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Lecture No. (1)

Design of Reinforced Concrete Structures

Syllabus:

- 1. Design of Continuous Beams and One Way Slabs.
- 2. Design of Two Way Edge's Supported Slabs.
- 3. Design of Flat Plate and Flat Slabs.
- 4. Design of One Way Ribbed Floor Slabs.
- 5. Design of Staircase.
- 6. Yield Line Theory for Slab Analysis.
- 7. Multistory Construction.
- 8. Precast Construction.
- 9. Prestressed Reinforced Concrete.
- 10.Design of Reinforced Concrete Bridges.

Note:

The ACI-Code recommendations will be adopted in designing the above structural elements.

References:

- 1. ACI Committee, Building Code Requirements for Structural Concrete (ACI 318M-11), an ACI Standard and Commentary, 2011.
- 2. Nilson, A.H., Darwin, D, Dolan, C.W., 2011, Design of Concrete Structures, McGraw Hill Higher Education, Boston.

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1- Introduction:

The general layout of frame multistory building is shown in Fig.(1)



Figure (1) The general layout of frame multistory building

The design of reinforced concrete frame structures may include the design of the following structural elements:

- a. Design of RC Slabs.
- b. Design of RC Beams.

c. Design of RC Columns.

Figure (2) showed the Reinforced Concrete Building Elements. Figure (3) showed several types of RC Slabs.

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Common types of floor systems: (a) Two way flat plate, (b) Two way waffle, (c) Two way flat slab with drops, (d) One-way beam and slab, (e) Skip joist wide module, (f) Two-way beam and slab, (g) One-way joist slab, and (h) One way flat slab.

Figure (3) Types of RC Slabs

<u>Slab Classifications:</u>

The arrangement of beams and floor system makes the possibility of forming several slab or plate types shown in Figures (3 & 4). If the ration between long to short span is more than two (Length/Width > 2) or the slab is supported by two edges only, then the loads applied to the roof and floor are assumed to be transmitted in one direction (in short direction), such slabs are referred to as one-way slab, as shown in Figure(4a&c). If ration between long to short span is less or equal to two (Length/Width \leq 2), the loads applied to the roof and the floor are assumed to be transmitted in two directions, such slabs are referred to as two way slabs, as shown in Figure(4b).



Figure (4) Arrangement of RC Slabs

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2- Design of Continuous Beams and One Way Slabs

Continuous beams and continuous one way slabs can be design according to ACI code provisions, by considering the following:

A) Effective Span Length:

- 1- Item 8.9.1: Span length of members <u>not built integrally</u> with supports shall be considered as the <u>clear span plus the depth of the</u> <u>member</u>, but need <u>not exceed</u> <u>distance between centers of</u> <u>supports</u>.
- 2- Item 8.9.3: For <u>beams built integrally with supports</u>, design on the basis of moments *at faces of support shall be permitted*.

B) Minimum Depth of Members

Table (1) Show the Minimum thickness of Non - prestressed beams or one-way slabs. Unless deflections are calculated should be as follow:

	Minimum thickness, h					
	Simply supported	One end continuous	Both ends continuous	Cantilever		
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections					
Solid one- way slabs <i>l</i> /20		<i>l</i> /24	ℓ/28	<i>ll</i> 10		
Beams or ribbed one- way slabs	<i>l</i> /16	ℓ/18.5	<i>ℓ</i> /21	<i>l</i> /8		

Notes:

Values given shall be used directly for members with normalweight concrete and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:

a) For lightweight concrete having equilibrium density, w_c , in the range of 1440 to 1840 kg/m³, the values shall be multiplied by $(1.65 - 0.0003w_c)$ but not less than 1.09.

b) For f_V other than 420 MPa, the values shall be multiplied by (0.4 + $f_V/700$).

