# *Structural Analysis*

by:

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Third Year

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# STRUCTURAL ANALYSIS

**Reference Book:**

- Hibbeler, R. C., "STRUCTURAL ANALYSIS", 8th ed. , Pearson Prentice Hall,2012.
- Kassimali, A. , " STRUCTURAL ANALYSIS",3ed ed. Nelson, a division of Thomson Canada Limited. 2005.
- Mau, S. T., " Fundamentals of Structural Analysis", The Library of Congress, 2003.
- مبادى نظرية المنشات، ترجمة د. عبدالحكيم حامد احمد، فريد نعموم مطلوب

## Syllabus:

- ***Introduction, Classification of Structures, Idealized Structure***
- ***Stability, Determinacy of Structures***
- ***statically determinat structures (Beam, Truss, Frame)***
- ***Deflection of structural analysis (Virtual work Method)***
- ***Moment Distribution Method***
- ***Moment Distribution Method with Side Sway***
- ***Influence line***

# Introduction:

A **structure** refers to a system of connected parts used to support a load Important examples related to civil engineering include buildings, bridges and towers. When designing a structure to serve a specified function for public use، the engineer must account for its safety, esthetics, and serviceability،while taking into consideration economic and environmental constraints.Often this requires several independent studies of different solutions before final judgment can be made as to which structural form is most appropriate.

***STRUCTURAL ANALYSIS:*** is the determination of the reactions, member forces, deformation of the structure and deflection of the joints due to applied loads and environmental effect.

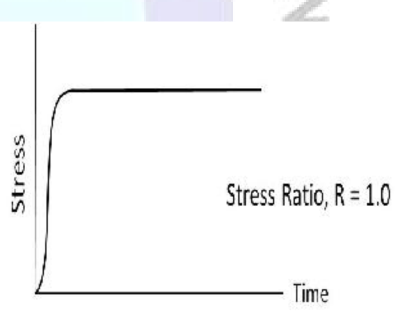
## *CLASSIFICATION OF STRUCTURES*

1- Dynamic structures, Static structures

depends on type of load applied to structures

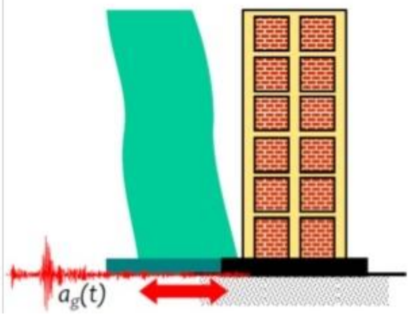
static load **(dead and live loads)**

(were loads remain constant with time)

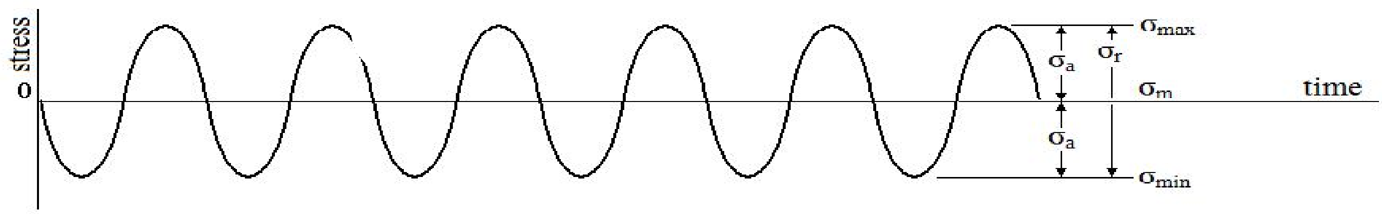



Dynamic load (load change with time) include:

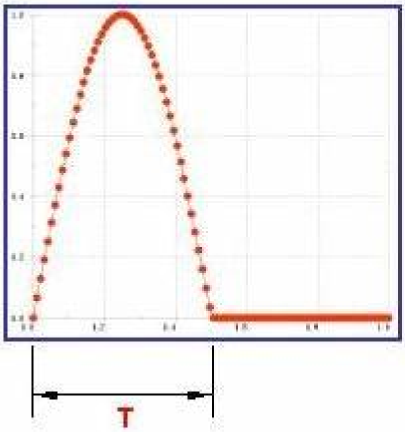
**earth quake load**



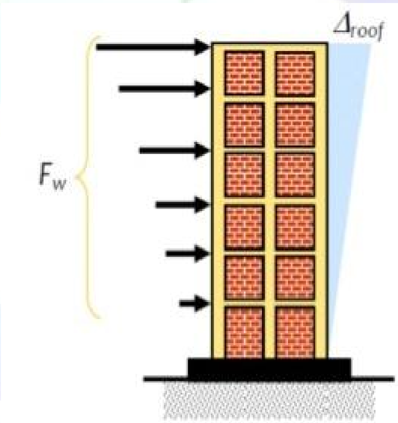

**cyclic load**



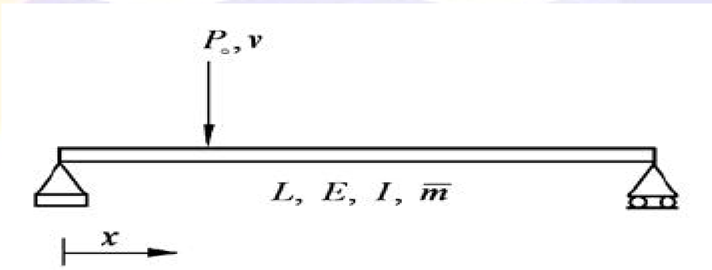
**, impact load** (increase of stress within short time)



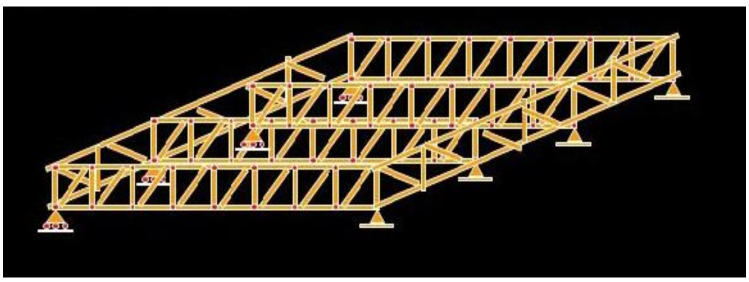

**,wind load,**

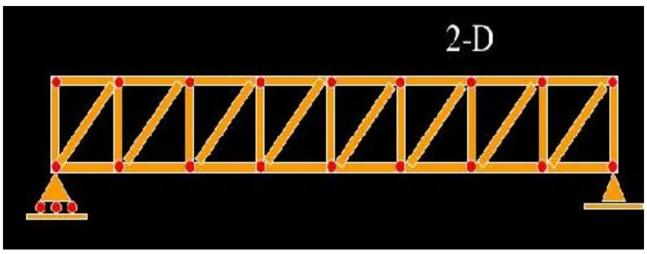



**moving load (**location of load change with time)

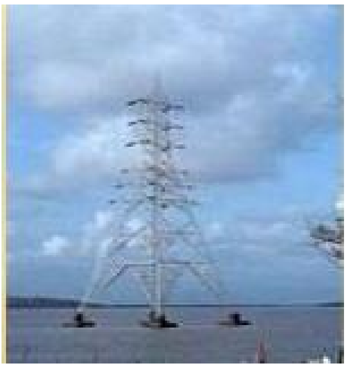



2- Space structures, Plane structures



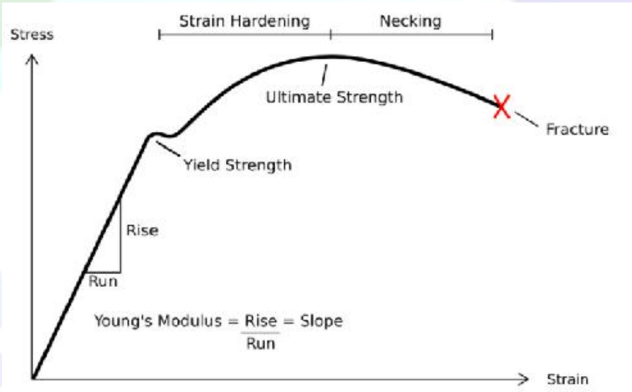


**space truss power transmission line tower**



3-Linear, non linear structures

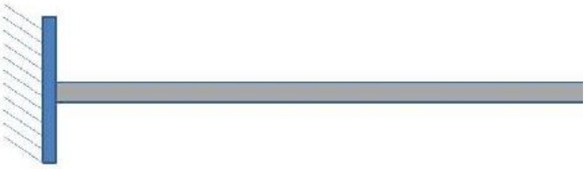
A linear static analysis is an analysis where a linear relation holds between applied forces and displacements. In practice, this is applicable to structural problems where stresses remain in the linear elastic range of the used material. A nonlinear analysis is an analysis where a nonlinear relation holds between applied forces and displacements. Nonlinear effects can originate from geometrical nonlinearity's (i.e. large deformations), material nonlinearity's (i.e. elasto-plastic material)




4- Determinate structures , non Determinate structures

In statics, a structure is statically indeterminate when the static equilibrium equations are insufficient for determining the internal forces and reactions on that structure. the equilibrium equations available for a two-dimensional body are

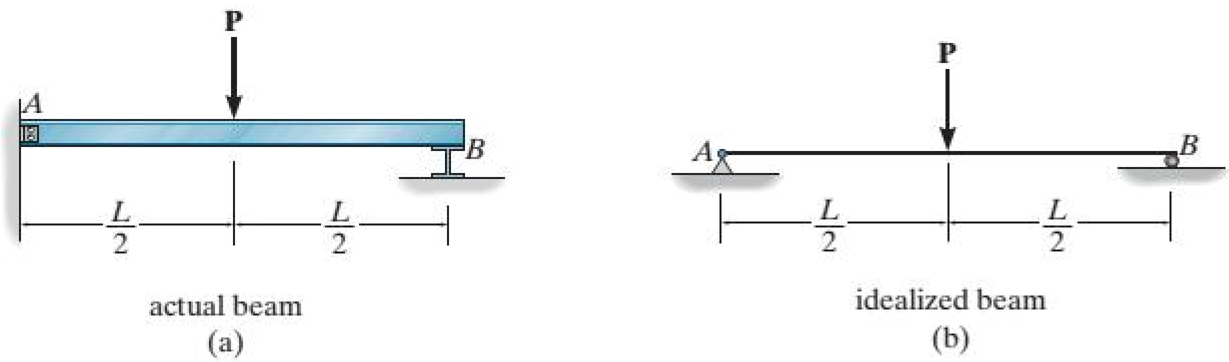
$\Sigma H = 0$: the sum of the horizontal components of the forces equals zero;
$\Sigma V = 0$: the sum of the vertical components of forces equals zero;
$\sum M$= the sum of the moments (about an arbitrary point) of all forces equals zero.



Fixed Beam

# Idealized Structure

An exact analysis of a structure can never be carried out, since estimates always have to be made of the loadings and the strength of the materials composing the structure. Furthermore, points of application for the loadings must also be estimated. It is important, therefore, that the structural engineer develop the ability to model or idealize a structure so that he or she can perform a practical force analysis of the members. In this section we will develop the basic techniques necessary to do this.

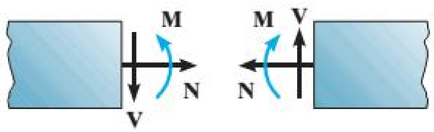



Type of structure to be studied

BEAM (structural element that primarily resists loads applied laterally to the beam's axis)

TRUSS (structure that consists of two-force members only )

FRAME (structural having flexural members)




## *STABILITY, DETERMINACY OF STRUCTURES*

## EQUILIBRIUM EQUATIONS

Any plane structure which is in a state of equilibrium under the action of an externally applied force system must satisfy the following three conditions:

•the sum of the horizontal components of all applied forces must equal zero

•the sum of the vertical components of all applied forces must equal zero

•the sum of the moments (about any point in the plane of the frame) of all applied forces must equal zero .

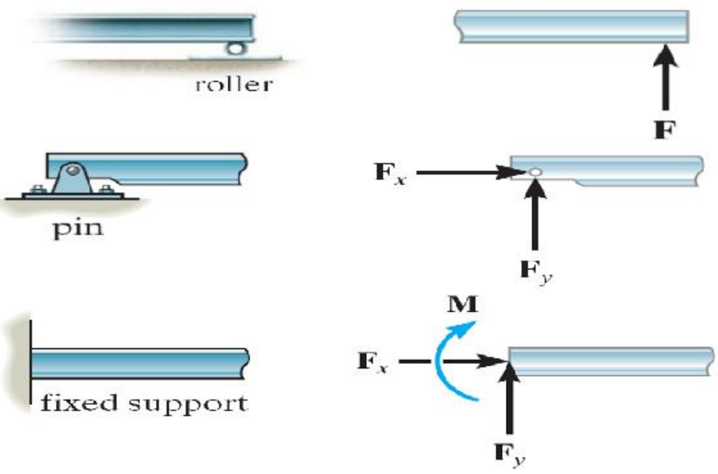
This is represented by the following 'three equations of static equilibrium'

Sum of the horizontal forces equals zero $\Sigma Fx=0$

Sum of the vertical forces equals zero $\Sigma Fy=0$

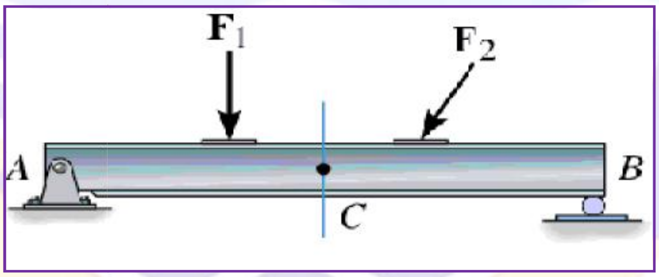
Sum of the moments about a point in the plane of the forces equals zero $\Sigma M=0$

