$CD = (R + y)\theta$$

- This layer is stretched because it becomes longer. Thus, it's been strained and strain, ε for this layer is;

$$\varepsilon = \frac{change\ in\ length}{original\ length} = \frac{CD - AB}{AB} = \frac{(R+y)\Theta - R\Theta}{R\Theta} = \frac{R\Theta - y\Theta - R\Theta}{R\Theta} = \frac{y}{R}$$

- # Hooke's Law, Modulus of Elasticity, E

From Hooke's Law, modulus of elasticity, E

$$E = \frac{\sigma}{\varepsilon}$$

Substitute $\varepsilon = \frac{y}{R}$

$$E = \frac{\sigma R}{y}$$

Rearrange;

$$\frac{E}{R} = \frac{\sigma}{y}$$

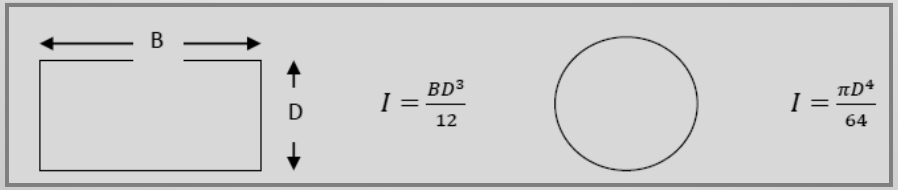
- Stress and strain vary along the length of the beam depending on the radius of curvature.

- Fibre stress at a distance *y* from neutral surface $\sigma = {}^{My}/_{I}$
- At any section of beam, the maximum fibre stress, $\sigma_{max} = {}^{Mc}/_{I} = {}^{M}/_{Z}$
  - Where as;
    - C is the distance from NA to outer fibre(the outer fibre will experienced maximum stress)
    - $Z = {}^{I}/_{C}$, section modulus of the beam

- Although the section modulus can be readily calculated for a given, value of the modulus are often included in tables to simplify calculations.

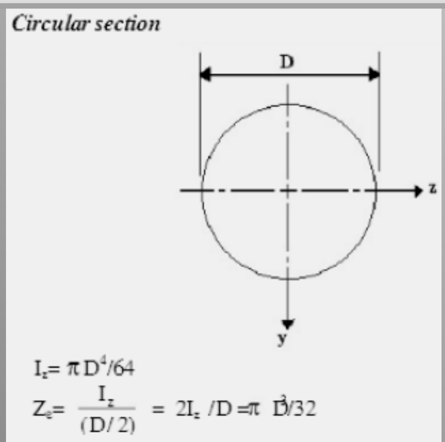
- **Standard Sections**

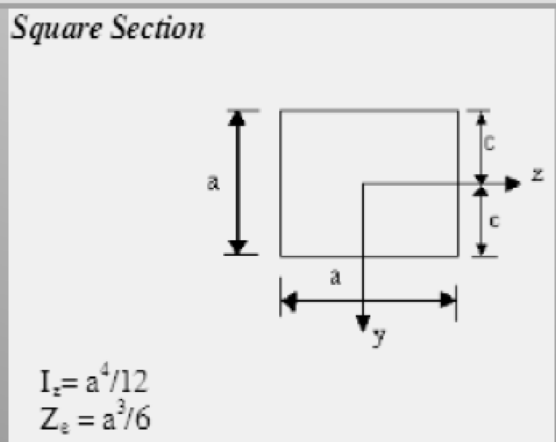
  - The value of ***I, second moment*** of area for a given section may be determined by formulae as in Figure. However, many beams are manufactured with standard section such as steel beams. British Standard, the properties of standard steel beams and joists have been given in the standard code.

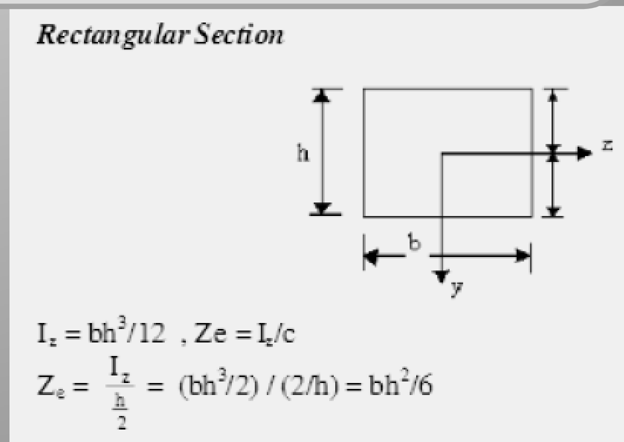


- **Elastic Section Modulus, Z**

  - The section modulus (Ze) is usually quoted for all standard sections and practically is of greater use than the second moment of area. Strength of the beam sections depends mainly on the second modulus. The section moduli of several shapes are calculated below