C) Moment Calculations

The moment and shear value can be exactly determined by the analytical method as moment distribution or slop deflection, etc. According to the ACI Code (8.3):

An approximate moment and shear shall be permitted when the following *conditions are satisfied*:

- 1. There are two or more spans.
- 2. Spans are approximately equal, with the larger of two adjacent spans not greater than the shorter by more than 20 percent.
- 3. Loads are uniformly distributed.
- 4. Unfactored live load (L) does not exceed three times unfactored dead load, D.
- 5. Members are prismatic.

Note:

- For calculating negative moments, ln is taken as the average of the adjacent clear span lengths.

- For Positive moment use the clear span as the effective span. According to ACI – Code the ultimate moment can be calculated as: $M_{ultimate} = coefficient \times w_u \times l_n^2$

Positive moment

Coefficient

End spans

Discontinuous end unrestrained ($w_u \times l_n^2$)/	11
Discontinuous end integral with support ($w_u \times l_n^2$) /	14
Interior spans ($w_u \times l_n^2$) /	16

Negative moments

At exterior face of first interior support	
Two spans	$(w_u \times l_n^2) / 9$
More than two spans	$(w_u \times l_n^2) / 10$
At other faces of interior supports	$(w_u \times l_n^2) / 11$

At interior face of exterior support for members built integrally with supports

Where support is spandrel beam ($w_u \times l_n^2$) / 24 Where support is a column ($w_u \times l_n^2$) / 16

Where as, the restrained conditions is shown Figure (5):





Figure (6) Distribution of moment coefficients of one continuous way slab and beam.

<u>Note:</u> At face of all supports for interior slabs with span ≤ 3.0 m, and beams where ratio of sum of column stiffness to beam stiffness>8 at each end of the span. The <u>negative moment</u> coefficient is "(wu x ln²)/12"



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D) Shear Calculations

Shear coefficients shown in Figure (7) can be used, whereas;

- Shear in end members at face of the first interior support $(1.15w_u \times L_n)/2$
- Shear at face of all other supports($w_u \times l_n$) / 2



Figure (7) Shear coefficients of one continuous way slab and beams

Information Required in Design

In order to complete the design processes, the following information are required.

1. Ultimate Load Factors:

In all design processes the load combinations suggested by ACI codes to get the ultimate loads, listed below, will be adopted.

$$U = 1.4D$$

$$U = 1.2D + 1.6L + 0.5(Lr \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.6(Lr \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$$

$$U = 1.2D + 1.0W + 1.0L + 0.5(Lr \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S$$

$$U = 0.9D + 1.0W$$

$$U = 0.9D + 1.0E$$

<u>Where</u>: D is dead load, L is live load, Lr is roof load, S is snow load, R is rain load, W is wind load, H is soil lateral pressure and E is the effect of earthquake.

2. Factors used in design:

The values of kn, β 1, and ρ max are given in Table (2) below

fc'=	20	22	25	28	30	35	Mpa(N/mm2)
B1=	0.85	0.85	0.85	0.85	0.836	0.801	
kn=	5.06	5.57	6.33	7.09	7.50	8.46	Mpa
fy=	276	276	276	276	276	276	Mpa(N/mm2)
r max	0.022438	0.024682	0.028047	0.031413	0.033102	0.037003	
fc'=	20	22	25	28	30	35	Mpa(N/mm2)
B1=	0.85	0.85	0.85	0.85	0.836	0.801	
kn=	5.06	5.57	6.33	7.09	7.50	8.46	Mpa
fy=	345	345	345	345	345	345	Mpa(N/mm2)
r max	0.01795	0.019745	0.022438	0.02513	0.026482	0.029602	
fc'=	20	22	25	28	30	35	Mpa(N/mm2)
B1=	0.85	0.85	0.85	0.85	0.836	0.801	
kn=	5.06	5.57	6.33	7.09	7.50	8.46	Mpa
fy=	400	400	400	400	400	400	Mpa(N/mm2)
r max	0.015482	0.01703	0.019353	0.021675	0.022841	0.025532	

3. Reinforcement Steel Arrangements:

The required reinforcement can be arranged according to Figure (8).