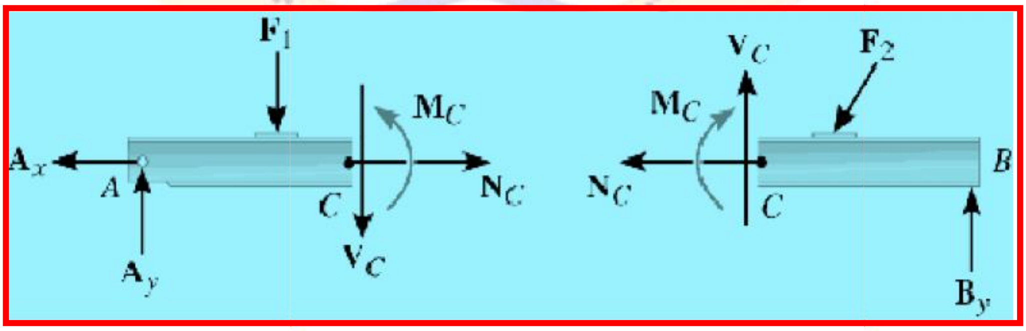
## SUPPORT REACTION

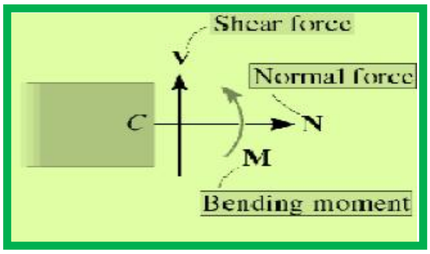



## INTERNAL FORCES

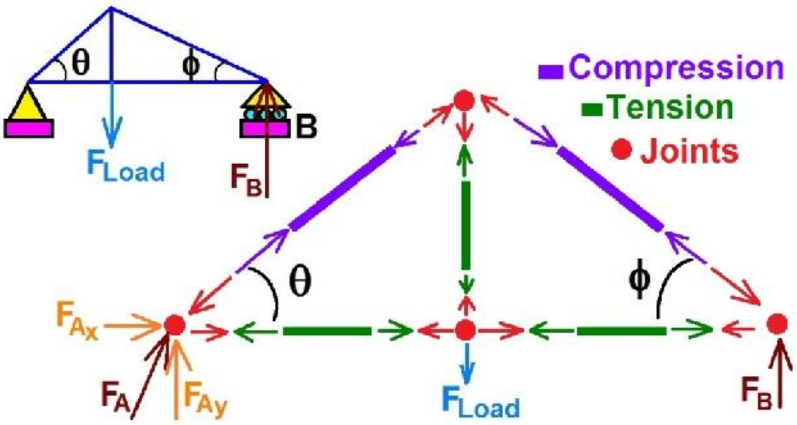
In beams and frames the members are flexural



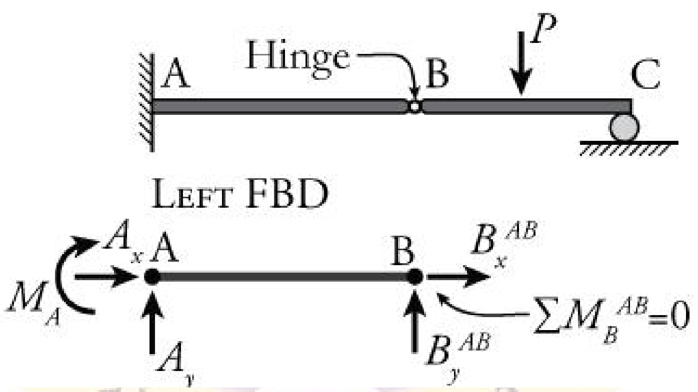




in truss the member having axial load

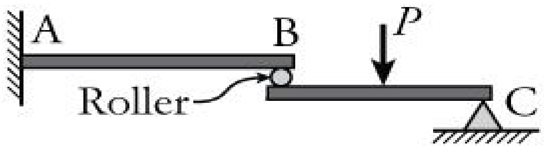



## CONDITION EQUATION

Additionally, c is the "number of equations of condition." These are release conditions within the structure that provide extra equilibrium equations beyond the three for global equilibrium. if an internal hinge is added to the structure, as shown, then there is one equation of condition. The addition of the hinge provides an additional equilibrium condition which forces the internal moment to be equal to 0 at point B ( $\sum MB=0$ ).



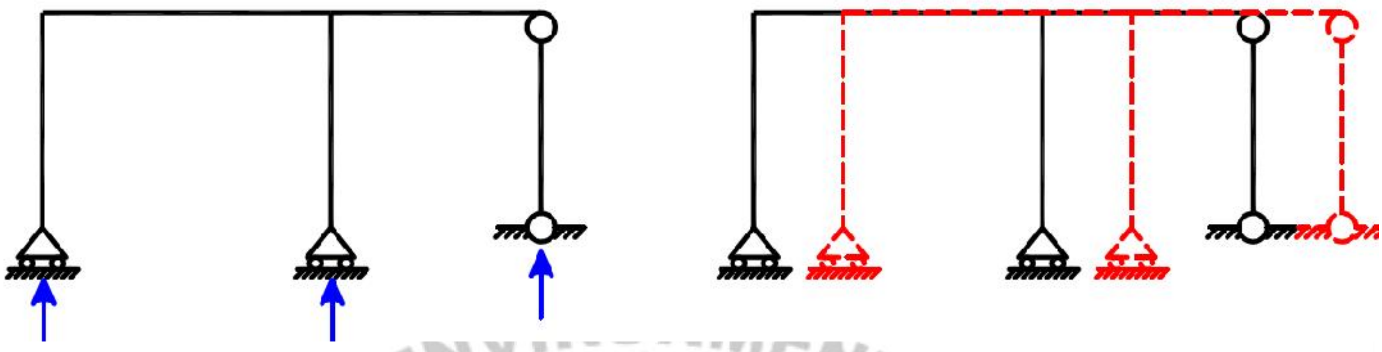

For a structure with an internal roller, both the force transfer in the direction of the roller and the moment are equal to zero at the location of the roller. This provides two extra equilibrium equations, and therefore two equations of condition. the extra equations are:



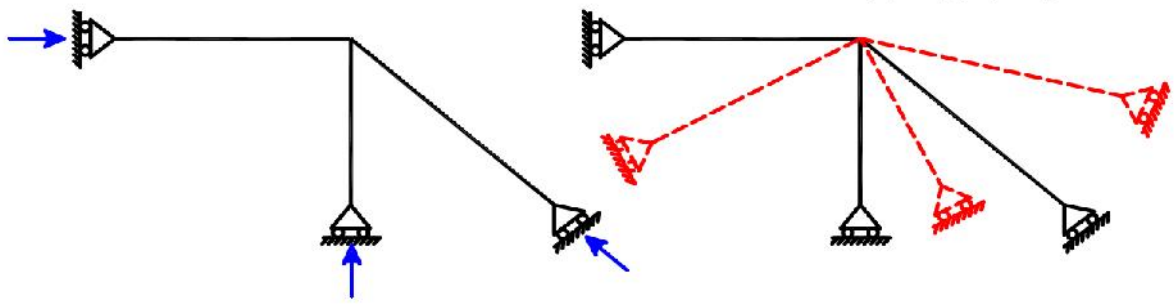

$\sum M@B=0$ and $\sum Bx=0$ .So, for each internal roller, there are two equations of condition: $c=2$.

## EXTERNAL STABILITY

A stable structure should have at least three reactive components, which are non-concurrent and non-parallel



(parllel reaction, Unstable)

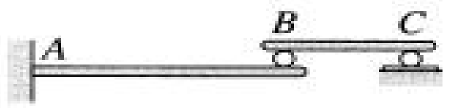
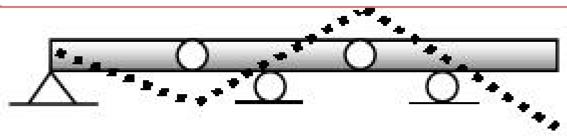
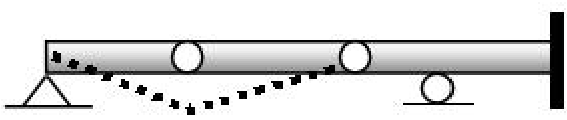
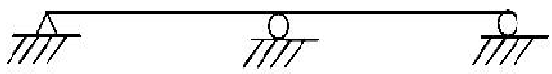


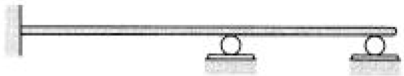
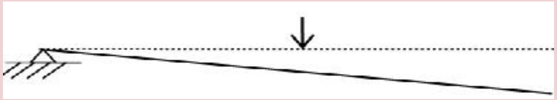
(concurrent reaction , unstable)

## ***BEAM***

**unknown** r =No. of reactions

**Equations** No. of Equations of equilibrium and condition equation= 3+c

| *beam* | *r* | *3+c* | *note* |
|---|---|---|---|
|  | 4 | 5 | Internally unstable |
|  | 5 | 5 | Stable and determinate |
|  | 4 | 5 | Internally unstable |
|  | 6 | 5 | Internally unstable |
|  | 4 | 3 | Stable and indeterminate to 1° |

| | | | |
|---|---|---|---|
|  | 5 | 3 | Stable and indeterminate to 2° |
|  | 1 | 3 | Externally unstable |

## ***TRUSS***

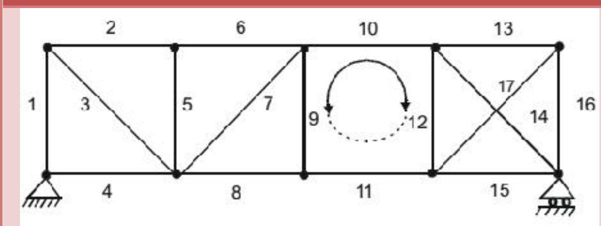
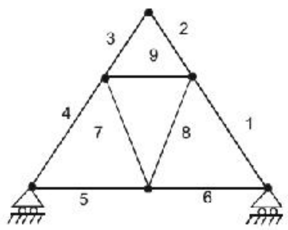
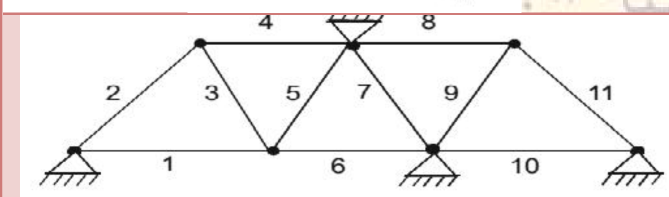
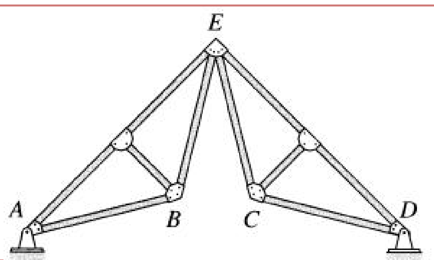
**unknown** r =No. of reactions, m= Total number of bars; *( m+ r )*

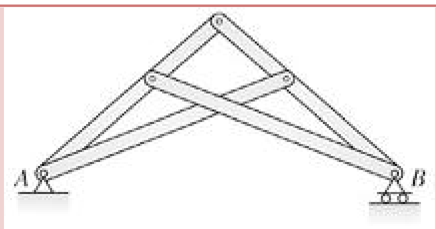
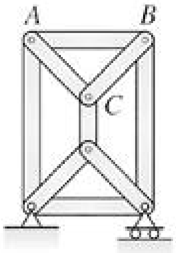
**Equations** 2j (**j = Total number of joints)**

**If m + r = 2 j Stable & internally determinate. Check the arrangement of members**
**If m + r > 2 j Stable & internally indeterminate degree of indeterminacy would be decided by the difference of these tow quantities**

**If m + r < 2 j Unstable.**

**A structure is said to have determinacy or indeterminacy only if it is stable**

| *truss* | *r+m* | *2j* | *note* |
|---|---|---|---|
|   | 20 | 20 | Internally unstable |
|   | 11 | 12 | Externally unstable |
|   | 19 | 14 | Stable and indeterminate to 5° |
|   | 14 | 14 | Stable and determinate |

| | | | |
|---|---|---|---|
|  | 9 | 10 | unstable |
|  | 12 | 12 | Unstable section ABC is supported by 3 parallel links |

## FRAME

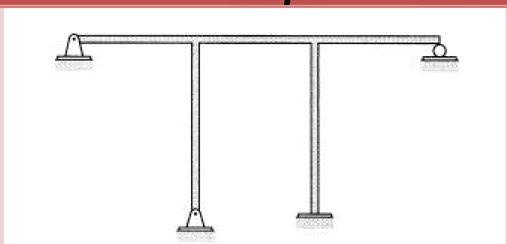
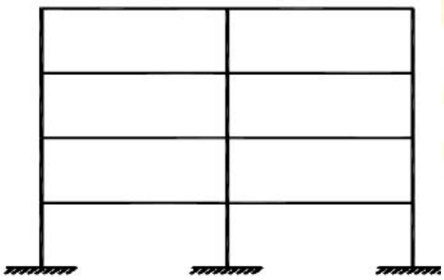
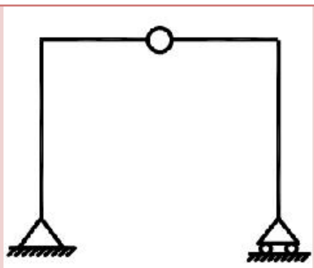
**unknown** r =No. of reactions, m= Total number of bars; *( 3m+ r )*

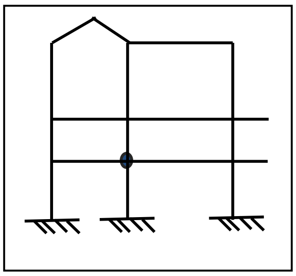
**Equations** **3j +c (j = Total number of joints, c= condition equation)**

**If 3m + r = 3 j+c Stable & internally determinate. Check the arrangement of members**
**If 3m + r > 3 j+c Stable & internally indeterminate degree of indeterminacy would be decided by the difference of these tow quantities**

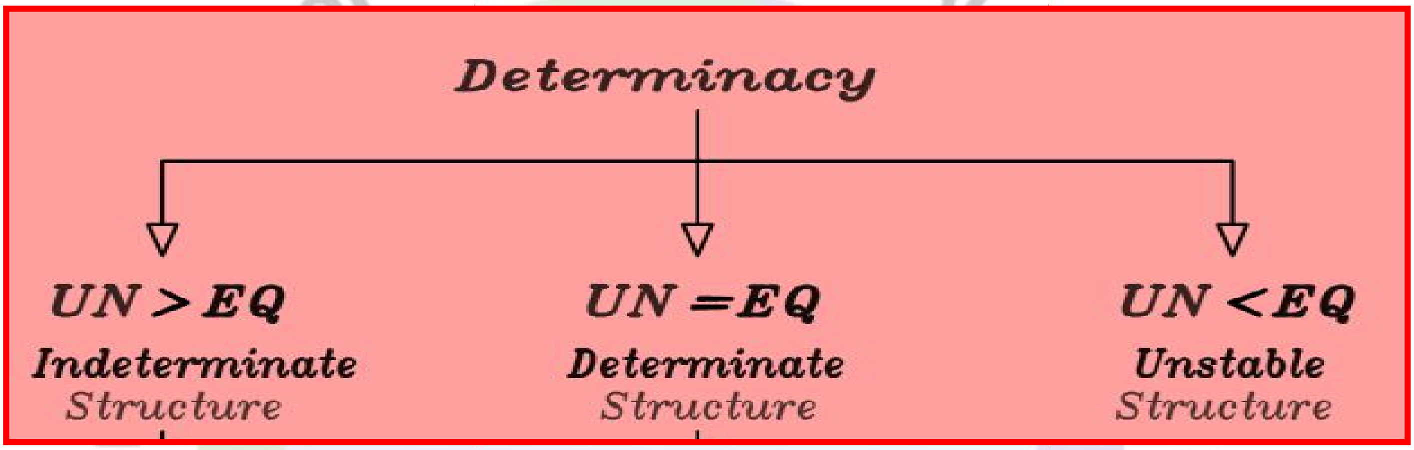
**If 3m + r < 3 j+c Unstable.**

**A structure is said to have determinacy or indeterminacy only if it is stable**

| *frame* | *r+3m* | *3j+c* | *Note* |
|---|---|---|---|
|  | 23 | 18 | Stable and indeterminate to $5^{o}$ |
|  | 29 | 15 | Stable and indeterminate to $24^{o}$ |
|  | 15 | 16 | Stable and indeterminate to $1^{o}$ |



Stable and indeterminate




## STATICALLY DETERMINAT STRUCTURES

## STATICALLY DETERMINAT TRUSS

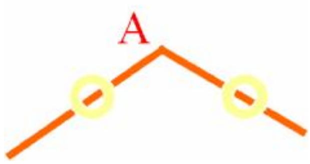
*Truss Analysis:* For truss analysis, it is assumed that:

- Bars are pin-connected.
- Joints are frictionless hinges.
- Loads are applied at the joints only.
- Stress in each member is constant along its length.

