# Mechanics of Dynamics

Definitions

# Mechanics of Dynamics

***Dynamics*** is that branch of mechanics which deals with the motion of bodies under the action of forces. The study of dynamics in engineering usually follows the study of statics, which deals with the effects of forces on bodies at rest.

# Dynamics has two parts

- ***kinematics*****,** which is the study of motion without reference to the forces which cause motion.

- ***kinetics*****,** which relates the action of forces on bodies to their resulting motions.

# Definitions

- ***Particle*** is a body of negligible dimensions.
- ***Time*** is the measure of the succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of static problems.
- ***Mass*** is the measure of the inertia or resistance to change in motion of a body.
- ***Force*** is the action of one body on another.
- ***Rigid body.*** A body is considered rigid when the change in distance between any two of its points is negligible.

# NEWTON'S LAWS

- ***Law I.*** A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.
- ***Law II.*** The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.
- ***Law III.*** The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

# Newton's second law

- ***Newton's second law*** forms the basis for most of the analysis in dynamics. For a particle of mass $m$ subjected to a resultant force $\mathbf{F}$, the law may be stated as:

$$\mathbf{F} = m\,\mathbf{a}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where $\mathbf{a}$ is the resulting acceleration measured in a nonaccelerating frame of reference

# Newton’s first and Third law

- Newton’s first law is a consequence of the second law since there is no acceleration when the force is zero, and so the particle is either at rest or is moving with constant velocity.
- The third law constitutes the principle of action and reaction with which you should be thoroughly familiar from your work in statics.

# UNITS

| QUANTITY | DIMENSIONAL SYMBOL | SI UNITS UNIT | SI UNITS SYMBOL |
|---|---|---|---|
| Mass | M | Base units: kilogram | kg |
| Length | L | Base units: meter* | m |
| Time | T | Base units: second | s |
| Force | F | newton | N |

*Also spelled *metre*.

# Apparent Weight

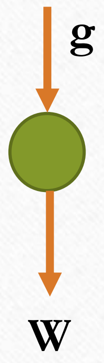
- The gravitational attraction of the earth on a body of mass *m* may be calculated from the results of a simple gravitational experiment. The body is allowed to fall freely in a vacuum, and its absolute acceleration is measured. If the gravitational force of attraction or true **weight of the body is *W***, then, because the body falls with an absolute acceleration ***g***, Eq. 2 gives

$$m = \mathbf{W} / \mathbf{g} \quad \rightarrow \quad \mathbf{W} = m\mathbf{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Sub Eq. 2 in Eq. 1

$$F = W.a/g \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$g = 9.8\ m/sec^2 \quad = 10\ m/sec^2$



# Rectilinear motion

- If we choose the $x$-direction, for example, as the direction of the rectilinear motion of a particle of mass $m$, the acceleration in the $y$- and $z$-directions will be zero and the scalar components of Eq. 1 become
- $F_x = ma_x$
- $F_y = ma_y$ ……………………………………………(4)
- $F_z = ma_z$

# Example 1

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension *T* in the hoisting cable is 8300 N. Find the reading *R* of the scale in newtons during this interval.

The total mass of the elevator, man, and scale is 750 kg.

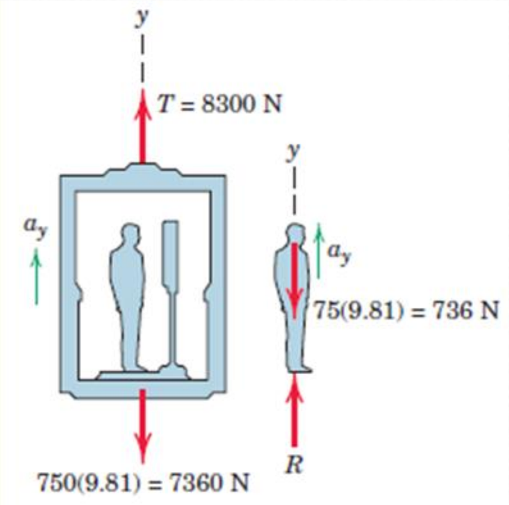



Figure 1

# Solution

The force registered by the scale depends on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

$$Fy = ma_y \qquad 8300 - 7360 = 750a_y \qquad a_y = 1.257 \text{ m/s2}$$

The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction *R* to this action is shown on the free-body diagram of the man alone together with his weight, and the equation of motion for him gives

$$Fy = ma_y \qquad R - 736 = 75(1.257) \qquad R = 830 \text{ N}$$

# Kinematics of Particles

Kinematics is the branch of dynamics which describes the motion of bodies without reference to the forces which either cause the motion or are generated as a result of the motion. Kinematics is often described as the "geometry of motion."