Figure (8) Cutoff and bend point for bars approximately equal spans with uniformly distributed loads. (a) Cutoff bars. (b) Bend bars.

4. Minimum Reinforcement Criterion:

10.5.1-At every section of a flexural member where tensile reinforcement is required by analysis, *As* provided shall not be less than that given by:

$$As_{min} = \frac{0.25\sqrt{f'c}}{fy} bw.d$$

and not less than

$$As_{min} = \frac{1.4}{fy} bw.d$$

According to ACI Code, <u>minimum slab reinforcement</u> will be: **7.12.2.1**-Area of shrinkage and temperature reinforcement shall provide at least the following ratios of reinforcement area to gross concrete area, but not less than 0.0014:

(a) Slabs where Grade 280 or 350 deformed bars are used0.0020

(b) Slabs where Grade 420 deformed bars or welded wire reinforcement are used......0.0018

(c) Slabs where reinforcement with yield stress exceeding 420MPa measured at a yield strain of 0.35 percent is used $\frac{0.0018 \times 420}{f_V}$

5. Maximum Spacing Criterion:

7.12.2.2 - Shrinkage and temperature reinforcement shall be spaced not farther apart than *five times the slab thickness*, nor farther apart than 450mm.

10.5.4 — For structural slabs and footings of uniform thickness, *As,min* in the direction of the span shall be the same as that required by 7.12.2.1. Maximum spacing of this reinforcement shall not exceed three times the thickness, nor 450 mm.

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Example (1):

Using the given floor plan shown in Figure below, design the slab (S1) and beam (B2), to support uniformly distributed service live loads of $W_L=5kN/m^2$ and finishing load $W_F=4.2kN/m^2$, in addition to self-weight. $fc'=20N/mm^2$, $fy=400N/mm^2$. Assume beam width bw=300mm and all columns are 300mm x 300mm.



Two Way Edge Supported Slabs

The slabs discussed in the previous literature, deform under load into an approximately cylindrical surface. The main structural action is one-way in such cases, in the direction normal to supports on two opposite edges of a rectangular panel. In many cases, however, rectangular slabs are of such proportions and are supported in such a way that two-way action results. When loaded, such slabs bend into a dished surface rather than a cylindrical one. This means that at any point the slab is curved in both principal directions, and since bending moments are proportional to curvatures, moments also exist in both directions. To resist these moments, the slab must be reinforced in both directions, by at least two layers of bars perpendicular, respectively, to two pairs of edges. The slab must be designed to take a proportionate share of the load in each direction.



Figure (9) Two way slab on simple edge support. (a) Bending of center strip of slab. (b) Grid model of slab

Figure 9 shows the two center strips of a rectangular plate with short span l_a and long span l_b . If the uniform load is q per square foot of slab, each of the two strips acts approximately as a simple beam, uniformly loaded by its share of q. Because these imaginary strips actually are part of the same monolithic slab, their deflections at the intersection point must be the same. Equating the center deflections of the short and long strips gives

$$\frac{5q_al_a^4}{384EI} = \frac{5q_bl_b^4}{384EI}$$

where q_a is the share of the load q carried in the short direction and q_b is the share of the load q carried in the long direction. Consequently,

$$\frac{q_a}{q_b} = \frac{l_b^4}{l_a^4} \qquad \qquad q_a = \frac{l_b^4}{l_a^4}$$

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1 - For rectangular panel
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3 x 6m. $w_a = w_b \times (6^4 / 3^4) = 16 w_b$ $w_a >> w_b (16 >> 1)$ $M_a > M_b$ $As_a > As_b$ So, it's a one way slab

2 – For square panel

 $l_{a} = l_{b}$ $w_{a} = w_{b} = w/2$ $Mu = (w_{a} \times l^{2} / 8)$ $Mu = ((w/2) \times l^{2} / 8)$ $Mu = 0.0625 wl^{2}$

The exact theory of bending of elastic plates shows that actually the maximum moment in such a square slab is only $0.048ql^2$, so that in this case the twisting moments relieve the bending moments by about 25 percent.