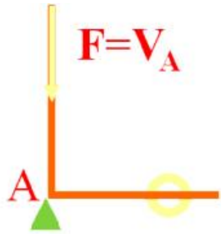
The objective of truss analysis is to determine the reactions and member forces.

### How to Determine Zero-Force Members in Truss

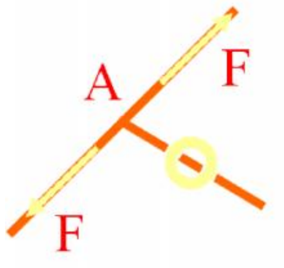
1. If a joint only has two members and no external load and/or no support, then those two members are zero-force members.



2. If a joint only has two members and is loaded, then if the line of action of resultant force from applied loads at the joint colinear with one of the members then the other member is a zero-force member. If the resultant force at the joint is not colinear with either member, then both members are not zero-force members.

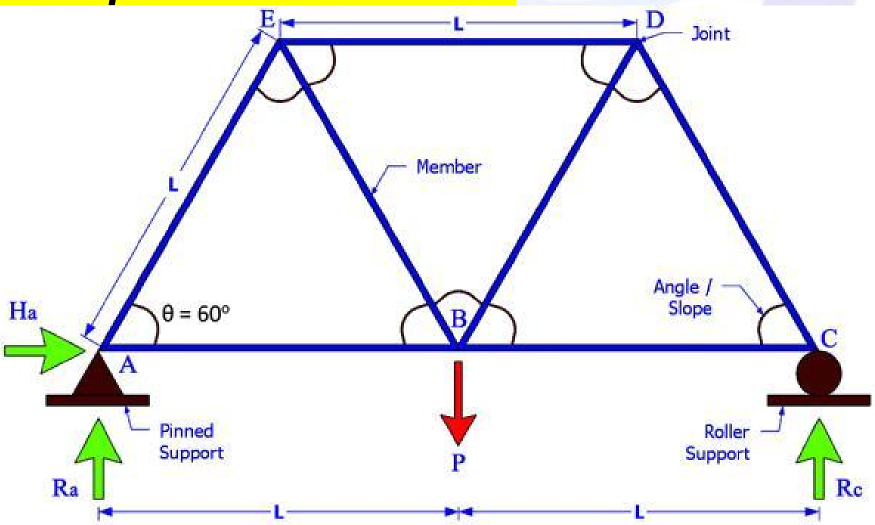


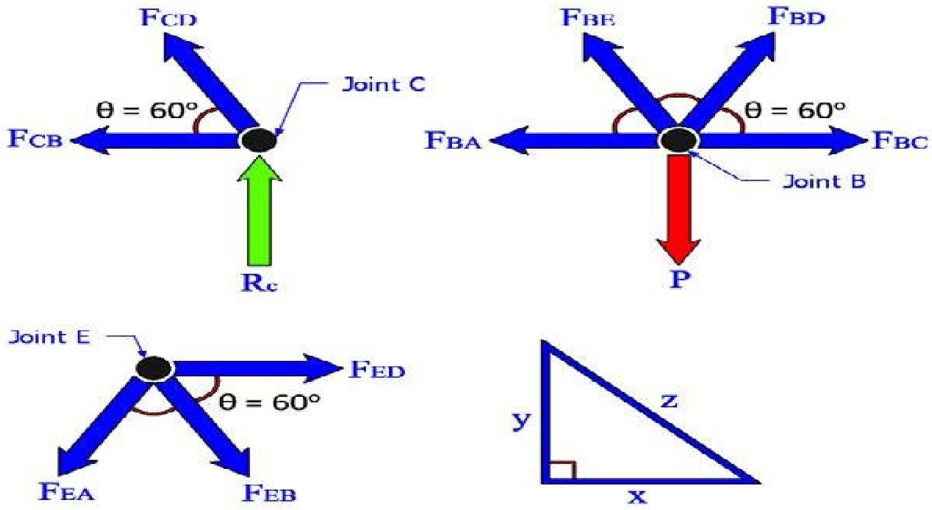

3. If a joint has three members and no external load and/or support, then if two of members are colinear then the non-colinear member is a zero-force




## *1. Method of Joints*

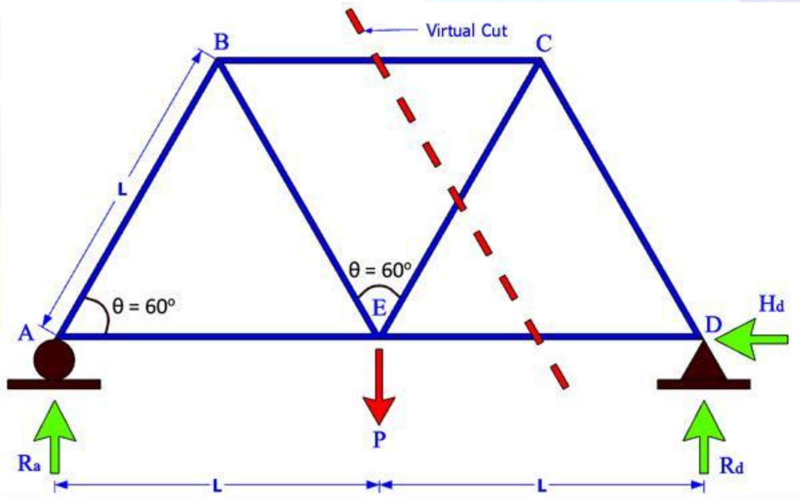
***Using the equilibrium equations of $\sum Fx = 0$ and $\sum Fy = 0$, the unknown member forces can be solved.***

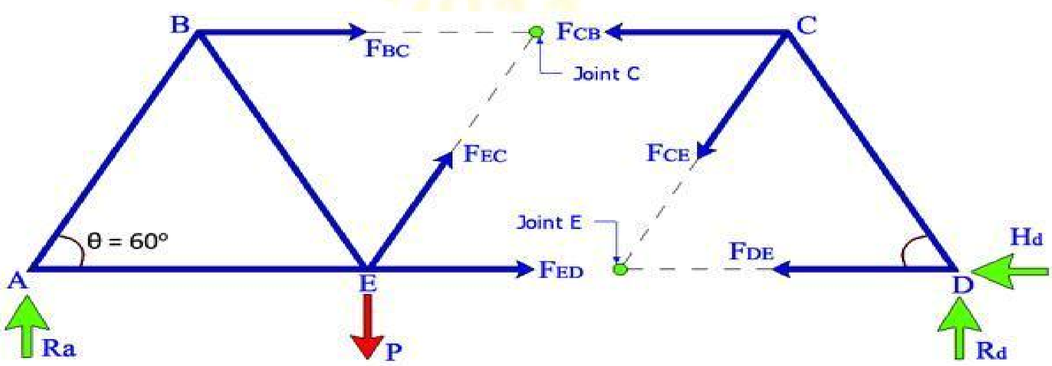


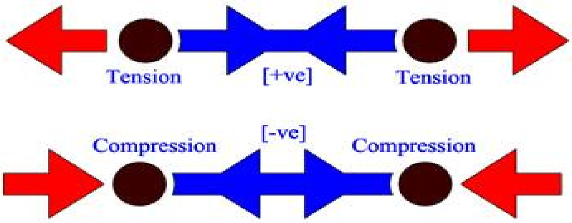



## *2. Section Method*

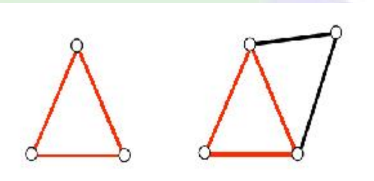
A section subdivides the truss into two separate parts. Since the entire truss is in equilibrium, any part of it must also be in equilibrium. Either of the two parts of the truss can be considered and the three equations of equilibrium $\sum Fx = 0$, $\sum Fy = 0$, and $\sum M = 0$ can be applied to solve for member forces.



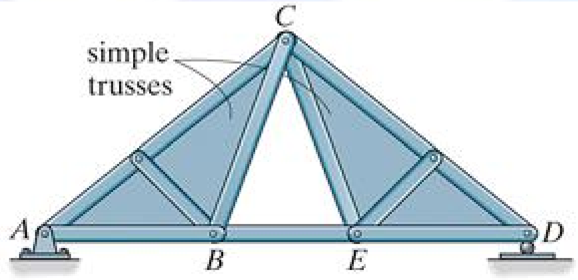



Simple Truss a simple truss is constructed starting with a basic triangular element and connecting two members to form additional elements. As each additional element of two members is placed on a truss, the number of joints is increased by one.

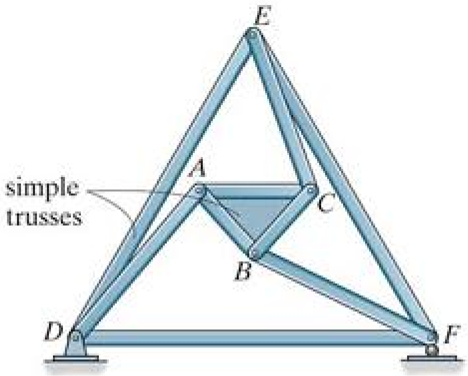


Compound Truss This truss is formed by connecting two or more simple trusses together, There are three ways in which simple trusses may be connected to form a compound truss:

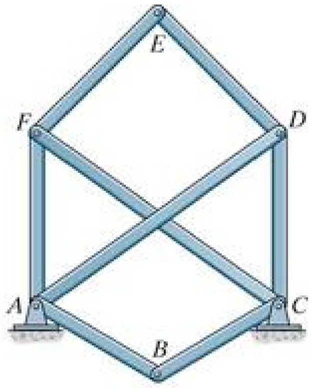
1.Trusses may be connected by a common joint and bar ..



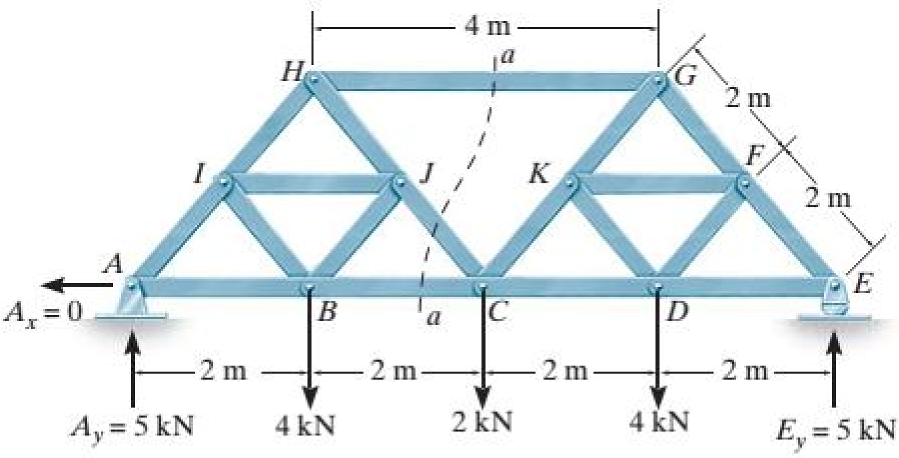

2.Trusses may be joined by three bars. Trusses may be joined by three bars.

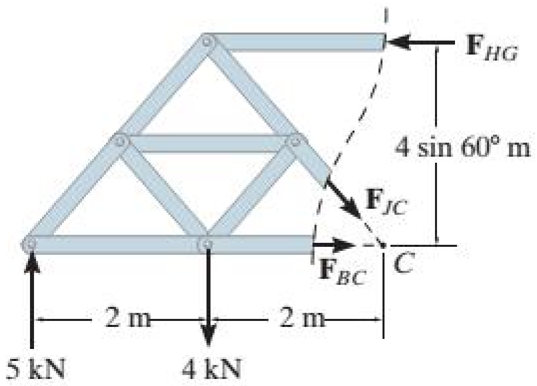



Complex Truss This is a truss that cannot be classified as being either simple or compound.




Example: Analyze the compound truss shown




$$\circlearrowleft +\Sigma M_C = 0; \quad -5(4) + 4(2) + F_{HG}(4 \sin 60^\circ) = 0$$

$$F_{HG} = 3.46 \text{ kN (C)}$$

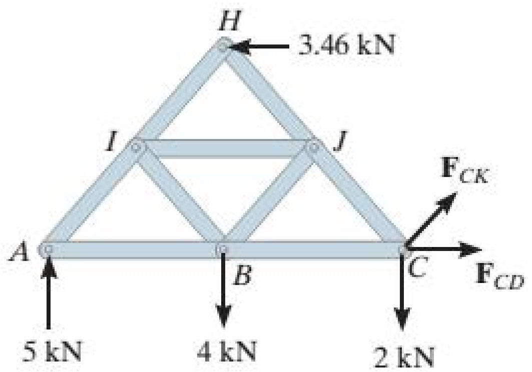
We can now proceed to determine the force in each member of the

simple trusses using the method of joints. For example, the free-body

diagram of ACH is shown The joints of this truss can be

analyzed in the following sequence:

Joint A:Determine the force in AB and AI.



Joint H:Determine the force in HI and HJ.

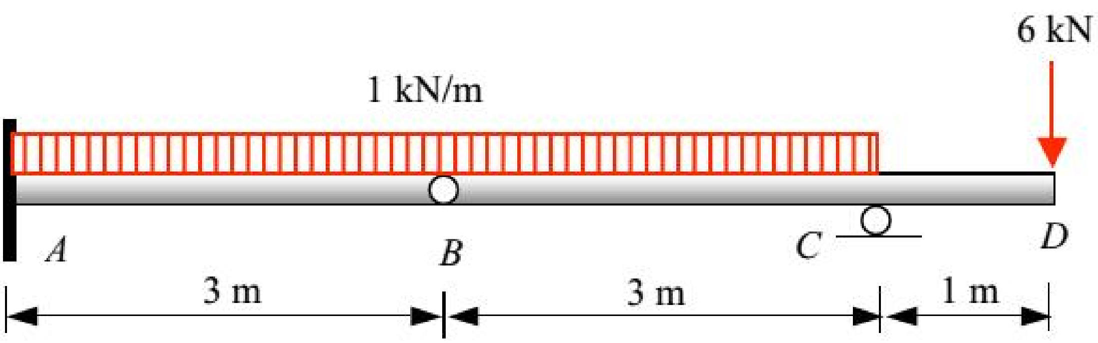
Joint I:Determine the force in IJ and IB.

