University of Mosul

College of Engineering

Civil Engineering Department

Physics

First level

Autumn course

Course Preparation

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2020-2021

Update rate equal 10%

Physics and Measurement

Like all other sciences, physics is based on experimental observations and quantitative measurements. The main objectives of physics are to identify a limited number of fundamental laws that govern natural phenomena and use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

1. Standards of Length, Mass, and Time:

In mechanics, the three fundamental quantities are length, mass and time. All other quantities in mechanics can be expressed in terms of these three. In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the **SI** (Système International), and its fundamental units of length, mass, and time are the meter, kilogram, and second, respectively. Other standards for SI fundamental units established by the committee are those for temperature (the kelvin), electric current (the ampere), luminous intensity (the candela), and the amount of substance (the mole), see Table 1.

Table 1: Fundamental (principle) dimension and their units in SI.

Quantity	length	mass	time	temperature	elec. current	amount of matter	amount of light
Unit Name	meter	kilogram	second	kelvin	ampere	mole	candle
Unit Symbol	m	kg	s	K	A	mol	cd

In addition to SI, another system of units, the U.S. customary system, is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the foot (ft), slug, and second, respectively.

In addition to the fundamental SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes *milli-* and *nano-* denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 2.

Table 2: Prefixes for Powers of Ten.

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	y	10^{3}	kilo	k
10^{-21}	zepto	z	10^{6}	mega	M
10^{-18}	atto	a	10^{9}	giga	G
10^{-15}	femto	f	10^{12}	tera	T
10^{-12}	pico	p	10^{15}	peta	P
10^{-9}	nano	n	10^{18}	exa	E
10^{-6}	micro	μ	10^{21}	zetta	Z
10^{-3}	milli	m	10^{24}	yotta	Y
10^{-2}	centi	c			
10^{-1}	deci	d			

The variables length, time, and mass are examples of fundamental quantities. Most other variables are derived quantities, those that can be expressed as a mathematical combination of fundamental quantities. Common examples are area (a product of two lengths) and speed (a ratio of a length to a time interval).

Another example of a derived quantity is density. The density ρ (Greek letter rho) of any substance is defined as its mass per unit volume:

$$\rho = \frac{m}{V}$$

In terms of fundamental quantities, density is a ratio of a mass to a product of three lengths.

2. Dimensional Analysis:

In physics, the word dimension denotes the physical nature of a quantity. The distance between two points, for example, can be measured in feet or meters, which are all different ways of expressing the dimension of length. The symbols we use in this lecture to specify the dimensions of length, mass, and time are L, M, and T, respectively. We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed is v, and in our notation, the dimensions of speed are written $[v]$ = L/T. As another example, the dimensions of area A are $[A] = L^2$. The dimensions and units of area, volume, speed, and acceleration are listed in Table 3.

Table 3: Dimensions and units of four derived quantities.

Quantity	Area (A)	Volume (V)	Speed (v)	Acceleration (a)	Density (ρ)
Dimensional	L^2	L^3	L/T	L/T^2	M/L^3
SI Units	m^2	m^3	m/s	m/s^2	kg/m^3
U.S. customary units	ft^2	ft^3	ft/s	ft/s^2	Ib/ft^3

Example:

Show that the expression $v = at$, where v represents speed, a acceleration, and t an instant of time, is dimensionally correct.

$$v = \frac{L}{T}$$

$$a = \frac{L}{T^2}T = \frac{L}{T}$$

Scalar and Vector Quantities

1. Scalars:

Many quantities in geometry and physics, such as area, volume, temperature, mass, and time, can be characterized by a single real number that is scaled to appropriate units of measure. These are called scalar quantities, and the real number associated with each is called a scalar. A scalar quantity is completely specified by a single value with an appropriate unit and has no direction.

Scalar quantities, require only a number (either positive or negative) and a unit for their description. For example, temperature, mass, length, area, volume, time, distance, speed, work, and energy are all scalars. The quantities listed above are usually represented by the symbols T, m, L, A, V, t, d, v, W, and E.

2. Vectors:

If you are preparing to pilot a small plane and need to know the wind velocity, you must know both the speed of the wind and its direction. Because direction is important for its complete specification, velocity is a vector quantity. A vector quantity is completely specified by a number with an appropriate unit (the magnitude of the vector) plus a direction.

Vector quantities, require a positive number, called a **vector magnitude**; a unit; and a direction for their description. For example, displacement, velocity, acceleration, force, momentum, angular momentum, impulse, and magnetic field are all vectors. The symbols for vector quantities are denoted using italic letters with arrows above them, bold letters, or bold letters with arrows above them. Throughout this textbook vector quantities will be denoted using bold italic letters with arrows above them, for example, $\vec{\boldsymbol{v}}$ for velocity, $\vec{\boldsymbol{p}}$ for momentum, $\vec{\boldsymbol{F}}$ for force, and $\vec{\boldsymbol{a}}$ for acceleration. The magnitude of a vector is always a positive scalar quantity and can be denoted using the absolute value sign or by using the same letter as the vector without the arrow above it and in light face italics. For example, the magnitude of the velocity vector $\vec{\boldsymbol{v}}$, also called speed, can be denoted as either $|\vec{\boldsymbol{v}}|$ or v.

To describe vectors we need a **frame of reference** and a coordinate system. For example, a two-dimensional (2-D) coordinate system can consist of an x-axis and a y-axis, as shown in Fig. 2. A three-dimensional (3-D) coordinate system has an additional axis, the z-axis. In Fig. 2, the velocity vector $\vec{\boldsymbol{v}}_1$ has a magnitude of 30 m/s ($v_1 = 30\ m/s$) and a direction of 25° above the x-axis ($\theta_1 = 25°$); the second velocity, $\vec{\boldsymbol{v}}_2$, has a magnitude of 15 m/s ($v_2 = 15\ m/s$) and a direction of $\theta_2 = -10° = 350°$.

A 2-D vector can be described in terms of its magnitude and the angle it makes with the positive x-axis measured in the counterclockwise direction. Such a description is referred to as a polar notation or polar coordinates. For example, the polar notation for vector $\vec{\boldsymbol{v}}_1$ in Fig. 2 is 30 m/s with a direction of 25° or 30 m/s [25°].