# Kinematics of Particles

- We can describe the motion of a particle in a number of ways, and the choice of the most convenient or appropriate way depends a great deal on experience and on how the data are given. Let us obtain an overview of the several methods developed in this chapter by referring to Fig. 2, which shows a particle *P* moving along some general path in space. If the particle is confined to a specified path, as with a bead sliding along a fixed wire, its motion is said to be ***constrained***. If there are no physical guides, the motion is said to be ***unconstrained.***

# Kinematics of Particles

The position of particle *P* at any time can be described by specifying its

1. rectangular coordinates *x*, *y*, *z*,
2. its cylindrical coordinates *r*, *θ*, *z*,
3. or its spherical coordinates *R*, *θ*,*Φ* .
4. The motion of *P* can also be described by measurements along the tangent *t* and normal *n* to the curve. The direction of *n* lies in the local plane of the curve.† These last two measurements are called *path variables*.

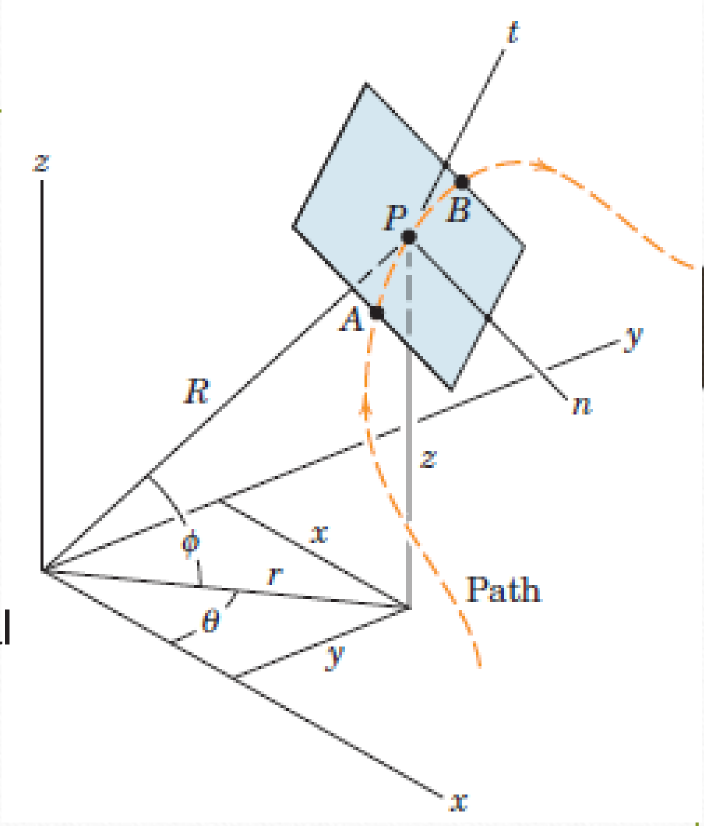


Figure 2

# plane motion

- In this study the case of *plane motion* will be considered, where all movement occurs in or can be represented as occurring in a single plane.
- We are going to deal with to type of plane motion:

1. Rectilinear motion
2. Curvilinear motion

# RECTILINEAR MOTION

- Consider a particle *P* moving along a straight line, Fig. 3.