Joint B:Determine the force in BC and BJ.

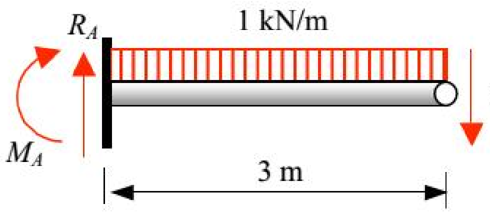
Joint J:Determine the force in JC.

## STATICALLY DETERMINAT BEAM

**Example**: Analyze the loaded beam shown and draw the shear and moment diagrams.




Define FBDs and find reactions and internal nodal forces




$\Sigma MC = 0$, $VB(3) - 3(1.5) + 6(1) = 0$

$\Longrightarrow$ $VB = -0.5$ kN

$\Sigma Fy = 0$, $-0.5 - 3 - 6 + RC = 0$

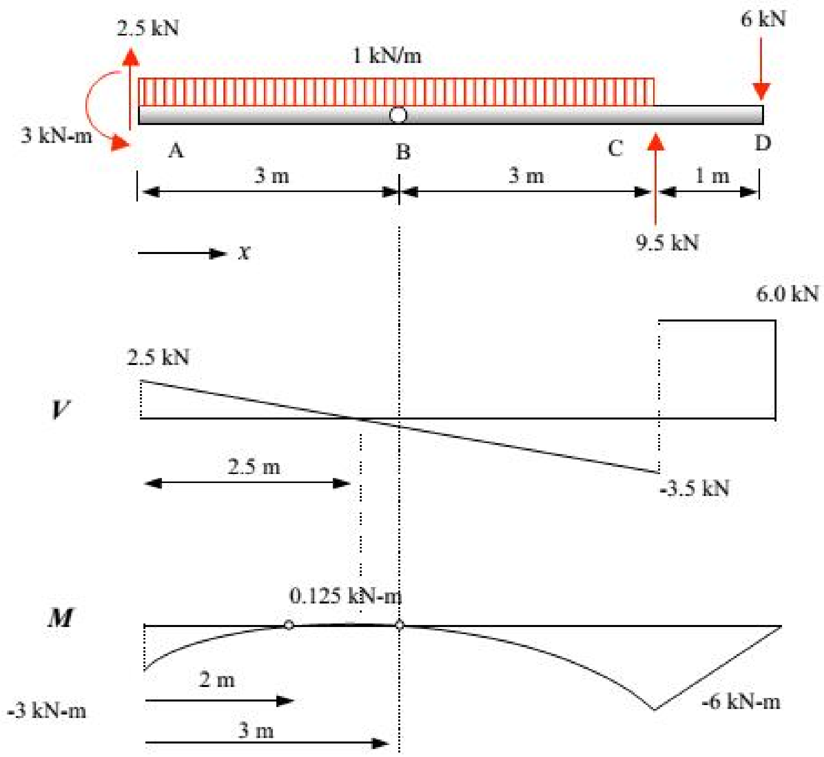
$\Longrightarrow$ $RC = 9.5$ kN

$\Sigma MA = 0$, $MA + 3(1.5) - 0.5(3) = 0$

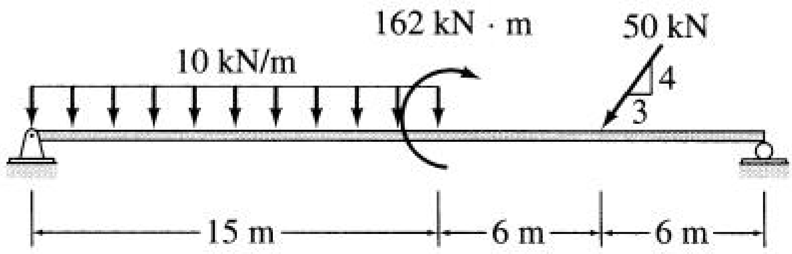
$\Longrightarrow$ $MA = -3$ kN-m

$\Sigma Fy = 0$, $0.5 - 3 + RA = 0$

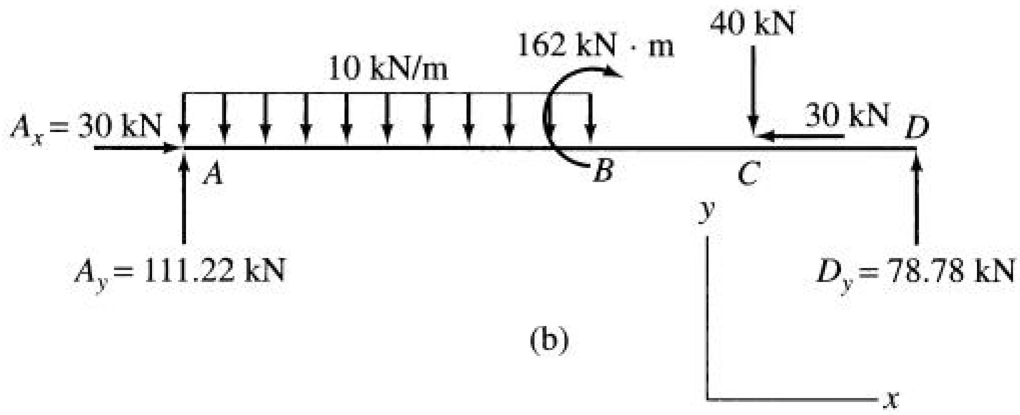
$\Longrightarrow$ $RA = 2.5$ kN



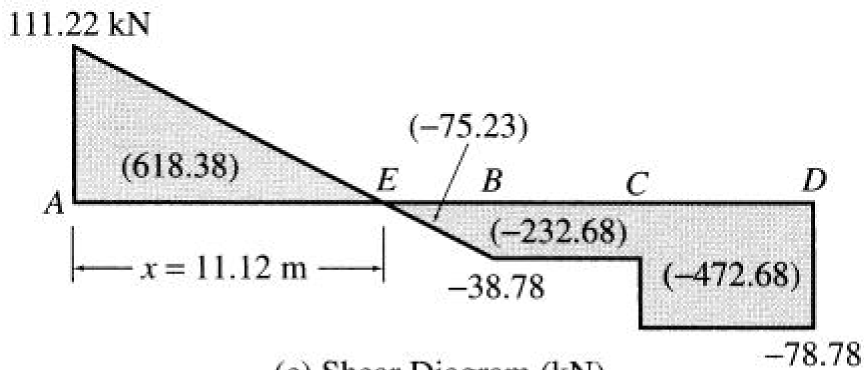
**Example**: Analyze the loaded beam shown and draw the shear and moment diagrams.



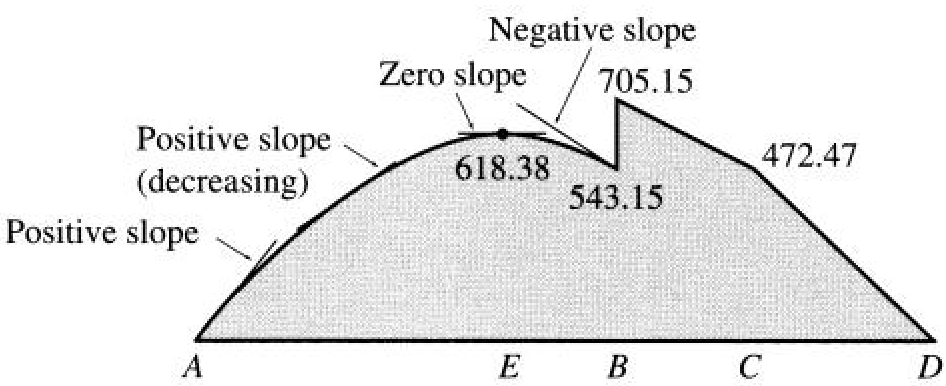

(a)




(b)

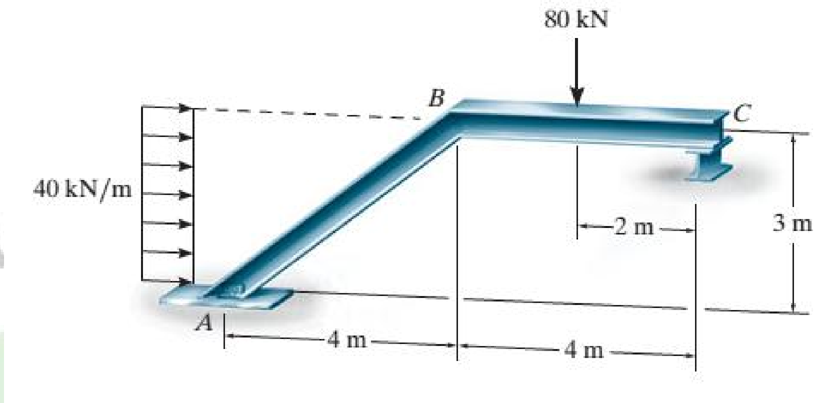


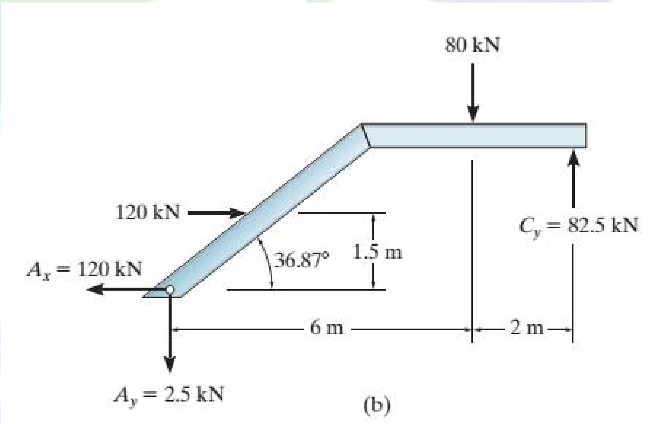

(c) Shear Diagram (kN)

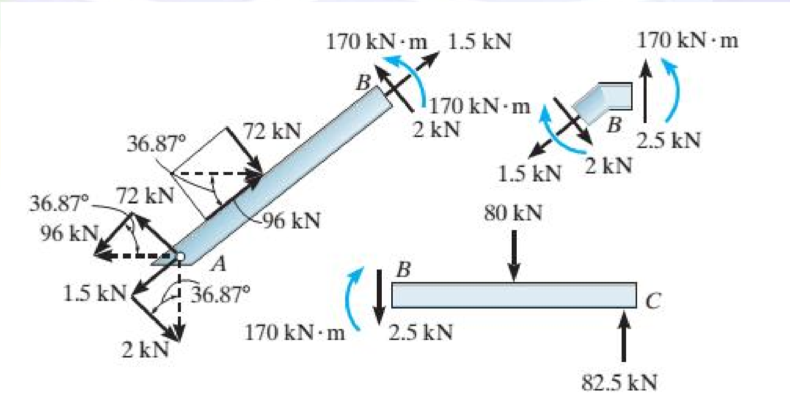


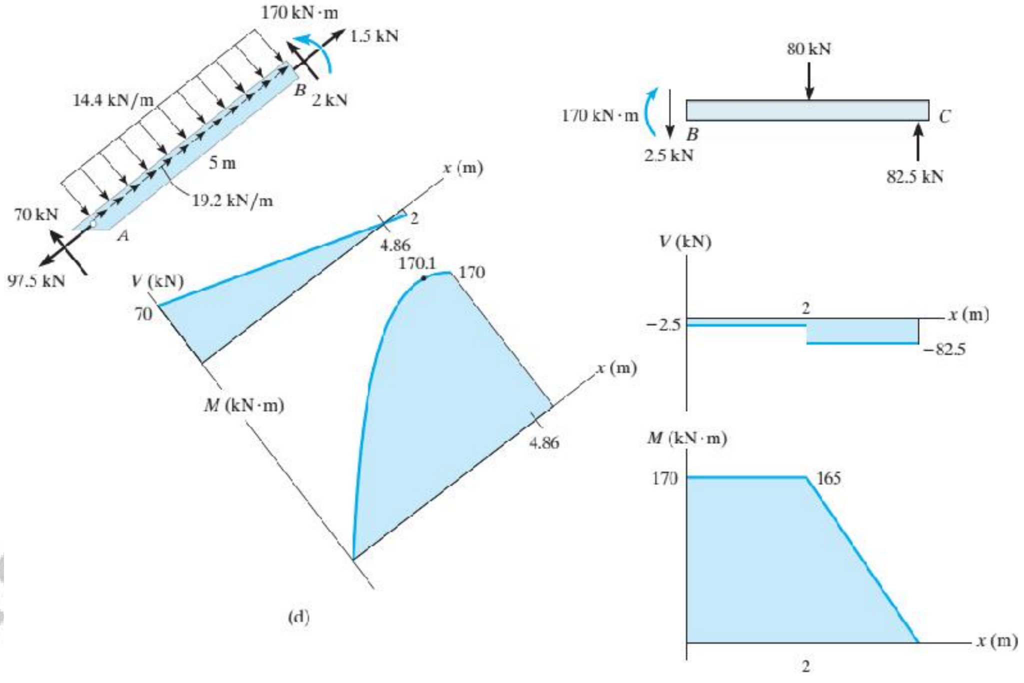

(d) Bending Moment Diagram (kN · m)

**Example:** Draw the shear and moment diagrams for the frame shown .






(d) (e)

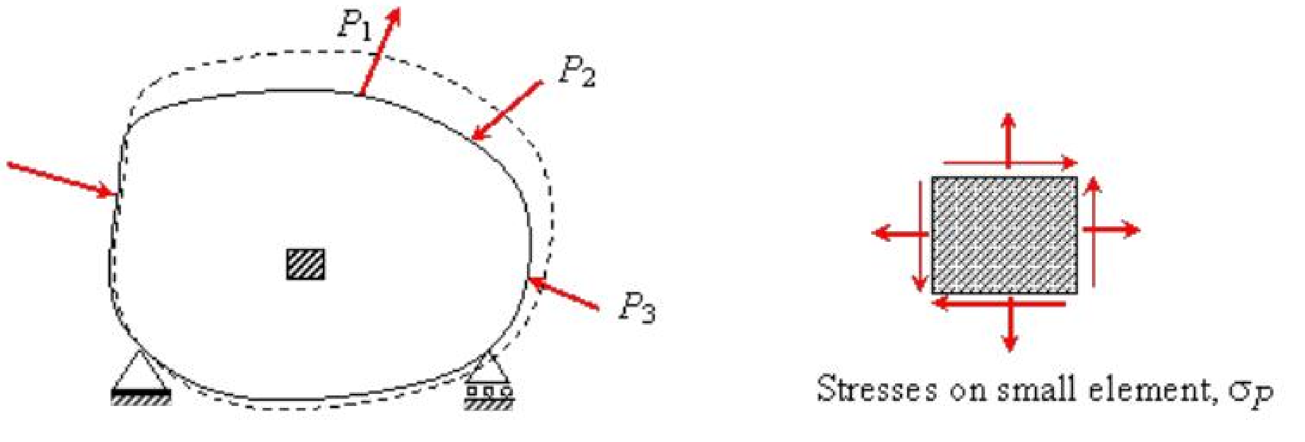
# *Deflection of structural analysis*

***Deflection*** is a term that is used to describe the degree to which a structural element is displaced under a load.

# *Deflection of structural analysis by using Virtual work method*

## Principle of Virtual Work

Consider a structural system subjected to a set of forces (P1, P2, P3 … referred as P force) under stable equilibrium condition as shown in Figure below Further, consider a small element within the structural system and stresses on the surfaces caused by the P forces are shown in Figure (b) and referred a σp.