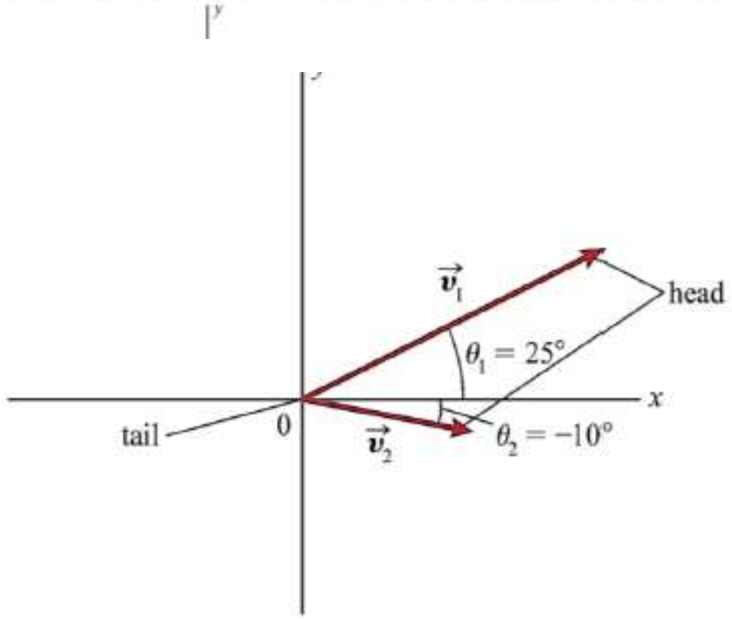


Fig. 2: Graphical representation of two 2-D velocity vectors. Given: $v_1 = 30$ m/s, $\theta_1 = 25°$, $v_2 = 15$ m/s, and $\theta_2 = -10° = 350°$.

In addition to polar coordinate systems, we will also use orthogonal coordinate systems, where the coordinate axes are perpendicular to each other. The word orthogonal means directed at right angles. You might encounter the term rectangular coordinate systems as well. Using orthogonal coordinate systems, vectors can also be described in terms of their components, which are scalar quantities. A 2-D vector has two components, v_x and v_y, as shown in Fig. 3. These components are projections of the vector onto the x- and y-axes, respectively, so vector components can be positive, negative or zero:

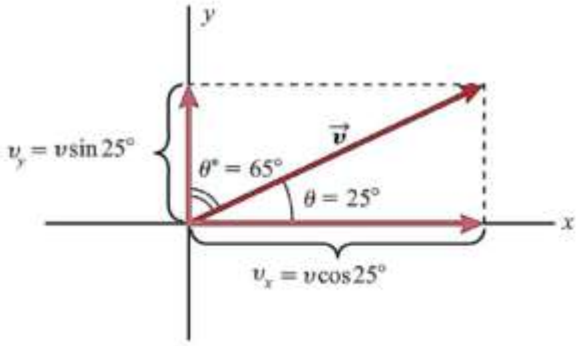


Fig. 3: Vector components of velocity vector $\vec{v}$ (30 m/s [25°]).

$$\begin{cases} v_x = v cos\theta \\ v_y = v sin\theta \end{cases} \Rightarrow \begin{cases} v = \sqrt{v_x^2 + v_y^2} \\ tan\theta = \dfrac{v_y}{v_x} \quad or \quad \theta = tan^{-1}\left(\dfrac{v_y}{v_x}\right) \end{cases}$$

where $|\vec{v}| \equiv v$ and the appropriate value of $\theta = tan^{-1}\left(\frac{v_y}{v_x}\right)$ can be determined from knowing the quadrant in which the vector is located.

Using the Pythagorean theorem, we can see that the magnitude of a vector is equal to the square root of the sum of the squares of its components. All angles are measured in a counterclockwise direction from the positive x-axis. However, it is some-times convenient to use the complementary angle θ^*, which measures the angle clockwise from the vertical y-axis, so that $\theta^* = 90° - \theta$. Hence,

$$\begin{cases} v_x = v sin\theta^* \\ v_y = v cos\theta^* \end{cases} \Rightarrow \begin{cases} v = \sqrt{v_x^2 + v_y^2} \\ tan\theta^* = \dfrac{v_x}{v_y} \end{cases}$$

Kinematics

Kinematics is the study of motion, which allows us to predict how an object will move, where it will be at a certain time, when it will arrive at a certain location, or how long it will take to cover a certain distance. In other words, in kinematics, we analyze how an object's position, velocity, and acceleration relate to one another, and how they change with time.

1. Position, Displacement and Distance:

To describe motion, we have to define the concepts of position, displacement, and distance.

1.1. Position:

Consider an object moving in one dimension. We denote the *position coordinate* of the center of mass of the object with respect to the choice of origin by $x(t)$. The position coordinate is a function of time and can be positive, zero, or negative, depending on the location of the object. The position has both direction and magnitude, and hence is a vector (Fig. 4). We denote the position coordinate of the center of the mass at $t = 0$ by the symbol $x_0 = x\ (t = 0)$. The SI unit for position is the meter [m].

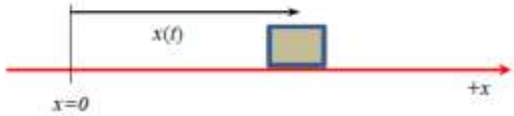


Fig. 4: The position vector, with reference to a chosen origin.

1.2. Displacement:

The change in position coordinate of the mass between the times t_1 and t_2 is

$$\Delta\vec{x} \equiv x(t_2) - x(t_1)$$

This is called the displacement between the times t_1 and t_2 (Fig. 5). Displacement is a vector quantity.

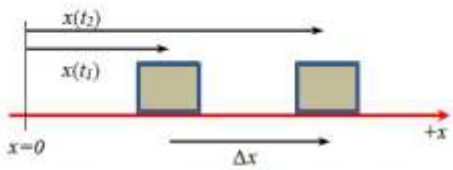


Fig. 5: The displacement vector of an object over a time interval is the vector difference between the two position vectors.