The largest moment occurs where the curvature is sharpest. Figure (9) shows this to be the case at midspan of the short strip s_1 . Suppose the load is increased until this location is overstressed, so that the steel at the middle of strip s_1 is yielding. If the strip were an isolated beam, it would now fail. Considering the slab as a whole, however, one sees that no immediate failure will occur. The neighboring strips (those parallel as well as those perpendicular to s_1), being actually monolithic with it, will take over any additional load that strip s_1 can no longer carry until they, in turn, start yielding. This inelastic redistribution will continue until in a rather large area in the central portion of the slab all the steel in both directions is yielding. Only then will the entire slab fail. From this reasoning, which is confirmed by tests, it follows that slabs need not be designed for the absolute maximum moment in each of the two directions (such as $0.048ql^2$ in the example given in the previous paragraph), but only for a smaller average moment in each of the two directions in the central portion of the slab. For instance, one of the several analytical methods in general use permits a square slab to be designed for a moment of $0.036ql^2$. By comparison with the actual elastic maximum moment $0.048ql^2$, it is seen that, owing to inelastic redistribution, a moment reduction of 25 percent is provided.



Figure (10) Moments and moment variations in a uniformly loaded slab with simple supports on four sides.

A) Direct Design Method For Two Way Slabs (Semi-empirical Method)

This method can be used for design of two way slabs supported on beams or walls or without beams or walls support as in flat slab or flat plate construction, and it can be used under the following conditions:

- 1. There shall be a minimum of three continuous spans in each direction.
- 2. Panels shall be rectangular, with a ratio of longer to shorter span center-to-center of supports within a panel not greater than 2.
- 3. Successive span lengths center to center of supports in each direction shall not differ by more than one-third the longer span.
- Offset of columns by a maximum of 10 percent of the span (in direction of offset) from either axis between centerlines of successive columns shall be permitted.
- 5. All loads shall be due to gravity only and uniformly distributed over an entire panel. The unfactored live load shall not exceed two times the unfactored dead load.
- 6. For a panel with beams between supports on all sides, Eq. (13-2) shall be satisfied for beams in the two perpendicular directions.

$$0.2 \le \frac{\alpha_{f1} \ell_2^2}{\alpha_{f2} \ell_1^2} \le 5.0 \tag{13-2}$$

where α_{f1} and α_{f2} are calculated in accordance with Eq. (13-3).

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s} \tag{13-3}$$

Where: Ecb = modulus of elasticity of beam concrete, MPa,

Ecs = modulus of elasticity of slab concrete, MPa,

Ib = moment of inertia of gross section of beam about centroidal axis, mm⁴.

Is = moment of inertia of gross section of slab about centroidal axis, mm⁴.

Steps Required for Design by Direct Design Method

The direct design method consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously.

Three fundamental steps are involved as follows:

- (a) Determination of the total factored static moment;
- (b) Distribution of the total factored static moment to negative and positive sections;
- (c) Distribution of the negative and positive factored moments to the column and middle strips and to the beams, if any.

(a) Total factored static moment for a span

According to ACI Code,13.6.2.1, Total factored static moment, *Mo*, for a span shall be determined in a strip bounded laterally by centerline of panel on each side of centerline of supports.

According to ACI Code, 13.6.2.2, Absolute sum of positive and average negative factored moments in each direction shall not be less than

$$M_o = \frac{q_u l_2 l_n^2}{8}$$

Where ln: is length of clear span in direction that moments are being determined. For purpose of calculating the total static moment M_0 in a panel, the clear span (ln) in the direction of moments is used. The clear span is defined to extend from face to face of the columns, column capitals, brackets, or walls but not to be less than $(0.65 \ lc/c)$. The total factored moment in a span (M_0) , given in the above equation, for a strip bounded laterally by a centerline of the panel on each side of the centerline of the supports.