Let the body undergoes to a set of compatible virtual displacement . These displacements are imaginary and fictitious as shown by dotted line. While the body is displaced, the real forces acting on the body move through these displacements. These forces and virtual displacements must satisfy the principle of conservation of energy.

$$\delta W_e = \delta W_i$$

$$\sum_{i=1}^{n} P_i(\delta D)_i = \int_V \sigma_p(\delta\varepsilon)dV$$

**This is the principle of virtual work**

If a system in equilibrium under a system of forces undergoes a deformation, the work done by the external forces ( P ) equals the work done by the internal stresses due to those forces, ($\sigma p$ ).

$$\delta F \times \Delta = \sum \delta f \times \Delta L$$

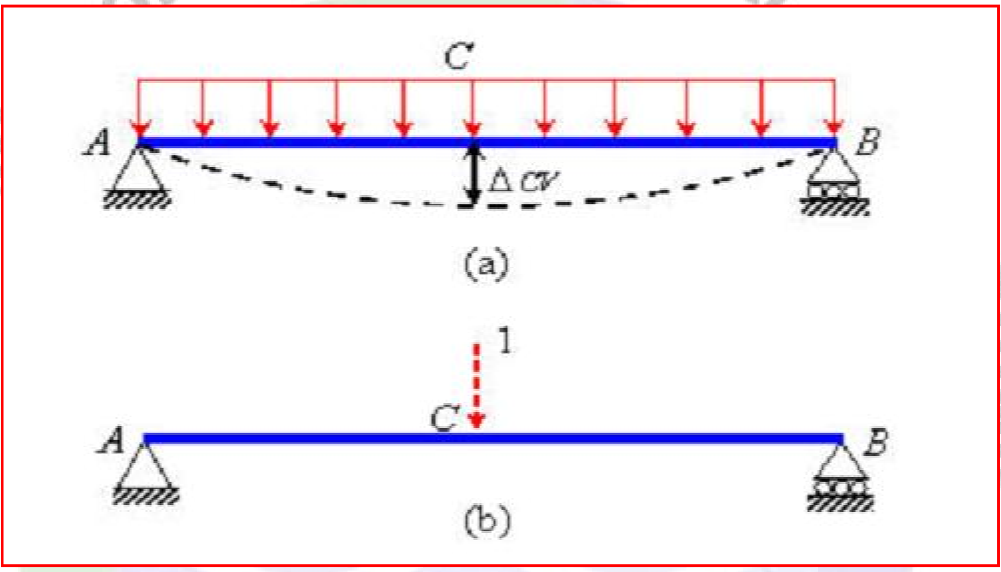
The left hand side of Eq. denotes the external work done by the virtual force $\delta F$ moving through the real displacement $\Delta$ . On the other hand, the right hand side of Eq. represents the internal work done by the virtual internal element forces d f moving through the displacement $\Delta L$ .

Since $\delta F$ is arbitrary and for convenience let $\delta F = 1$ (i.e. unit load). The Eq. above can be re-written as:

$$1 \times \Delta = \sum f \times \Delta L$$

## Application of virtual work to beams and frames

In order to find out the vertical displacement of C of the beam shown in Figure (a) below, apply a unit load as shown in Figure (b).




The internal virtual work is considered mainly due to bending moment *Mu* and caused due to internal moments under going the rotation *dθ* due to the applied loading.

$$d\theta = M_p dx / EI$$

where *MP* is the moment due to applied loading, the Eq. for the displacement of C will take a shape of

$$1 \times \Delta_{CV} = \int_0^L M_u \times d\theta$$

$$\Delta_{CV} = \int_0^L M_u \frac{M_p dx}{EI}$$

The basic steps to be followed for finding the displacement or slope of a beams and frames are summarized as:

1. Compute the bending moment *MP* due to applied external forces.

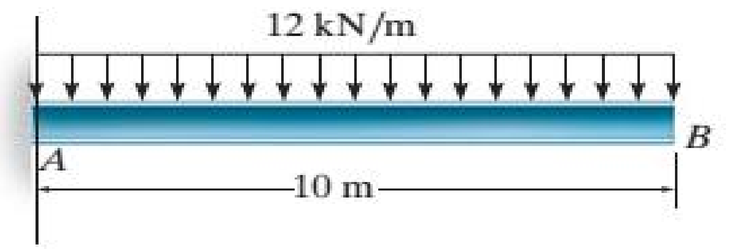
2. Compute the bending moment *mu* due to unit load applied in the direction of required displacement or slope.

3. Compute the integral $\int_0^l Mp.mu\, dx/EI$ over the entire members of the beam or frame which will provide the desired displacement.

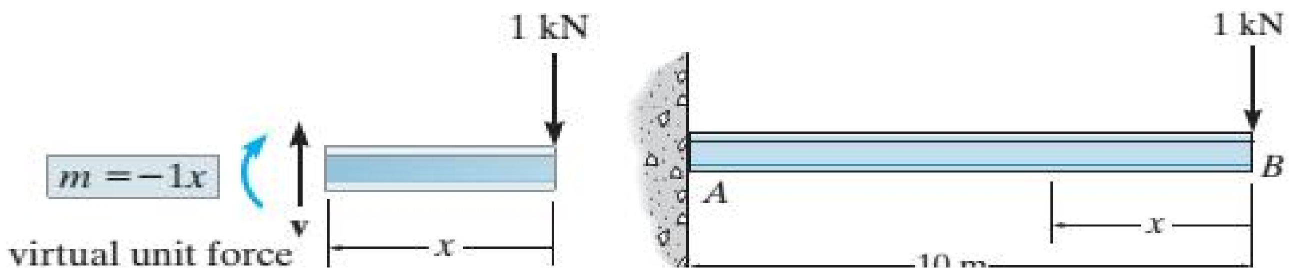
4. The bending moment shall be taken as positive if sagging and negative if hogging (in case of

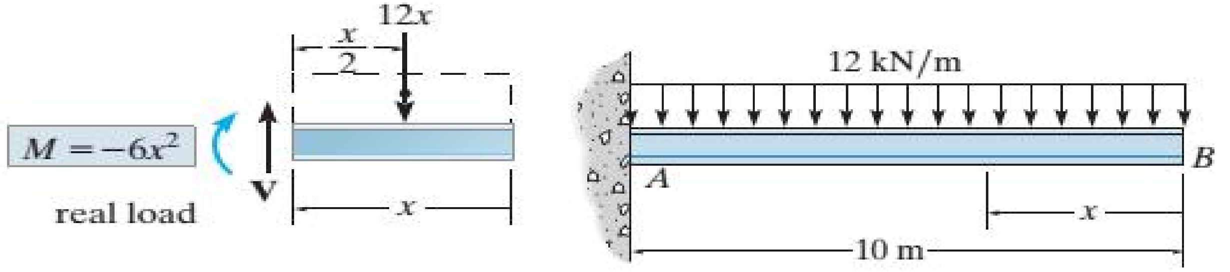
beams).

Example Determine the displacement of point B of the steel beam shown , Take E=200 GPa, $I=500(10)^6 mm^4$




The vertical displacement of point B is obtained by placing a virtual unit load of 1 kN at B



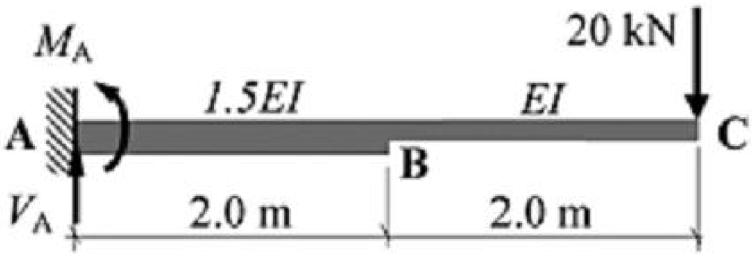
$$1\,\text{kN}\cdot\Delta_B = \int_0^L \frac{mM}{EI}dx = \int_0^{10} \frac{(-1x)(-6x^2)\,dx}{EI}$$

$$1\,\text{kN}\cdot\Delta_B = \frac{15(10^3)\,\text{kN}^2\cdot\text{m}^3}{EI}$$

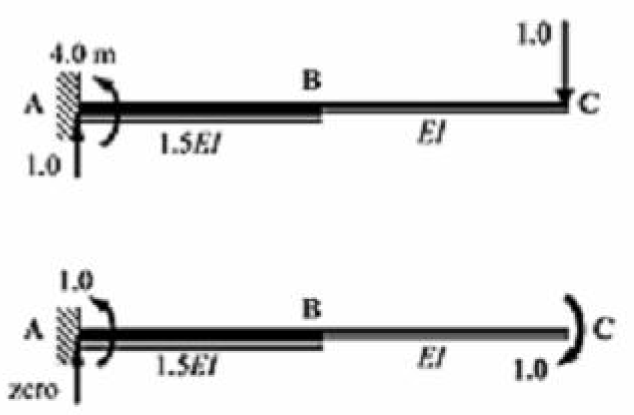
$$\Delta_B = \frac{15(10^3)\,\text{kN}\cdot\text{m}^3}{200(10^6)\,\text{kN/m}^2(500(10^6)\,\text{mm}^4)(10^{-12}\,\text{m}^4/\text{mm}^4)}$$

$$= 0.150\,\text{m} = 150\,\text{mm}$$

**Example:** Determine the magnitude and direction of the deflection and slope at C:




applied a unit point load at C and a unit moment at C



$$\delta_C = \int_0^L \frac{Mm}{EI}dx = \int_C^B \frac{Mm}{EI}dx + \int_B^A \frac{Mm}{1.5EI}dx$$

$$M = -20x \qquad m = -x \qquad \therefore Mm = +20x^2$$

$$\int_C^B \frac{Mm}{EI}dx = \int_0^2 \frac{20x^2}{EI}dx = \left[\frac{20x^3}{3EI}\right]_0^2 = +\frac{53.33}{EI}\text{ m}$$

$$M = -20x \qquad m = -x \qquad \therefore Mm = +20x^2$$

$$\int_B^A \frac{Mm}{EI}dx = \int_2^4 \frac{20x^2}{1.5EI}dx = \left[\frac{20x^3}{4.5EI}\right]_2^4 = \left[\frac{20\times 4^3}{4.5EI} - \frac{20\times 2^3}{4.5EI}\right] = +\frac{248.89}{EI}\text{ m}$$

$$\therefore \delta_C = +\frac{53.33}{EI} + \frac{248.89}{EI} = \frac{302.22}{EI}\text{ m} \downarrow$$

**Similarly to determine the slope:**

$$\theta_C = \int_0^L \frac{Mm}{EI}dx = \int_C^B \frac{Mm}{EI}dx + \int_B^A \frac{Mm}{1.5EI}dx$$

$$M = -20x \qquad m = -1.0 \qquad \therefore Mm = 20x$$

$$\int_C^B \frac{Mm}{EI}dx = \int_0^2 \frac{20x}{EI}dx = \left[\frac{20x^2}{2EI}\right]_0^2 = +\frac{40.0}{EI}\text{ rad.}$$

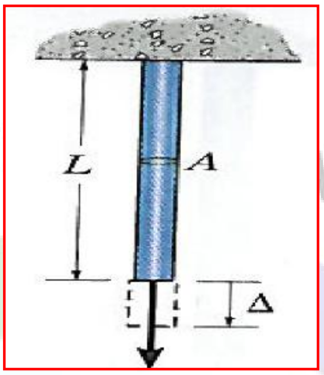
$$M = -20x \qquad m = -1.0 \qquad \therefore Mm = 20x$$

$$\int_B^A \frac{Mm}{EI}dx = \int_2^4 \frac{20x}{1.5EI}dx = \left[\frac{20x^2}{3.0EI}\right]_2^4 = \left[\frac{20\times 4^2}{3.0EI} - \frac{20\times 2^2}{3.0EI}\right] = +\frac{80.0}{EI}\text{ rad.}$$

$$\therefore \theta_C = +\frac{40.0}{EI} + \frac{80.0}{EI} = +\frac{120.0}{EI}\text{ rad.} \searrow$$

# Virtual Work Method for Trusses Deflection

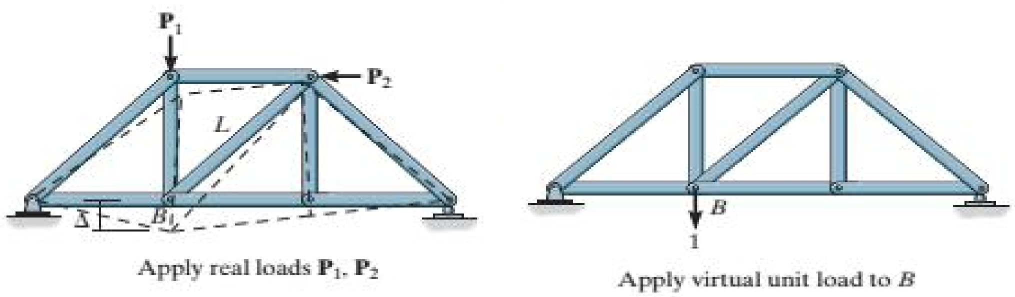
We can use the method of virtual work to determine the displacement of a truss joint when the truss is subjected to an external loading, temperature change, or fabrication errors.




- Hooke's Law σ = EЄ
- Normal Stress σ = N/A
- A=Constant Cross-Sectional Area
- Final Strain Є=Δ/L
- L=Length
- Δ = NL/AE
- $U_i = N^2L/2AE$

## EXTERNAL LOADING:

For the purpose of explanation let us consider the vertical displacement




of joint B of the truss in Fig. shown Here atypical element of the truss would be one of its members having a length L . If the applied loadings and cause a linear elastic material response ,then this element deforms an amount where Nis the normal or axial force in the member, caused by the loads. the virtual-work equation for the truss will be:

$$1.\Delta=\sum\frac{SuL}{AE}$$

where:

**1**: external virtual unit load acting on the truss joint in the stated direction of Δ

u: internal virtual normal force in a truss member caused by the external virtual unit load.