1.3. Distance:

It is very important to recognize the difference between displacement and distance traveled. Distance is the length of a path followed by a particle. Let's consider an example to help differentiate the terms distance and displacement. You are sitting at your desk at home, your coffee machine indicates a fresh brew is ready, you walk 10 steps to fill your coffee mug, and then you bring it back to your desk. Your displacement in this case is zero, since your final position is the same as your initial position. However, the distance you have covered, there and back, is a total of 20 steps.

For one-dimensional motion, that displacement can take on positive or negative values. The distance, on the other hand, is a positive scalar representing the actual distance travelled.

2. Speed, Velocity and Acceleration:

As we discussed earlier, one of the main goals of kinematics is to describe how the object's position changes with time. We can do this by plotting an $x(t)$ graph. Let us imagine a person walking along a straight line oriented in the positive x-direction. As shown in Fig. 6, If a person walks at a constant pace (Fig. 6(a)), the distances between his adjacent positions on the diagram will be equal. However, when he is speeding up, the distances between his adjacent positions will continuously increase (Fig. 6(b)). Fig. 6(c) shows the case when the person is slowing down; in this case the distances will decrease. The motion diagram illustrates the pace of the person's motion. In physics, to describe the pace, or the rate of change of the object's position, we use the concepts of velocity and speed.

Speed is a scalar quantity describing how fast an object is moving, while velocity is a vector indicating not only how fast an object is moving, but also where it is headed (the direction of its motion).

Average speed (a scalar) is the distance covered by the object divided by the time.

Average velocity (a vector) is the displacement divided by the elapsed time.

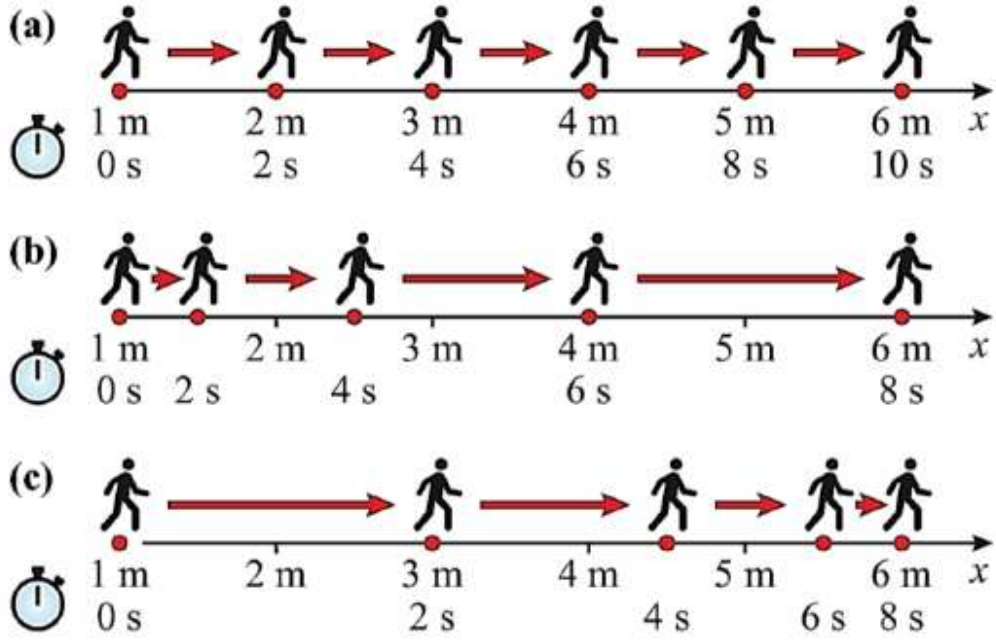


Fig. 6: Motion diagrams including average velocity vectors representing the one-dimensional motion of a person walking in the positive x-direction: (a) Walking at a constant pace. (b) Speeding up. (c) Slowing down.

For motion along the x-axis, with initial position x_0 and final position x, the average speed and the average velocity are given by the following expressions, where t_0 and t are the initial and final clock readings, respectively:

$$Average\ speed\ (v_{avg}) = \frac{d}{\Delta t} = \frac{d}{t - t_0}$$

$$Average\ velocity\ (v_{x,avg}) = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0}$$

Here d is the distance, a scalar quantity that is always positive; Δx is the displacement, a vector quantity that can take on positive or negative values in one-dimensional motion; and Δt is the elapsed time. When the two cars are moving in opposite directions with the same speed, their velocities will be equal in magnitude but opposite in sign. The SI units for both speed and velocity are metres per second (m/s), but in everyday life it is often convenient to use kilometres per hour (km/h) or miles per hour (mph or mi/h).

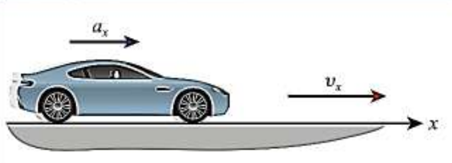
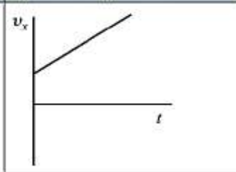
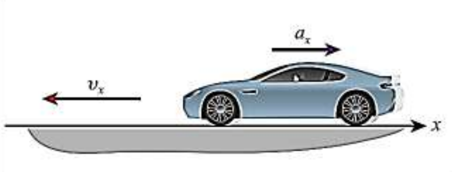
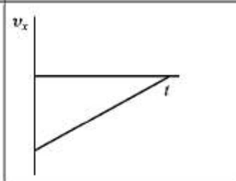
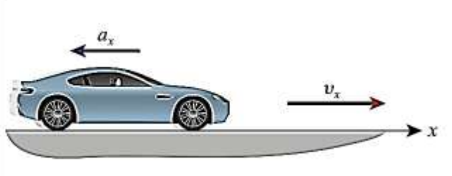
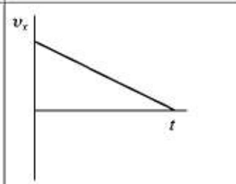
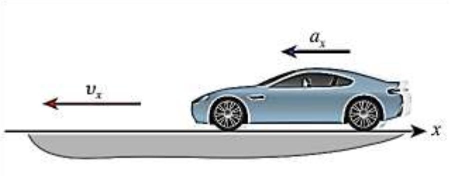
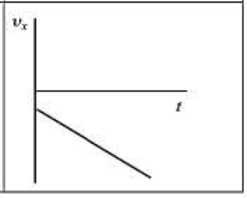
Just as velocity represents the rate of change of position in time, acceleration represents the rate of change of velocity in time. Since acceleration indicates by how much the velocity is changing every second, its units are metres per second per second, or m/s^2.