Note: For irregular column shape, use equivalent square column.



Examples of equivalent square section for supporting members.

(b) Distribution of the total static moment (Mo) to (M^{-ve}) and (M^{+ve})

moment sections

i) Interior Span:

In an interior span, total static moment, Mo, shall be distributed as follows, as shown in Figure (11):

Negative factored moment.....0.65

Positive factored moment0.35

ii) <u>Exterior Span:</u>

For end span spans, the apportionment of total static moment among the three critical moment sections (interior negative, positive, and exterior negative, as illustrated in Figure (11)), depends upon the flexural restrained provided for the slab by the exterior column or exterior wall. The total static moment is distributed according to Table (2).



Fig. (11) Distribution of total static moment (Mo) to critical sections for positive and negative bending



Figure (12) Conditions of Edge Restrained

Table (2) Distribution factors applied to static moment (Mo) for positive and
negative moments in end span

•								
· · · · · · · · · · · · · · · · · · ·	(a)	(b)	(c)	(d)	(e)			
	Exterior Edge Unrestrained	Slab with Beams between All Supports	Slab without Beams between Interior Supports		Exterior Edge			
			Without Edge Beam	With Edge Beam	Fully Restrained			
Interior negative moment	0.75	0.70	0.70	0.70	0.65			
Positive moment	0.63	0.57	0.52	0.50	0.35			
Exterior negative moment	0	0.16	0.26	0.30	0.65			

(c) Distribution of the total negative and total positive factored moments to the column and middle strips and beam.

It's assumed that the moment in middle strip and the moment in the column strip is constant unless there is a beam in the column strip, because the beam will take larger share of moments in column strip.

The distribution of moment will depends on ratio l_2/l_1 , $\alpha_1 \cdot l_2/l_1$, and the degree of torsional restrained by edge beam (M^{-ve} at exterior support) depends on the parameter (β), as shown in Table (3).

 I_2/I_1 0.5 1.0 2.0 Interior negative moment 75 75 75 $\alpha_{f1}l_2/l_1 = 0$ 90 75 45 $\alpha_{f1} l_2 / l_1 \ge 1.0$ Exterior negative moment 100 100 $\beta_r = 0$ 100 $\alpha_{f1}l_2/l_1 = 0$ 75 75 $\beta_i \ge 2.5$ 75 100 100 100 $\beta_{i} = 0$ $\alpha_{f1} l_2 / l_1 \ge 1.0$ 45 $\beta_i \ge 2.5$ 90 75 Positive moment 60 60 60 $\alpha_{f1} l_2 / l_1 = 0$ $\alpha_{l_1} l_2 / l_1 \ge 1.0$ 90 75 45

Table (3) Column Strip moment percentage of total moment at critical section

Where:

 l_1 is the span in direction of moment.

 l_2 is the span in the other direction.

$$\alpha = \frac{\text{flexural stiffness of beam}}{\text{flexural stiffness of slab}} = \frac{E_{cb}.I_{b}}{E_{cs}.I_{s}}$$

$$\beta 1 = \frac{E_{cb}.C}{2E_{cs}.I_{s}}$$

 $C=2I_b$.

 α_1 and α_2 is used to identify (α) computed in direction of (l_1) (bending span) and (l_2) (transverse span).

where I_s , as before, is calculated for the slab spanning in direction l_1 and having width bounded by panel centerlines in the l_2 direction. The constant C pertains to the torsional rigidity of the effective transverse beam, which is defined according to ACI Code 13.7.5 as the largest of the following:

- 1. A portion of the slab having a width equal to that of the column, bracket, or capital in the direction in which moments are taken
- 2. The portion of the slab specified in 1 plus that part of any transverse beam above and below the slab
- **3.** The transverse beam defined as in Fig. (13)

The constant C is calculated by dividing the section into its component rectangles, each having smaller dimension x and larger dimension y, and summing the contributions of all the parts by means of the equation

$$C = \sum \left(1 - 0.63 \, \frac{x}{y} \right) \frac{x^3 y}{3}$$

The subdivision can be done in such a way as to maximize C.