**Δ**: external joint displacement caused by the real loads on the truss.

**S**: internal normal force in a truss member caused by the real loads.

**L**: length of the member.

**A**: cross section area of a member.

**E**:modulus of elasticity of a member.

**TEMPERATURE**:. In some cases, truss members may change their length due to temperature. If α is the coefficient of thermal expansion for a member and is ΔT the change in its temperature, the change in length of a member is $\Delta L=\alpha\ L\ \Delta T\ u$ Hence, we can determine the displacement of a selected truss joint due to this temperature change from:

$$1.\Delta=\sum\alpha\ L\ \Delta T\ u$$

**1 =**external virtual unit load acting on the truss joint in the stated direction of Δ

**u=**internal virtual normal force in a truss member caused by the external virtual unit load.

Δ=external joint displacement caused by the temperature change.

α=coefficient of thermal expansion of member.

**ΔT=**change in temperature of member.

**L=**length of member.

## FABRICATION ERRORS AND CAMBER.

Occasionally, errors in fabricating the lengths of the members of a truss may occur. Also, in some cases truss members must be made slightly longer or shorter in order to give the truss a camber. Camber is often built into a bridge truss so that the bottom cord will curve upward by an amount equivalent to the downward deflection of the cord when subjected to the bridge's full dead weight. If a truss member is shorter or longer than intended, the displacement of a truss joint from its expected position can be determined from direct application of equation:

**1.Δ=∑u ΔL**

were:

**1 =**external virtual unit load acting on the truss joint in the stated direction of Δ

**u=**internal virtual normal force in a truss member caused by the external virtual unit load.

Δ=external joint displacement caused by the temperature change.

ΔL=difference in length of the member from its intended size as caused by a fabrication error.

# Procedure for Analysis

The following procedure may be used to determine a specific displacement of any joint on a truss using the method of virtual work.

## Virtual Forces u

•Place the unit load on the truss at the joint where the desired displacement is to be determined. The load should be in the same direction as the specified displacement, e.g., horizontal or vertical.

•With the unit load so placed, and all the real loads removed from the truss, use the method of joints or the method of sections and calculate the internal u force in each truss member. Assume that **tensile forces are positive** and **compressive forces are negative**.

## Real Forces S

•Use the method of sections or the method of joints to determine the S force in each member. These forces are caused only by the real loads acting on the truss. Again, assume **tensile forces are positive** and **compressive forces are negative.**

## Virtual-Work Equation

•Apply the equation of virtual work, to determine the desired displacement. It is important to retain the algebraic sign for each of the corresponding u and S forces when substituting these

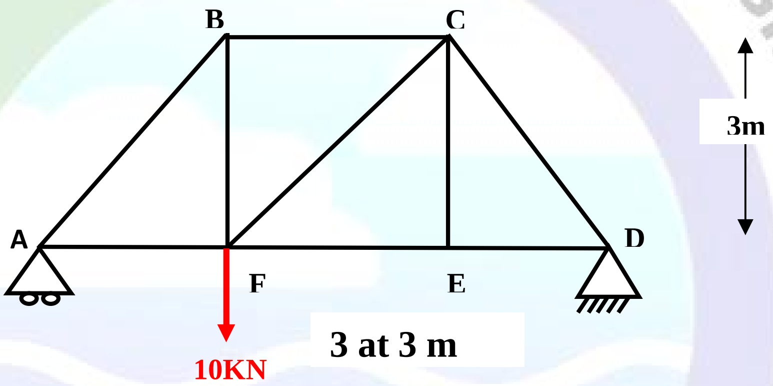
terms into the equation.

•If the resultant sum $\sum SuL/AE$ is positive, the displacement $\Delta$ is in the same direction as the unit load. If a negative value results, $\Delta$ is opposite to the unit load.

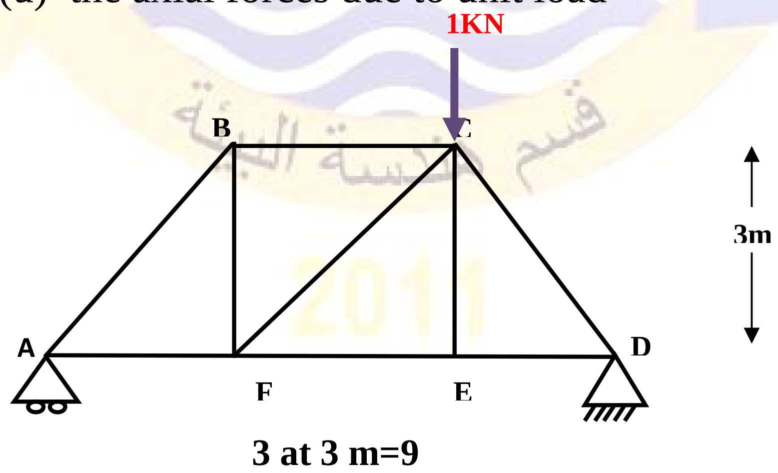
•When applying $\Delta L=\alpha L \Delta T$ u realize that if any of the members undergoes an increase in temperature ,$\Delta T$ will be positive ,whereas a decrease in temperature results in a negative value for $\Delta T$

•For 1.$\Delta=\sum u \Delta L$ when a fabrication error **increases the length** of a member, is**$\Delta L$** positive, whereas **a decrease in length** is **negative**.

Example: for the truss shown determine the vertical deflection of joint (C) , the relative deflection between joint B and E along the line joining them, the rotation of member EF



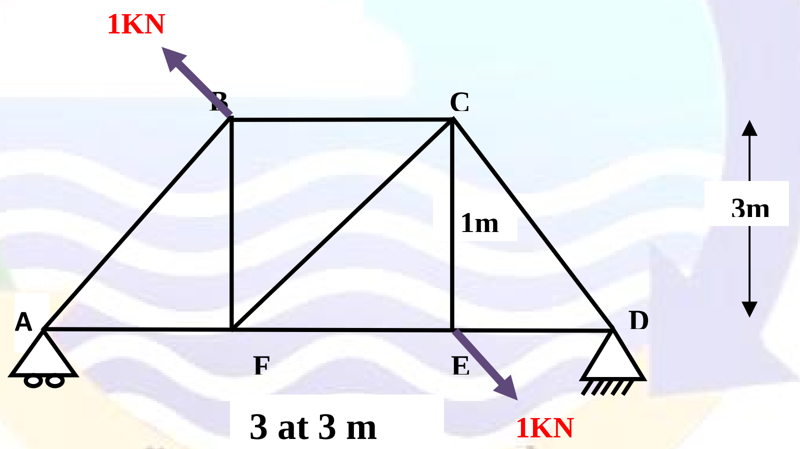

to fined *vertical deflection of joint (C),* unit load (1KN) shall be applied at joint (c), determine (S) the axial forces in members due to external applied load , then determine (u) the axial forces due to unit load



| member | Length(m) | S | $u_c$ | SuL |
|---|---|---|---|---|
| AB | 4.24 | -9.43 | -0.47 | 18.79 |
| BC | 3 | -6.67 | -0.33 | 6.66 |
| CD | 4.24 | -4.71 | -0.943 | 18.83 |
| DE | 3 | 3.33 | 0.667 | 6.66 |
| EF | 3 | 3.33 | 0.667 | 6.66 |
| FA | 3 | 6.67 | 0.33 | 6.66 |
| BF | 3 | 6.67 | 0.33 | 6.66 |
| FC | 4.24 | 4.71 | -0.47 | -9.39 |
| CE | 3 | 0 | 0 | 0 |
| | | | ∑ SuL | 61.53 |

*vertical deflection of joint (C)*= ∑ SuL/AE= 61.53/AE

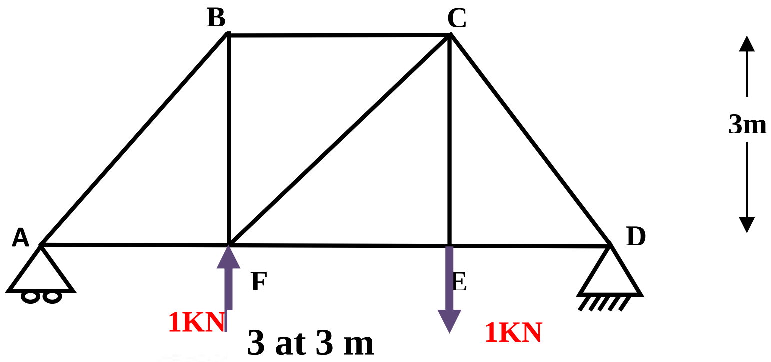
To determine the relative deflection between joint B and E along the line joining them, unit load shall be applied along the line BE




| member | Length(m) | S | $U_{BE}$ | SuL |
|---|---|---|---|---|
| AB | 4.24 | -9.43 | 0 | 0 |
| BC | 3 | -6.67 | 0.707 | -14.15 |
| CD | 4.24 | -4.71 | 0 | 0 |
| DE | 3 | 3.33 | 0 | 0 |
| EF | 3 | 3.33 | 0.707 | 7.06 |
| FA | 3 | 6.67 | 0 | 0 |
| BF | 3 | 6.67 | 0.707 | 14.15 |
| FC | 4.24 | 4.71 | -1 | -19.97 |
| CE | 3 | 0 | 0.707 | 0 |
| | | | ∑ SuL | -12.9 |

Relative deflection between joint B and E= ∑ SuL/AE=-12.9/AE

To fined the rotation of member EF applied 1KN on member EF as shown




| member | Length(m) | S | $U_{fE}$ | SuL |
|---|---|---|---|---|
| **AB** | 4.24 | -9.43 | 0.47 | 18.79 |
| **BC** | 3 | -6.67 | 0.33 | -6.6 |
| **CD** | 4.24 | -4.71 | -0.47 | 9.39 |
| **DE** | 3 | 3.33 | 0.33 | 3.297 |
| **EF** | 3 | 3.33 | 0.33 | 3.297 |
| **FA** | 3 | 6.67 | -0.33 | -6.603 |
| **BF** | 3 | 6.67 | -0.33 | -6.603 |
| **FC** | 4.24 | 4.71 | -0.947 | -18.91 |
| **CE** | 3 | 0 | 1 | 0 |
| | | | **∑ SuL** | -41.522 |

Rotation of member B E=( ∑ SuL/AE)/L=-41.522/3AE=13.8/AE

**IF** the member BC subjected to an increase in temperature of $\Delta T=60^{o}$ C ,$\alpha=1.08*10^{-5}$ , and member AB 5 mm too short . Take E=200 GPa,A=900mm$^2$ fined vertical deflection of joint C

$$1.\Delta=\sum\frac{SuL}{AE}+\sum\alpha\, L\, \Delta T\, u+\sum u\delta$$

$1.\Delta=61.53/AE+1.08*10^{-5}*60*3*(-0.33)+-0.005*-0.47$

$1.\Delta=2.05mm$

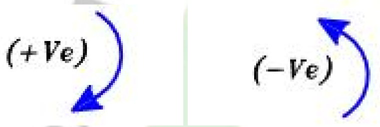
# MOMENT DISTRIBUTION METHOD

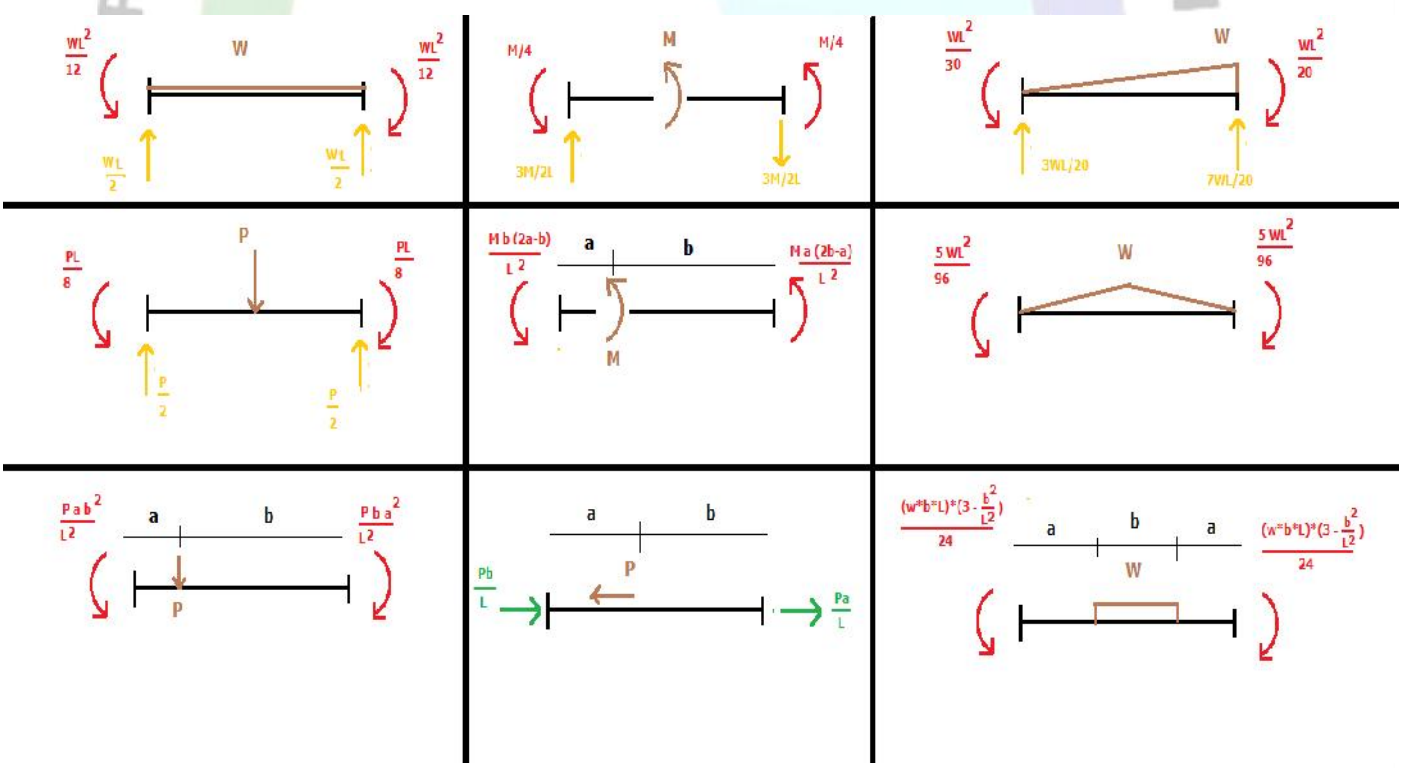
The moment distribution method is a structural analysis method for ***statically indeterminate beams and frames*** In the moment distribution method, every joint of the structure to be analysed is fixed so as to develop the fixed-end moments. Then each fixed joint is sequentially released and the fixed-end moments (which by the time of release are not in equilibrium) are distributed to adjacent members until equilibrium is achieved.