The average acceleration of an object is the change in its velocity divided by the elapsed time:

$$Average\ acceleration\ (a_{x,avg}) = \frac{\Delta v_x}{\Delta t} = \frac{v_x - x_{x,0}}{t - t_0}$$

The relative directions of an object's velocity and acceleration indicate whether the object is speeding up, slowing down, or moving at a constant velocity. These relationships for one-dimensional motion are summarized in Table 4. The right column shows $v_x(t)$ graph for the case of constant acceleration.

Table 4: The Effect of the Relative Direction of Velocity and Acceleration on the Object's Motion in the Case of Constant Acceleration

Visual Representation of Motion	Description of Motion	v_x	a_x	$v_x(t)$, when a_x = const
	Speeding up in the positive x-direction	+	+	
	Slowing down in the negative x-direction	−	+	
	Slowing down in the positive x-direction	+	−	
	Speeding up in the negative x-direction	−	−	

Forces and Motion

1. Mass and Gravity Force:

The attractive gravitational force, $\vec{F}_g$, that Earth exerts on an object of mass m is given by:

$$\vec{F}_g = m\vec{g}$$

Here m is the mass of the object and $\vec{g}$ is the acceleration due to gravity. Near the surface of the earth, g = 9.81 m/s^2.

2. Newton's Laws of Motion:

English physicist Isaac Newton (1642–1726) developed three laws that provide the basis for the dynamics of mechanical situations. These three laws of motion now carry his name.

2.1. Newton's First Law:

If no net force is exerted on an object, the object's velocity will not change. The net force refers to the vector sum of all the forces acting on that object.

2.2. Newton's Second Law:

The acceleration of an object depends inversely on its mass and directly on the net applied force. Newton's second law is normally presented in equation form as:

$$\vec{F}_{net} = \sum_{all\,forces} \vec{F} = m\vec{a}$$

2.3. Newton's Third Law:

The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction.

$$\vec{F}_{1\to2} = -\vec{F}_{2\to1}$$

$$or \quad \vec{F}_{12} = -\vec{F}_{21}$$

The third law is illustrated in Fig. 7. The force that object 1 exerts on object 2 is popularly called the *action force,* and the force of object 2 on object 1 is called the *reaction force.*

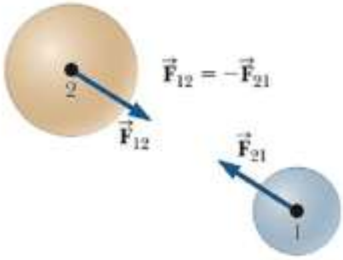


Fig. 7: Newton's third law. The force $\vec{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{F}_{21}$ exerted by object 2 on object 1.

3. Spring Forces and Hooke's Law:

Consider the spring–mass system of Fig. 8. In position (a), the mass is in the **equilibrium position**, where the spring is unstretched and exerts no force on the mass. If you move the mass to the right to stretch the spring by a displacement $\vec{x}$ from the equilibrium position, the spring will pull the mass with a force opposite in direction to the displacement, as shown in Fig. 8(b). When the mass is moved to the left, as in Fig. 8(c), the compressed spring will push the mass to the right, again in the opposite direction to the extension of the spring.

The Hooke's law can be expressed as:

$$\vec{F} = -k\vec{x}$$

where k is the spring constant, $\vec{F}$ is the force exerted by the spring on the mass, and $\vec{x}$ is the extension or compression of the spring from the unstretched equilibrium state.

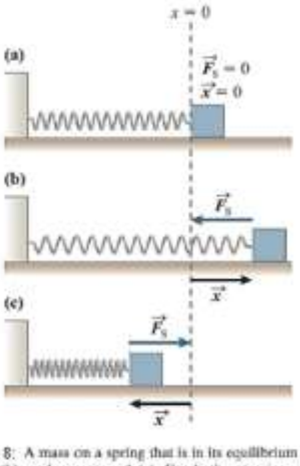


Fig. 8: A mass on a spring that is in its equilibrium state (a), stretched (b), and compressed (c). For both extension and compression, the force on the mass is in the opposite direction to the displacement

4. Friction Forces:

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a **force of friction.**

If we apply an external horizontal force $\vec{F}$ to the book, acting to the right, the book remains stationary when $\vec{F}$ is small (see Fig. 9(a)). The force on the book that counteracts $\vec{F}$ and keeps it from moving acts toward the left and is called the **force of static friction $\vec{\mathbf{f}}_s$**. As long as the book is not moving, $f_s = F$. Therefore, if $\vec{F}$ is increased, $\vec{\mathbf{f}}_s$ also increases. Likewise, if $\vec{F}$ decreases, $\vec{\mathbf{f}}_s$ also decreases.

If we increase the magnitude of $\vec{F}$ as in Fig. 9(b), the book eventually slips. When the book is on the verge of slipping, f_s has its maximum value $f_{s,max}$ as shown in Fig. 9(c). When F exceeds $f_{s,max}$, the book moves and accelerates to the right. We call the friction force for an object in motion the **force of kinetic friction $\vec{\mathbf{f}}_k$**. When the book is in motion, the force of kinetic friction on the can is less than $f_{s,max}$ (Fig. 9(c)). The net force $F - f_k$ in the x direction produces an acceleration to the right, according to Newton's second law. If $F = f_k$, the acceleration is zero and the trash can moves to the right with constant speed.