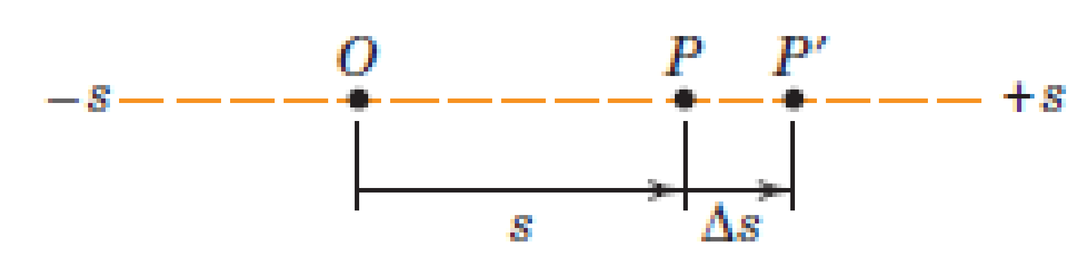



Figure 3

- At time $t + \Delta t$ the particle has moved to *P*′ and its coordinate becomes
- $s + \Delta s$. The change in the position coordinate during the interval $\Delta t$ is
- called the *displacement* $\Delta s$ of the particle.

# Velocity and Acceleration

- The average velocity of the particle during the interval $\Delta t$ is the displacement divided by the time interval.

  $v = ds/dt = \dot{s}$…………………………………5

  The average acceleration of the particle during the interval $\Delta t$ is the change in its velocity divided by the time interval.

  $a = dv/dt = \dot{v}$ or $a = d^2s/dt^2 = \ddot{s}$……………………………..6

# The sign of velocity and acceleration

- The velocity is positive or negative depending on whether the corresponding displacement is positive or negative.
- The acceleration is positive or negative depending on whether the velocity is increasing or decreasing. Note that the acceleration would be positive if the particle had a negative velocity which was becoming less negative.

# Equation relating displacement, velocity, and acceleration

- By eliminating the time *dt* from equations 5 and 6, we obtain a differential equation relating displacement, velocity, and acceleration. This equation is

$$v\,dv = a\,ds \quad or \qquad \dot{s}\,d\dot{s} = \ddot{s}\,ds \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

# Constant Acceleration.

- When $a$ is constant, the Eqs. 5 and 6 can be integrated directly. For simplicity with $s = s_0$, $v = v_0$, and $t = 0$ designated at the beginning of the interval, then for a time interval $t$ the integrated equations become

$$\int_{v_0}^{v} dv = a \int_0^t dt \qquad \text{or} \qquad v = v_0 + at$$

$$\int_{v_0}^{v} v\, dv = a \int_{s_0}^{s} ds \qquad \text{or} \qquad v^2 = v_0{}^2 + 2a(s - s_0)$$

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + at)\, dt \qquad \text{or} \qquad s = s_0 + v_0 t + \tfrac{1}{2}at^2$$

# Example 2

The position coordinate of a particle which is confined to move along a **straight line** is given by $s = 2t^3 - 24t + 6$, where $s$ is measured in meters from a convenient origin and $t$ is in seconds. Determine

(a) the time required for the particle to reach a velocity of 72 m /s from its initial condition at $t = 0$,

(b) the acceleration of the particle when $v = 30$ m /s, and

(c) the net displacement of the particle during the interval from $t = 1$ s to $t = 4$ s.

# Solution

**(a)** The velocity and acceleration are obtained by successive differentiation of $s$ with respect to the time. Thus,

$$v = \dot{s}\,, \qquad v = 6t^2 - 24 \text{ m/s}$$

$$a = \dot{v}\,, \qquad a = 12t \text{ m/s2}$$

Substituting $v = 72$ m /s into the expression for $v$ gives us $72 = 6t2 - 24$, from which $t = \pm 4$ s.

The negative root describes a mathematical solution for $t$ before the initiation of motion, so this root is of no physical interest. Thus, the desired result is

$$t = 4 \text{ s}$$

# Solution

- **(b)** Substituting $v$ = 30 m /s into the expression for $v$ gives $30 = 6t2 - 24$, from which the positive root is $t$ = 3 s, and the corresponding acceleration is

$$a = 12(3) = 36 \text{ m/s2}$$

# Solution

The net displacement during the specified interval is

$$\Delta s = s_4 - s_1 \text{ or}$$

$$\Delta s = (32(43) - 24(4) + 64) - (32(13) - 24(1) + 64)$$

$$= 54 \text{ m}$$

# Example 3

A particle moves along the $x$-axis with an initial velocity $v_x = 50$ ft /sec at the origin when $t = 0$. For the first 4 seconds it has no acceleration, and thereafter it is acted on by a retarding force which gives it a constant acceleration $a_x = -10$ ft /sec2.