In order to apply the moment distribution method to analyse a structure, the following things must be considered.

## Fixed end moments

Fixed end moments are the moments produced at member ends by external loads.



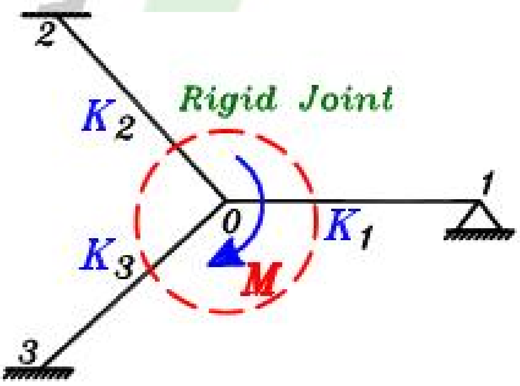
## Flexural stiffness

The flexural stiffness (EI/L) of a member is represented as the product of the modulus of elasticity (E) and the second moment of area (I) divided by the length (L) of the member. What is needed in the moment distribution method is not the exact value but the ratio of flexural stiffness of all members.

## Distribution factors

When a joint is being released and begins to rotate under the unbalanced moment, resisting forces develop at each member framed together at the joint. Although the total resistance is equal to the unbalanced moment, the magnitudes of resisting forces developed at each member differ by the members' flexural stiffness. Distribution factors can be defined as the proportions of the unbalanced moments carried by each of the members. In mathematical terms, distribution factor of member

where n is the number of members framed at the joint.




$K_1 \Rightarrow$ member (01) Stiffness

$K_2 \Rightarrow$ member (02) Stiffness

$K_3 \Rightarrow$ member (03) Stiffness

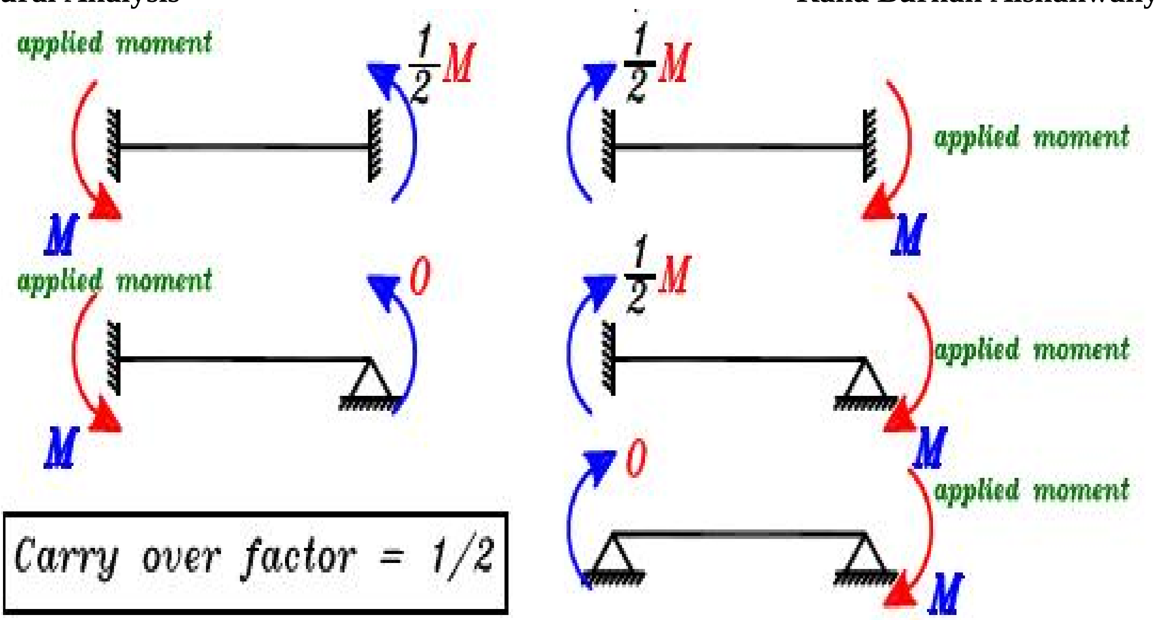
$D.F_1 \Rightarrow$ member (01) Distribution factor $= \frac{K_1}{K_1+K_2+K_3} = \frac{K_1}{\Sigma K}$

$D.F_2 \Rightarrow$ member (02) Distribution factor $= \frac{K_2}{K_1+K_2+K_3} = \frac{K_2}{\Sigma K}$

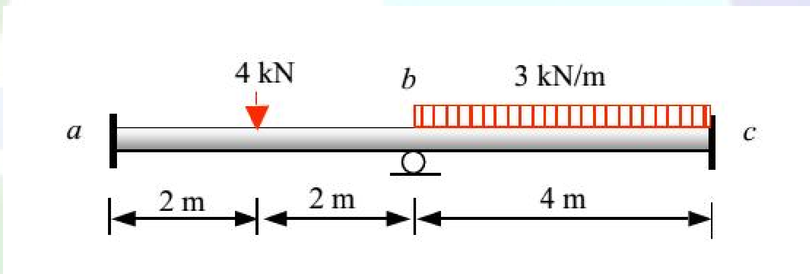
$D.F_3 \Rightarrow$ member (03) Distribution factor $= \frac{K_3}{K_1+K_2+K_3} = \frac{K_3}{\Sigma K}$

## Carryover factors

When a joint is released, balancing moment occurs to counterbalance the unbalanced moment which is initially the same as the fixed-end moment. This balancing moment is then carried over to the member's other end. The ratio of the carried-over moment at the other end to the fixed-end moment of the initial end is the carryover factor.




**Example:** **Find all the member-end moments of the beam shown. EI is constant for all members.**




FEM for member ab. The concentrated load of 4 kN creates FEMs at end a and end b.

$$M^F{}_{ab} = -\frac{(P)(Length)}{8} = -\frac{(4)(4)}{8} = -2 \text{ kN-m}$$

$$M^F{}_{ba} = \frac{(P)(Length)}{8} = \frac{(4)(4)}{8} = 2 \text{ kN-m}$$

FEM for member bc. The distributed load of 3 kN/m creates FEMs at end b and end c.

$$M^F{}_{bc} = -\frac{(w)(Length)^2}{12} = -\frac{(3)(4)^2}{12} = -4 \text{ kN-m}$$

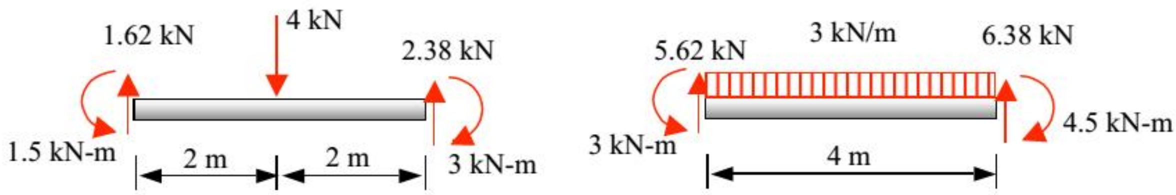
$$M^F{}_{cb} = \frac{(w)(Length)^2}{12} = \frac{(3)(4)^2}{12} = 4 \text{ kN-m}$$

Compute DF at b:

$$DF_{ba} : DF_{bc} = 4EK_{ab} : 4EK_{bc} = 4\left(\frac{EI}{L}\right)_{ab} : 4\left(\frac{EI}{L}\right)_{bc} = \frac{1}{4} : \frac{1}{4} = 0.5 : 0.5$$

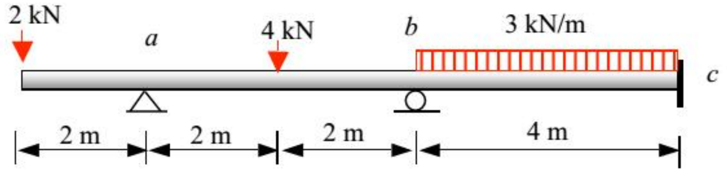
Assign DF at a and c: DFs are zero at a and c.

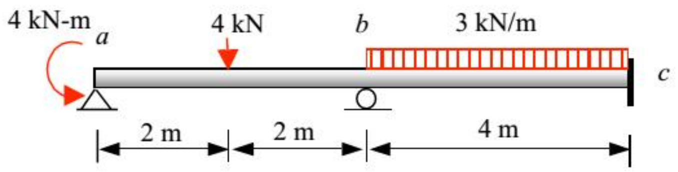
| Node | *a* | *b* | | *c* |
|---|---|---|---|---|
| Member | *ab* | | *bc* | |
| DF | 0 | 0.5 | 0.5 | 0 |
| EAM | | | | |
| MEM | $M_{ab}$ | $M_{ba}$ | $M_{bc}$ | $M_{cb}$ |
| FEM | –2 | +2 | –4 | +4 |
| DM | | +1 | +1 | |
| COM | +0.5 | | | +0.5 |
| Sum | –1.5 | +3 | –3 | +4.5 |




*FBDs of the two members.*

**Example: Find all the member-end moments of the beam shown. EI is constant for all members.**




$$M^F{}_{ab} = -\frac{(P)\,(Length)}{8} = -\frac{(4)\,(4)}{8} = -\ 2\ \text{kN-m}$$

$$M^F{}_{ba} = \frac{(P)\,(Length)}{8} = \frac{(4)\,(4)}{8} = 2\ \text{kN-m}$$

$$M^F{}_{bc} = -\frac{(w)(Length)^2}{12} = -\frac{(3)(4)^2}{12} = -4 \text{ kN-m}$$

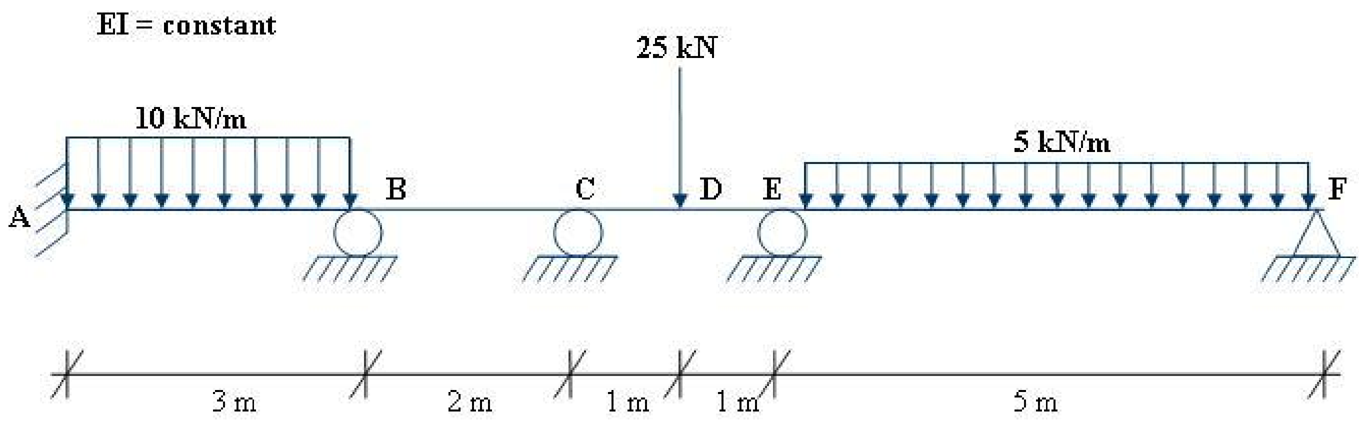
$$M^F{}_{cb} = \frac{(w)(Length)^2}{12} = \frac{(3)(4)^2}{12} = 4 \text{ kN-m}$$

$$DF_{ba} : DF_{bc} = 4EK_{ab} : 4EK_{bc} = 4(\frac{EI}{L})_{ab} : 4(\frac{EI}{L})_{bc} = \frac{1}{4} : \frac{1}{4} = 0.5 : 0.5$$

Assign DF at a and c: DFs are one at a and zero at c.

| Node | *a* | *b* | | *c* |
|---|---|---|---|---|
| Member | *ab* | | *bc* | |
| DF | 1 | 0.5 | 0.5 | 0 |
| MEM | $M_{ab}$ | $M_{ba}$ | $M_{bc}$ | $M_{cb}$ |
| EAM | −4 | | | |
| FEM | −2 | +2 | −4 | +4 |
| DM | −2 | | | |
| COM | | −1 | | |
| DM | | +1.5 | +1.5 | |
| COM | +0.8 | | | +0.8 |
| DM | -0.8 | | | |
| COM | | −0.4 | | |
| DM | | +0.2 | +0.2 | |
| COM | +0.1 | | | +0.1 |
| DM | −0.1 | | | |
| COM | | 0.0 | | |
| Sum | −4 | +2.3 | −2.3 | +4.9 |

## Example:

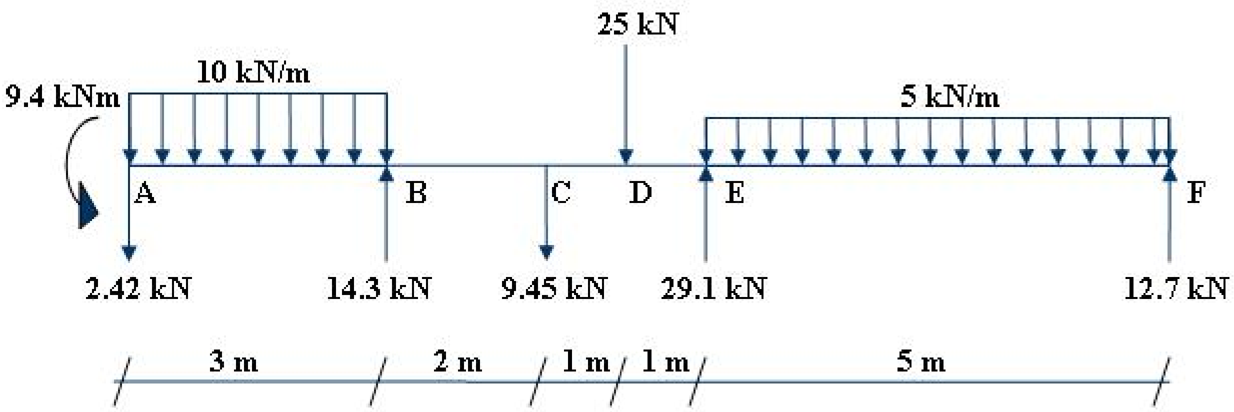



**Step 1**: Determine the distribution factors at joint B,C,E,F.

**Step 2**: Determine the fixed-end moments.

| | | AB | BA | BC | CB | CE | EC | EF | FE |
|---|---|---|---|---|---|---|---|---|---|
| | DF | | 0.4 | 0.6 | 0.5 | 0.5 | 0.714 | 0.286 | 1 |
| 1 | FEM | 7.50 | -7.50 | | | 6.25 | -6.25 | 10.4 | -10.4 |
| 2 | Balance joint B,C,E | | 3.00 | 4.50 | -3.13 | -3.125 | -2.96 | -1.187 | |
| 3 | Carryover | 1.50 | | -1.56 | 2.25 | -1.482 | -1.56 | | -0.593 |
| 4 | Balance joint B,C,E | | 0.625 | 0.94 | -0.38 | -0.384 | 1.12 | 0.447 | |
| 5 | Carryover | 0.313 | | -0.19 | 0.47 | 0.558 | -0.19 | | 0.223 |
| 6 | Balance joint B,C,E | | 0.077 | 0.12 | -0.51 | -0.513 | 0.14 | 0.055 | |
| 7 | Carryover | 0.038 | | -0.26 | 0.06 | 0.069 | -0.26 | | 0.027 |
| 8 | Balance joint B,C,E | | 0.103 | 0.15 | -0.06 | -0.063 | 0.18 | 0.073 | |
| 9 | Carryover | 0.051 | | -0.03 | 0.08 | 0.092 | -0.03 | | 0.037 |
| 10 | Balance joint B,C,E | | 0.013 | 0.02 | -0.08 | -0.084 | 0.02 | 0.009 | |
| 11 | Carryover | 0.01 | | -0.04 | 0.01 | 0.011 | -0.04 | | 0.005 |
| 12 | Final moments | 9.4 | -3.7 | 3.6 | -1.3 | 1.3 | -9.8 | 9.8 | -10.7 |

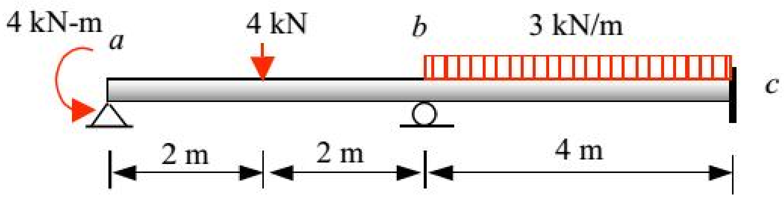
**Step 3**: Determine the member shear forces by equilibrium equations and show the Free Body Diagram.