The magnitude of the force of static friction between any two surfaces in contact can have the values

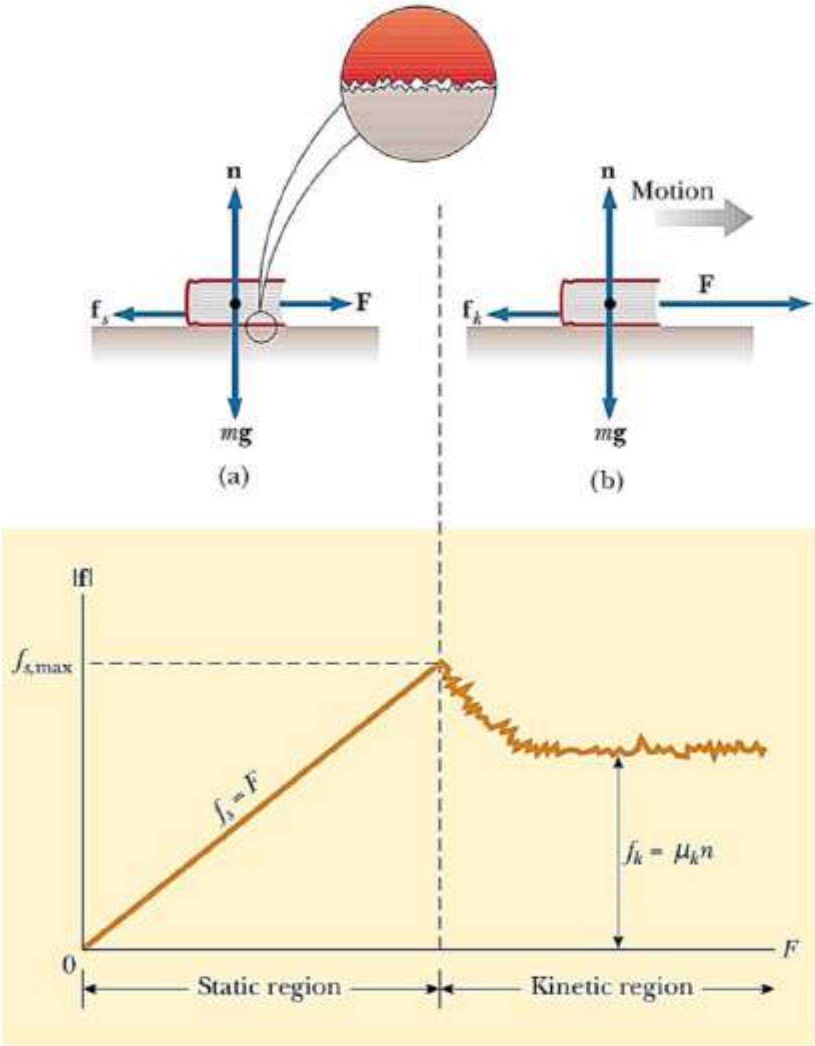
$$f_s \leq \mu_s n$$

where the dimensionless constant μ_s is called the **coefficient of static friction** and n is the magnitude of the normal force exerted by one surface on the other.

The magnitude of the force of kinetic friction acting between two surfaces is

$$f_k \leq \mu_k n$$

where μ_k is the **coefficient of kinetic friction.**



n
Motion
F
f_s
f_k
mg
(a)
(b)
|f|
$f_{s,\max}$
$f_s = F$
$f_k = \mu_k n$
0
F
Static region
Kinetic region

Fig. 9: The direction of the force of friction **f** between a book and a rough surface is opposite the direction of the applied force **F**. Because the two surfaces are both rough, contact is made only at a few points, as illustrated in the "magnified" view. (a) The magnitude of the force of static friction equals the magnitude of the applied force. (b) When the magnitude of the applied force exceeds the magnitude of the force of kinetic friction, the book accelerates to the right. (c) A graph of frictional force versus applied force. Notice that $f_{s,max} > f_k$.

5. Uniform Circular Motion:

An object moving with uniform speed v along a circular trajectory (path) of radius r has a radially inward **centripetal acceleration** with a magnitude of

$$a_r = \frac{v^2}{r}$$

If we apply Newton's second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated as:

$$\sum F_r = ma_r = m\frac{v^2}{r}$$

According to Newton's second law, an object moving in a circular path must be experiencing a net force that points toward the centre of the circle as well. This net force is the vector sum of all the forces acting on the object and it must point radially inward.

Consider a ball of mass m that is tied to a string of length r and moves at constant speed in a horizontal, circular path as illustrated in Fig. 10. Its weight is supported by a frictionless table. Why does the ball move in a circle? According to Newton's first law, the ball would move in a straight line if there were no force on it; however, the string prevents motion along a straight line by exerting on the ball a radial force F_r that makes it follow the circular path. This force is directed along the string toward the center of the circle.

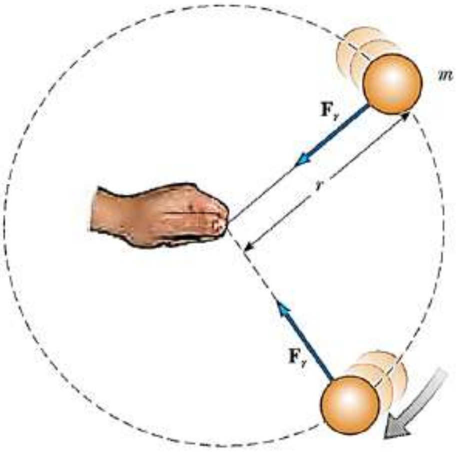


Fig. 10: Overhead view of a ball moving in a circular path in a horizontal plane. A force $\mathbf{F}_r$ directed toward the center of the circle keeps the ball moving in its circular path.

Work

If a force is applied to a body, which then moves, we say the force does work.

Work is the product of the force exerted on an object and the distance through which the point of action of the force moves.

Let us start with the simplest situation of a rigid body on a frictionless surface that moves a distance d in a straight line under the influence of a force of constant magnitude, F, that is acting parallel to the direction of motion of the object (see Fig. 11). Since the object is rigid, each point of the object moves by the same distance. In one dimension, if the force is constant with magnitude F, and the body moves a distance d, the work done is:

Work done by a force on an object = Force applied × Distance the object moves

$$W_F = \pm Fd$$

Fig. 11: Force $\vec{F}$ pushing an object over a distance $\vec{d}$.