(a) Symmetric slab

(b) Single side slab

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Fig. (13) The Portion of Slab to be Included with Beam.



Using the interpolation charts for lateral distribution of slab moments, shown in Fig. (14), and Table (3), the following conclusions can be stated:

- When beam of $(\alpha_l > 1.0)$ between column centerline. The beam in the column strip spanning in direction of (l_1) is to be proportional to 85% of the column strip moment when $\alpha_1 \frac{l_2}{l} \ge 1.0$.
- When $\alpha_I = 0.0$, (no beam or flat slab), the slab in the column strip will take all the resisting moment.
- When $0.0 < \alpha_1 \frac{l_2}{l_1} < 1.0$ the resisting moment in the beam shall be



determined by proportions.

Figure (14) Interpolation charts for lateral distribution of slab moments.

✤ Minimum Slab Thickness of Two Way Edge Supported Slabs:

For slabs with beams spanning between the supports on all sides, the minimum thickness, h, shall be as follows:

- (a) For α_m equal to or less than 0.2, (Flat Slab requirements should be applied).
- (b) For α_m greater than 0.2 but not greater than 2.0 (0.2< $\alpha_m \le 2.0$), *h* shall not be less than:

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)}$$
 and not less than 125mm

(c) For α_m greater than 2.0 ($\alpha_m > 2.0$), *h* shall not be less than:

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$$
 and not less than 90mm

Where:

 $\beta = \frac{Clear \ Long \ Span}{Clear \ Short \ Span}$

 α_m average values of α for all beams supporting the slab panel.

$$\alpha m = \frac{\alpha 1 + \alpha 2 + \alpha 3 + \alpha 4}{4}$$

lu Clear span in long direction face to face of support in long

direction.

(d) Also minimum thickness requirements for two way slabs is given as:

 $h\min = \frac{\text{Clear Perimeters}}{180}$

(If slab is supported by beams from four sides)

***** <u>The strip width limitations according to ACI-Code are given as;</u>

- Column strip is a design strip with a width on each side of a column centerline equal to 0.25.l₂ or 0.25.l₁, whichever is less. Column strip includes beams, if any.
- Middle strip is a design strip bounded by two column strips.
- A **panel** is bounded by column, beam, or wall centerlines on all sides.

* Shear in slab systems with beams

- Beams with $\alpha_1 . l_2 / l_1$ equal to or greater than 1.0 shall be proportioned to resist shear caused by factored loads on tributary areas which are bounded by 45-degree lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides, as shown in Figure (15).
- Beams with α₁.1₂/l₁ between (1.0 and 0.0), (shallow beams or flat slab), proportion of load on the slab carried by beam is to be found by linear interpolations, assuming beams carry no load at α₁= 0.
- The remaining ratio of load to be carried by beam.



Figure (15) Tributary area for shear on an interior beam.

Special Corner Reinforcement:

- Corner reinforcement shall be provided for a distance in each direction from the corner equal to one-fifth the longer span.
- Corner reinforcement shall be placed parallel to the diagonal in the top of the slab and perpendicular to the diagonal in the bottom of the slab (Fig. 16, Option 1). Alternatively, reinforcement shall be placed in two layers parallel to the sides of the slab in both the top and bottom of the slab (Fig. 16, Option 2).



Figure (16) Special Reinforcement at Exterior Corners of a Beam Supported Two-Way Slab.

Example (2):

Using the given floor plan shown in Figure below, design the two way exterior slab (6.0m x 7.5m c/c) between shallow beams (300mm x 500mm), to support uniformly distributed service live loads of $W_L=6 \ kN/m^2$. $fc'=25 \ N/mm^2$, $fy=300N/mm^2$, $Kn=6.33N/mm^2$.