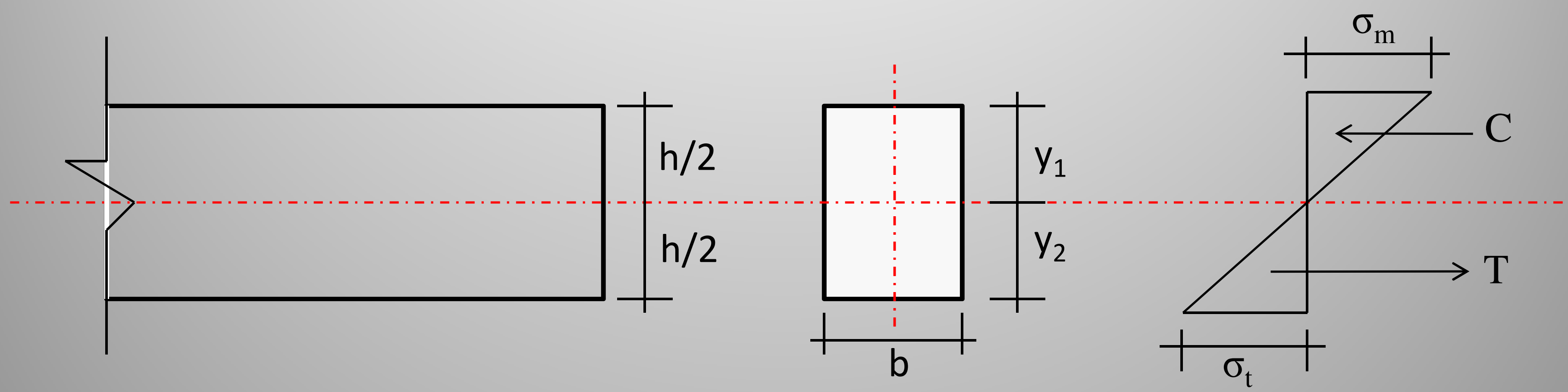




Maximum stress in beam

Negative bending moment

Positive bending moment



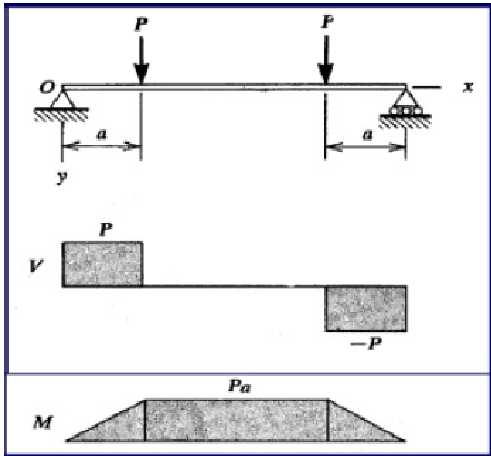
Maximum bending stress on cross section occurred at the lowest and highest part of the beam, in which y_{max}
is the farthest distance from Neutral Axis (NA)

Stress in compression $$\sigma_{top} = \sigma_{comp} = \frac{-My_{max}}{I} = Mc/I$$

Stress in tension $$\sigma_{bottom} = \sigma_{tension} = \frac{My_{max}}{I} = Mc/I$$

Pure Bending

- Pure bending
  - is a condition of stress where a bending moment is applied to a beam without the simultaneous application of **axial, shear**, or **torsional forces**.
  - Beam that is subjected to pure bending means the shear force in the particular beam is **zero**, and **no torsional** or **axial loads** .
  - is also the **flexure (bending)** of a beam that under a constant *bending moment (M)* therefore pure bending only occurs when the *shear force (V)* is equal to zero since dM/dx= V.



Pure bending in mid section (no shearing force)

- This equation indicates that for a given allowable stress and a max bending moment the section modulus Ze must not be less than the ratio (Mmax/σallowable).
- If the allowable stress of the material in tension is the same as in compression the use of a section which is symmetrical about the neutral axis is preferred and the material from which the beam is made should be ductile

$$\sigma_{allowable} \geq \frac{M_{max}}{Z_e}$$

Example 1

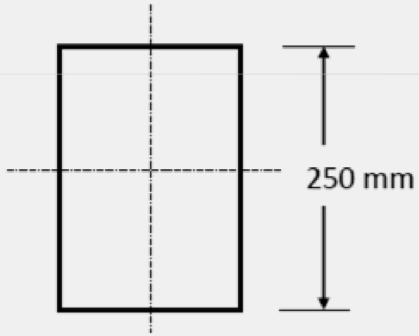
A 250 mm (depth) x 150 mm (width) rectangular beam is subjected to maximum bending moment of 750 kNm , find;

i. The maximum stress in the beam
ii. The value of the longitudinal stress at a distance of 65 mm from the top surface of the beam.

Solution:

i. maximum stress in the beam;

$$\sigma = \frac{My}{I} = {}^{Mc}/_{I}$$



Distance of N.A. from the top surface of the beam;

$$y = \frac{h}{2} = \frac{250}{2} = 125\ mm = 0.125\ m$$

Moment of inertia, *I*

$$I = \frac{bh^3}{12} = \frac{150 \times 250^3}{12} = 195312500\ mm^4 = 0.0001953\ m^4$$

Using relationship

$$\sigma = \frac{My}{I} = \frac{750 \times 10^3 \times 0.125}{0.0001953} = 4.8 \times 10^8 Nm^{-2}$$

ii. longitudinal stress at a distance of 65 mm from the top surface

Using relationship

$$\sigma = \frac{My}{I} = \frac{My_1}{I}$$

Distance y = y1 = 60 mm = 0.06 m

$$\sigma = \frac{My_1}{I} = \frac{750 \times 10^3 \times 0.06}{0.0001953} = 230\ \boldsymbol{MNm^{-2}}$$

Tutorial

1) A beam has a rectangular cross section 80 mm wide x 100 mm deep. It is subjected to maximum bending moment of 15 kNm. The beam is made from metal that has a modulus of elasticity of 180 GPa. Calculate the maximum stress on the section.

2) A symmetrical section with 200 mm, deep has a moment of inertia of $2.26x10^5$ m^4 about its N.A. Find the longest span over which , when simply supported the beam would carry a uniformly distributed load 4 kN/m run without the stress due to bending exceeding 125 MN/m

3) Find the dimensions of a timber beam with length 8 m to carry a brick wall of 200 mm thick and 5 m high. If the density of brick work is1850 Kg/m^3 and the maximum permissible stress is limited to 7.5 MN/m^2 given that the depth of beam is twice the width.

Thanks

- END -