The SI unit for work is the joule (J), which equals the product of a newton (units of force) and a metre (unit of displacement).

1 J = 1 N.m

$(Note\text{: } 1\ newton\ =\ 1\ \frac{kg.m}{s^2}, \quad so\ 1\ \text{joule}\ =\ 1\ \frac{kg.m^2}{s^2})$

Energy

If a body has the capacity (or ability) to do work we say it has energy. The energy of the body is the amount of work it can do. In other words, energy is anything that can be converted into work; i.e., anything that can exert a force through a distance.

When the body does some work it uses up some of its energy. But if work is done on the body its energy increases. Energy comes in many different forms like mechanical, thermal, light, electrical, magnetic, sound and nuclear; but we consider only mechanical energy.

Mechanical energy is the capacity of doing work that a body possesses by virtue of its motion (kinetic energy) or by virtue of its position (potential energy). Mechanical energy is of two types:

(a) Kinetic energy – ability to do work by virtue of motion.

Suppose a particle of mass m is accelerated from rest to velocity v in a distance x by a constant force F. Here $u = 0$, so

$$v^2 = 0 + 2ax$$

But also $F = ma$, so

$$v^2 = 2\frac{F}{m}x$$

$$\therefore\ Fx = \frac{1}{2}mv^2$$

Force times distance is work done, so the work done in getting to speed v from speed 0 is $\frac{1}{2}mv^2$. This is called the kinetic energy of the particle, since if we now reverse the process the particle can do this amount of work in slowing down to rest.

$$\text{kinetic energy (KE)} = \frac{1}{2}mv^2$$

Note: Kinetic Energy *cannot* be negative. Mass can't be negative and even if velocity is negative, it is square and the square of a negative number is positive.

The work-kinetic energy theorem:

The work-kinetic energy theorem states that if work is done on a system by external forces and the only change in the system is in its speed.

Consider a system consisting of a single object. Fig. 12 shows a block of mass m moving through a displacement directed to the right under the action of a net force $\sum \boldsymbol{F}$, also directed to the right. We know from Newton's second law that the block moves with an acceleration **a**. If the block moves through a displacement $\Delta r = \Delta x\hat{\mathbf{i}} = (x_f - x_i)\hat{\mathbf{i}}$, the work done by the net force $\sum \boldsymbol{F}$ is

$$\sum W = \int_{x_i}^{x_f} \sum \boldsymbol{F}\, dx$$

$$\sum W = KE_f - KE_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

For a nonisolated system, we can equate the change in the total energy stored in the system to the sum of all transfer of energy across the system boundary. For an isolated system, the total energy is constant-this is a statement of conservation of energy.

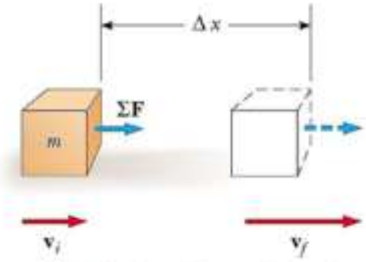


Fig. 12: An object undergoing a displacement $\Delta\mathbf{r} = \Delta x\hat{\mathbf{i}}$ and a change in velocity under the action of a constant net force $\Sigma\mathbf{F}$.

(b) Potential energy – ability to do work by virtue of position.

i) Gravitational potential energy:

Gravitational potential energy is the energy stored in an object as the result of the elevation of that object. Suppose we lift a book of mass m from height 0 to height h (see Fig. 13). The force needed is mg, and the distance moved is h, so the work done is mgh.

Again, if we reverse the process the particle can do this amount of work in coming down (very slowly!).

$\therefore$ Gravitational potential energy (PE_g) (due to gravity) $= mgh$

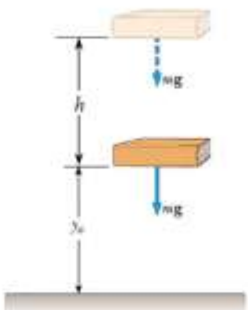


Fig. 13: The work done by an external agent on the system of the book and the earth.

ii) Elastic Potential Energy:

Elastic potential energy is the energy stored in an object due to the temporary deformation of that object.

$$\text{Elastic potential energy } (\text{PE}_e) = \frac{1}{2}kx^2$$

Spring Constant, k, usually in N/m, is how much force it takes to compress or expand the spring per meter.

x is displacement from equilibrium position (or rest position). Equilibrium position (or rest position) is where the force of the spring equals zero.

• Because k can't be negative and x is squared, PE_e can never be negative.

• Like Kinetic Energy and Gravitational Potential Energy, PE_e is a scalar.

Conservation of Total Mechanical Energy

If no work is done on a body, then its energy is unchanged. We say:

If the total work done by external forces acting on a body is zero, there is no change in the total mechanical energy of the body, this is called the principle of conservation of mechanical energy.

$$W = \Delta KE + \Delta PE$$

In other word, energy cannot be created or destroyed (i.e. it is "conserved"). It can only be changed from one form to another.

We can graph the mechanical energies as a function of time (see Fig. 14):