Calculate the velocity and the $x$-coordinate of the particle for the conditions

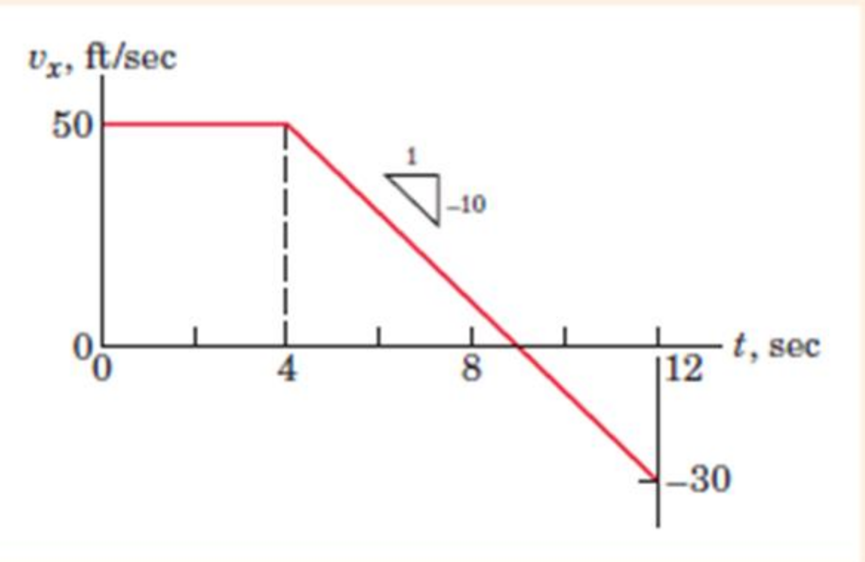
of $t = 8$ sec and $t = 12$ sec and find the maximum positive $x$-coordinate reached by the particle.

# Solution

$\int dv = \int a\,dt$ , $\int_{50}^{v_x} dv_x = -10 \int_4^t dt$, $v_x = 90 - 10t$ ft/sec

$t = 8$ sec, $v_x = 90 - 10(8) = 10$ ft/sec

$t = 12$ sec, $v_x = 90 - 10(12) = -30$ ft/sec



# Solution

$\int ds = \int v \, dt \qquad x = 50(4) + \int_4^t (90 - 10t)\, dt \qquad = -5t^2 + 90t - 80$ ft

For the two specified times,

$t = 8$ sec, $x = -5(82) + 90(8) - 80 = 320$ ft

$t = 12$ sec, $x = -5(122) + 90(12) - 80 = 280$ ft

# Solution

- The $x$-coordinate for $t = 12$ sec is less than that for $t = 8$ sec since the motion is in the negative $x$-direction after $t = 9$ sec. The maximum positive $x$-coordinate is, then, the value of $x$ for $t = 9$ sec which is
- $x_{max} = -5(9^2) + 90(9) - 80 = 325$ ft
- These displacements are seen to be the net positive areas under the $v$-$t$ graph up to the values of $t$ in question

# Example 4

- An airplane accelerates down a runway at 3.20 $m/s^2$ for 32.8 s until is finally lifts off the ground. Determine the distance traveled before takeoff.
- $a = +3.2\ m/s^2$
- $t = 32.8\ s$
- $v_i = 0\ m/s$
- $s = ??$

# solution

- $s = v_i*t + 0.5*a*t^2$
- $s = (0\ m/s)*(32.8\ s)+ 0.5*(3.20\ m/s^2)*(32.8\ s)^2$
- $s = 1720\ m$

# Example 5

- A car starts from rest and accelerates uniformly over a time of 5.21 seconds for a distance of 110 m. Determine the acceleration of the car.
- $s = 110$ m
- $t = 5.21$ s
- $v_i = 0$ m/s
- $a = ??$

# solution

- $s = v_i*t + 0.5*a*t^2$
- $110\ m = (0\ m/s)*(5.21\ s) + 0.5*(a)*(5.21\ s)^2$
- $110\ m = (13.57\ s^2)*a$
- $a = (110\ m)/(13.57\ s^2)$
- $a = 8.10\ m/\ s^2$