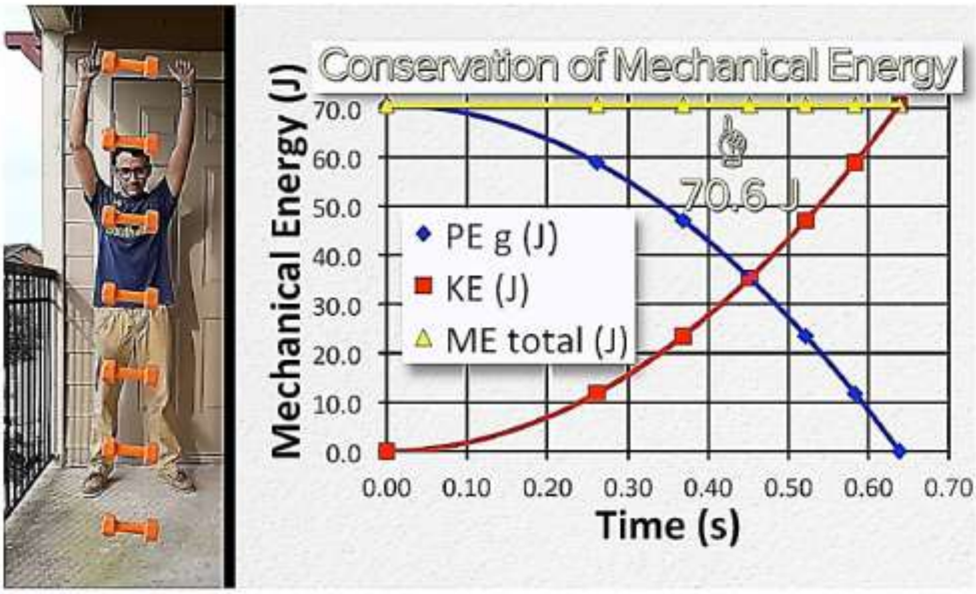


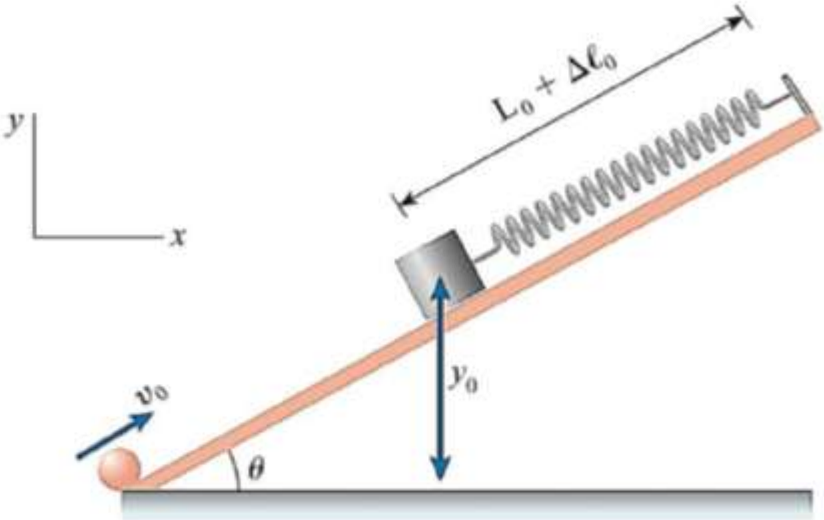
Fig. 14: The change of mechanical energy with time.

Consider the system shown in Fig. 15. A mass M, attached to a spring, is resting on an inclined ramp. The other end of the spring is fixed to the top end of the slope. A ball of mass m is at rest at the bottom of the slope. We choose a coordinate system with the positive y-axis pointing vertically up, and for this discussion we assume the ball and the block to be point objects. Let the unstretched length of the spring be L_0 and the change in length when the mass M is attached as shown be Δl_0. Also, let the initial height of mass M be y_0.

Now suppose that the ball is given an initial velocity v_0 that is sufficient for the ball to reach the top of the ramp in the absence of the mass–spring system. For this system, the initial kinetic energy is the kinetic energy of the ball and the potential energy is the sum of the gravitational potential energy of the block and the elastic potential energy of the stretched spring.

$$KE_i = \frac{1}{2}mv_0^2$$

$$PE_i = Mgy_0 + \frac{1}{2}k(\Delta l_0)^2$$



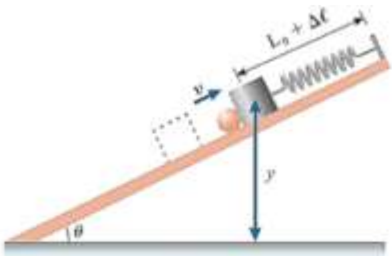


Fig. 15: A ball rolls up a ramp at velocity v_0 and hits a spring .

Suppose at a later time the ball strikes the block and pushes the block up the ramp. Let the height of the block at some instant be y, the change in the spring's length be Δl, and the speed of the ball be v. Since the ball and the block are in contact, the speed of the block is also v. At this instant, the kinetic energy is the sum of the kinetic energies of the ball and the block and the total potential energy is the sum of the gravitational potential energies of the ball and the block plus the elastic potential energy of the spring. Therefore,

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}Mv^2 = \frac{1}{2}(m+M)v^2$$

$$PE = mgy + Mgy + \frac{1}{2}k(\Delta l)^2 = (m+M)gy + \frac{1}{2}k(\Delta l)^2$$

Then, the law of conservation of energy states that

$$\frac{1}{2}mv_0^2 + Mgy_0 + \frac{1}{2}k(\Delta l_0)^2 = \frac{1}{2}(m+M)v^2 + (m+M)gy + \frac{1}{2}k(\Delta l)^2$$

Power

Power is defined as the rate at which work is done

$$P = Fv$$

where F is the force on a body, and v is its velocity. This definition applies even if the force and/or velocity are changing. If the force is constant then $W = Fx$ and

$$\frac{dW}{dt} = F\frac{dx}{dt} = Fv = P$$

so in this case the power is the 'rate of doing work'.

(One of the reasons why power is important in mechanics is that, for example, a car engine working at a fixed rate - at a fixed r.p.m. - generates (approximately) a fixed power; the force the engine generates will however vary with the speed of the car. As a car goes up a steep hill at constant power, it will slow down. As the velocity decreases the force produced by the engine will increase, until it is sufficient to maintain a constant (lower) velocity.)

The unit of power is joules/sec; this also has the name 'watt' (symbol W).

$$1\,watt = 1\,\frac{\text{joule}}{sec} = 1\,\frac{N.m}{s} = 1\frac{kg.m^2}{s^3}$$

(Note: 1 kW = 1000W.)

A unit power in the U.S. customary system is the horsepower (hp):

1 hp = 746 W

Linear Momentum

The linear momentum of an object is defined as the product of its mass and its velocity:

$$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$$

Momentum is a vector quantity because it equals the product of a scalar quantity, m, and a vector quantity, $\boldsymbol{v}$, and has units of kilogram metres per second ($\frac{kg.m}{s}$).

Momentum and Kinetic Energy:

From the definition of momentum, we get:

$$\vec{\boldsymbol{v}} = \frac{\vec{\boldsymbol{p}}}{m}$$

$$v^2 = \frac{p^2}{m^2}$$

If we substitute this into kinetic energy equation, we find that

$$K = \frac{1}{2}\frac{p^2}{m} \quad or \quad p = \sqrt{2mK}$$

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Rate of Change of Linear Momentum and Newton's Laws:

Taking the derivative of linear momentum Equation with respect to time gives:

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

If we consider an object whose mass does not change, then $\frac{dm}{dt} = 0$

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

We can combine the Newton's second law equation ($\vec{F}_{net} = m\vec{a}$) and above equation to write Newton's second law as:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Here we see that the net force is equal to the rate of change of linear momentum of the object with respect to time.

Law of Conservation of Linear Momentum:

Consider two particles 1 and 2 that can interact with each other but are isolated from their surroundings (Fig. 16). That is, the particles may exert a force on each other, but no external forces are present. It is important to note the impact of Newton's third law on this analysis. If an internal force from particle 1 (for example, a gravitational force) acts on particle 2, then there must be a second internal force-equal in magnitude but opposite in direction-that particle 2 exerts on particle 1.

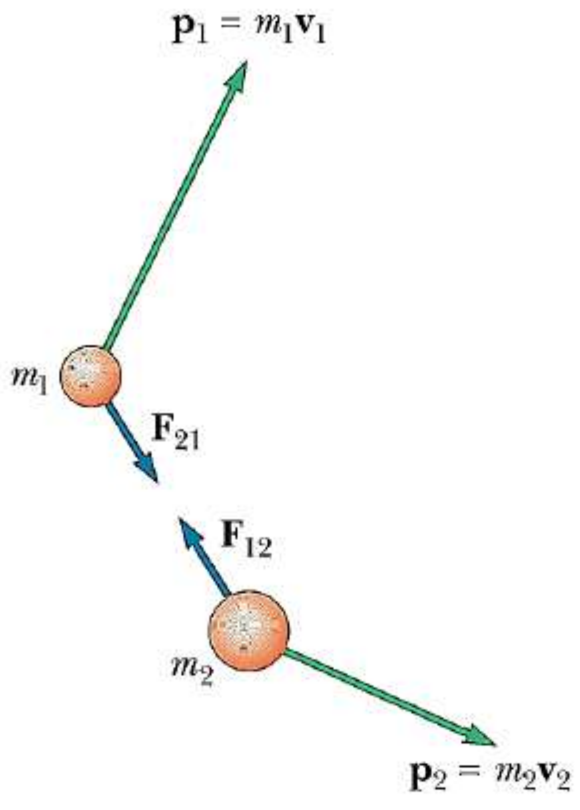

Fig. 16: At some instant, the momentum of particle 1 is $p_1 = m_1 v_1$ and the momentum of particle 2 is $p_2 = m_2 v_2$. Note that $\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}$. The total momentum of the system $\boldsymbol{p}_{tot}$ is equal to the vector sum $\boldsymbol{p}_1 + \boldsymbol{p}_2$.

Suppose that at some instant, the momentum of particle 1 is $\boldsymbol{p}_1$ and that of particle 2 is $\boldsymbol{p}_2$. Applying Newton's second law to each particle, we can write

$$\boldsymbol{F}_{21} = \frac{dp_1}{dt} \qquad and \qquad \boldsymbol{F}_{12} = \frac{dp_2}{dt}$$

where $\boldsymbol{F}_{21}$ is the force exerted by particle 2 on particle 1 and $\boldsymbol{F}_{12}$ is the force exerted by particle 1 on particle 2. Newton's third law tells us that $\boldsymbol{F}_{12}$ and $\boldsymbol{F}_{21}$ are equal in magnitude and opposite in direction. That is, they form an action-reaction pair $\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}$. We can express this condition as

$$\boldsymbol{F}_{21} + \boldsymbol{F}_{12} = 0$$

or as:

$$\frac{dp_1}{dt} + \frac{dp_2}{dt} = \frac{d}{dt}(\boldsymbol{p}_1 + \boldsymbol{p}_2) = 0$$

Because the time derivative of the total momentum $\boldsymbol{p}_{tot} = \boldsymbol{p}_1 + \boldsymbol{p}_2$ is zero, we conclude that the total momentum of the system must remain constant:

$$\boldsymbol{p}_{tot} = \sum_{system} \boldsymbol{p} = \boldsymbol{p}_1 + \boldsymbol{p}_2 = constant$$

or, equivalently, $\boldsymbol{p}_{1i} + \boldsymbol{p}_{2i} = \boldsymbol{p}_{1f} + \boldsymbol{p}_{2f}$

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where $\boldsymbol{p}_{1i}$ and $\boldsymbol{p}_{2i}$ are the initial values and $\boldsymbol{p}_{1f}$ and $\boldsymbol{p}_{2f}$ are the final values of the momenta for the two particles for the time interval during which the particles interact. This equation in component form demonstrates that the total momenta in the x, y, and z directions are all independently conserved:

$$\boldsymbol{p}_{1ix} + \boldsymbol{p}_{2ix} = \boldsymbol{p}_{1fx} + \boldsymbol{p}_{2fx} \qquad \boldsymbol{p}_{1iy} + \boldsymbol{p}_{2iy} = \boldsymbol{p}_{1fy} + \boldsymbol{p}_{2fy} \qquad \boldsymbol{p}_{1iz} + \boldsymbol{p}_{2iz} = \boldsymbol{p}_{1fz} + \boldsymbol{p}_{2fz}$$

This result, known as the **law of conservation of linear momentum**, can be extended to any number of particles in an isolated system. It is considered one of the most important laws of mechanics.

Impulse

The impulse $\boldsymbol{I}$ of a force is defined as the change in momentum $\Delta\boldsymbol{p}$ caused by that force.

From Newton's Second Law, if $\boldsymbol{F}$ is constant

$$\boldsymbol{F} = \frac{dp}{dt}$$

Then $\boldsymbol{I} = d\boldsymbol{p} = \boldsymbol{F}dt$

Integrating equation above to find the change in the momentum of a particle when the force acts over some time interval as:

$$\vec{I} = \Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}_{net}\, dt$$