**Modified stiffness**

The standard stiffness factor might be defined simply as K= 1/ L. The modified stiffness factor at the pinned ends of the elements, denoted as k*, must now be defined as 3/4 of the standard stiffness factor and for symmetrical structure be defined as 1/2 of the standard stiffness factor and for anti symmetrical structure be defined as 2/3 of the standard stiffness factor

**Example**: Find all the member-end moments of the beam shown. EI is constant for all members. Use the modified stiffness to account for the hinged end at node a.




$$M^F_{ab} = -\frac{(P)(Length)}{8} = -\frac{(4)(4)}{8} = -2 \text{ kN-m}$$

$$M^F_{ba} = \frac{(P)(Length)}{8} = \frac{(4)(4)}{8} = 2 \text{ kN-m}$$

$$M^F_{bc} = -\frac{(w)(Length)^2}{12} = -\frac{(3)(4)^2}{12} = -4 \text{ kN-m}$$

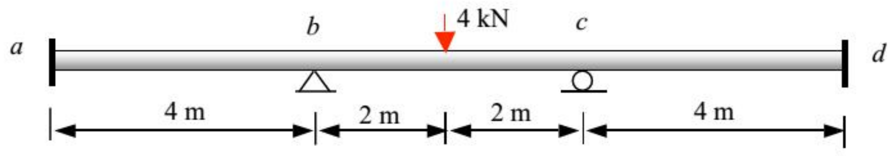
$$M^F_{cb} = \frac{(w)(Length)^2}{12} = \frac{(3)(4)^2}{12} = 4 \text{ kN-m}$$

$$\mathbf{DF}_{ba} : \mathbf{DF}_{bc} = 3EK_{ab} : 4EK_{bc} = 3\left(\frac{EI}{L}\right)_{ab} : 4\left(\frac{EI}{L}\right)_{bc} = \frac{3}{7} : \frac{4}{7} = 0.43 : 0.57$$

Assign DF at a andc: DFs are one at a and zero c.

| Node | a | b | | c |
|---|---|---|---|---|
| Member | ab | | bc | |
| DF | 0 | 0.43 | 0.57 | 0 |
| MEM | $M_{ab}$ | $M_{ba}$ | $M_{bc}$ | $M_{cb}$ |
| EAM | -4 | | | |
| FEM | -2 | +2 | -4 | +4 |
| DM | -2 | | | |
| COM | | -1 | | |
| DM | | +1.3 | +1.7 | |
| COM | 0.0 | | | +0.8 |
| SUM | -4.0 | +2.3 | -2.3 | +4.8 |

**Example: Find all the member-end moments of the beam shown. EI is constant for all members. Use the modified stiffness to account for the symmetric span between nodes b and c.**

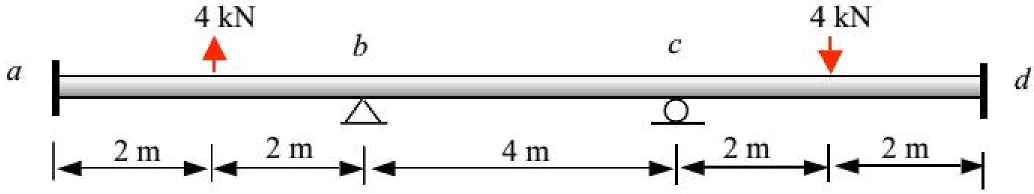



Compute DF at b

$$DF_{ba} : DF_{bc} = 4EK_{ab} : 2EK_{bc} = 4\left(\frac{EI}{L}\right)_{ab} : 2\left(\frac{EI}{L}\right)_{bc} = \frac{4}{6} : \frac{2}{6} = 0.67 : 0.33$$

| Node | a | b | | c |
|---|---|---|---|---|
| Member | ab | | bc | |
| DF | 0 | 0.67 | 0.33 | 0 |
| MEM | $M_{ab}$ | $M_{ba}$ | $M_{bc}$ | $M_{cb}$ |
| EAM | | | | |
| FEM | | | -4 | +4 |
| DM | | +2.67 | +1.33 | −1.33 |
| COM | +1.33 | | | |
| SUM | +1.33 | +2.67 | −2.67 | +2.67 |

**Example:** Find all the member-end moments of the beam shown. EI is constant for all members. Use the modified stiffness to account for the anti-symmetric span between nodes band c.

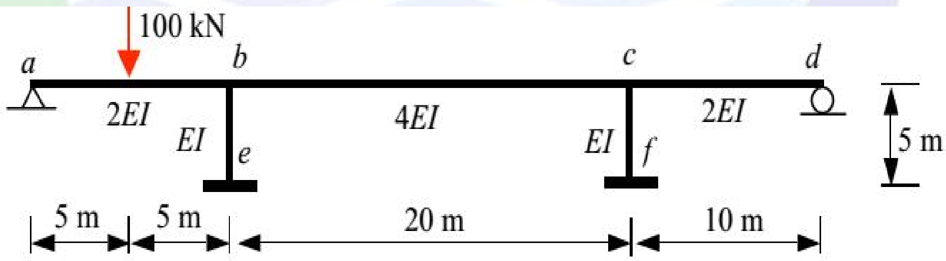



Compute DF at b

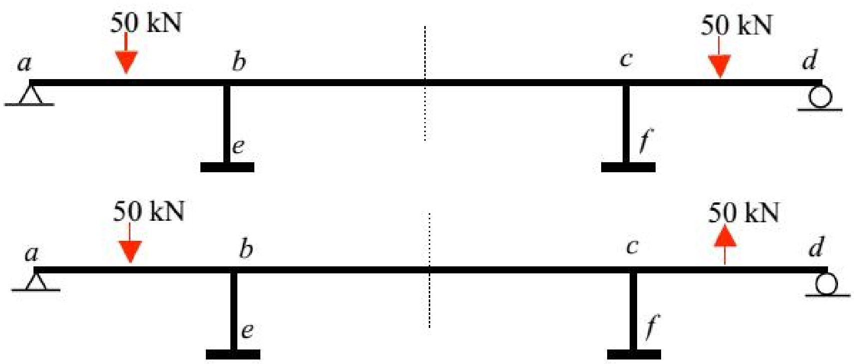
$$\mathbf{DF}_{ba} : \mathbf{DF}_{bc} = 4EK_{ab} : 6EK_{bc} = 4\left(\frac{EI}{L}\right)_{ab} : 6\left(\frac{EI}{L}\right)_{bc} = \frac{4}{10} : \frac{6}{10} = 0.4 : 0.6$$

| Node | $a$ | $b$ | | $c$ |
|---|---|---|---|---|
| Member | $ab$ | | $bc$ | |
| DF | 0 | 0.4 | 0.6 | 0 |
| MEM | $M_{ab}$ | $M_{ba}$ | $M_{bc}$ | $M_{cb}$ |
| EAM | | | | |
| FEM | +2 | −2 | | |
| DM | | +0.8 | +1.2 | 1.2 |
| COM | +0.4 | | | |
| SUM | +0.4 | −1.2 | +1.2 | +1.2 |

Example: Find all the member-end moments of the frame shown.




Symmetry of the structure calls for the decomposition of the load into a symmetric component and an anti-symmetric component.




*Symmetric and anti-symmetric loads.*

$$M^F{}_{ab} = -\frac{(P)\,(Length)}{8} = -\frac{(50)\,(10)}{8} = -\,62.5 \text{ kN-m}$$

$$M^F{}_{ba} = \frac{(P)\,(Length)}{8} = \frac{(50)\,(10)}{8} = 62.5 \text{ kN-m}$$

$$\mathbf{DF_{ba} : DF_{bc} : DF_{be} = 3EK_{ab} : 2EK_{bc} : 4EK_{bc}}$$

$$= 3(\frac{2EI}{10})_{ab} : 2(\frac{4EI}{20})_{bc} : 4(\frac{EI}{5})_{bc} = \frac{6}{10} : \frac{8}{20} : \frac{4}{5}$$

$$= \frac{3}{5} : \frac{2}{5} : \frac{4}{5} = \frac{3}{9} : \frac{2}{9} : \frac{4}{9} = 0.33 : 0.22 : 0.45$$

Compute DF at $b$ (Anti-symmetric Case):

$$\mathbf{DF_{ba} : DF_{bc} : DF_{be} = 3EK_{ab} : 6EK_{bc} : 4EK_{bc}}$$

$$= 3(\frac{2EI}{10})_{ab} : 6(\frac{4EI}{20})_{bc} : 4(\frac{EI}{5})_{bc} = \frac{6}{10} : \frac{24}{20} : \frac{4}{5}$$

$$= \frac{3}{5} : \frac{6}{5} : \frac{4}{5} = \frac{3}{13} : \frac{6}{13} : \frac{4}{13} = 0.23 : 0.46 : 0.21$$

Moment Distribution Table for a Symmetric Case and an Anti-symmetric Case

| | *Symmetric Case* | | | | | *Anti-symmetric Case* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Node | *a* | *b* | | | *e* | *a* | *b* | | | *e* |
| Member | *ab* | | *bc* | *be* | | *ab* | | *bc* | *be* | |
| DF | 1 | 0.33 | 0.22 | 0.45 | 0 | 1 | 0.23 | 0.46 | 0.31 | 0 |
| MEM | $M_{ab}$ | $M_{ba}$ | $M_{bc}$ | $M_{be}$ | $M_{eb}$ | $M_{ab}$ | $M_{ba}$ | $M_{bc}$ | $M_{be}$ | $M_{eb}$ |
| EAM | | | | | | | | | | |
| FEM | −62.5 | 62.5 | | | | −62.5 | 62.5 | | | |
| DM | 62.5 | | | | | 62.5 | | | | |
| COM | | 31.3 | | | | | 31.3 | | | |
| DM | | −31.0 | −20.6 | −42.2 | | | −21.6 | −43.2 | −29.0 | |
| COM | 0.0 | | | | −21.1 | 0.0 | | | | −14.5 |
| SUM | 0.0 | 62.8 | −20.6 | −42.2 | −21.1 | | 72.2 | −43.2 | −29.0 | −14.5 |

$M_{ab} = 0.0 + 0.0 = 0.0$ kN-m

$M_{ba} = 62.8 + (72.2) = 135.0$ kN-m

$M_{bc} = -20.6 + (-43.2) = -63.8$ kN-m

$M_{be} = -42.2 + (-29.0) = -71.2$ kN-m

$M_{eb} = -21.1 + (-14.5) = -35.6$ kN-m

$M_{dc} = 0.0 + 0.0 = 0.0$ kN-m

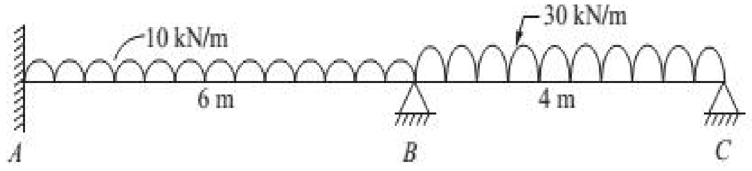
$M_{cd} = -62.8 + (72.2) = 9.4$ kN-m

$M_{cb} = 20.6 + (-43.2) = -22.6$ kN-m

$M_{cf} = 42.2 + (-29.0) = 13.2$ kN-m

$M_{fc} = 21.1 + (-14.5) = 6.6$ kN-m

Example




**Evaluate D.F**

| Joint | Members | Relative Stiffness (k) | Sum Σk | Distribution Factors (k/Σ |
|---|---|---|---|---|
| | BA | $I/6 = 0.167I$ | | 0.47 |
| B | | | $0.3555I$ | |
| | BC | $\frac{3}{4} \times \frac{I}{4} = 0.188I$ | | 0.53 |

**Determine F.E.M.**

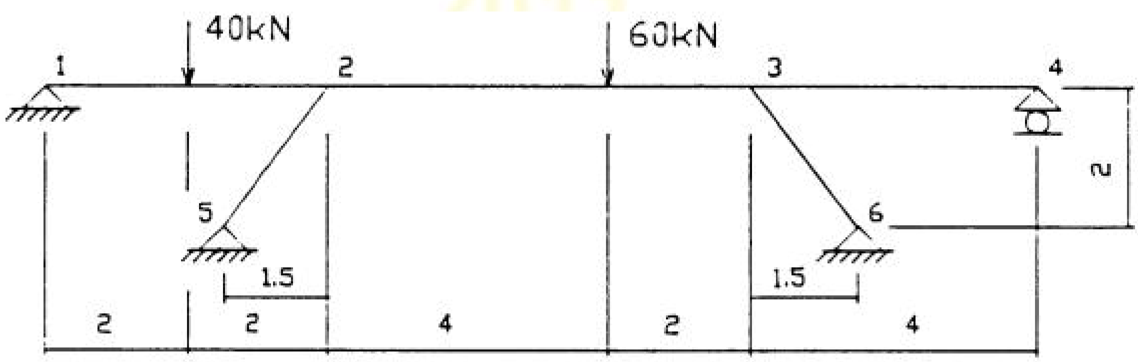
$$M_{FAB} = \frac{-10 \times 6^2}{12} = -30 \text{ kNm}, \; M_{FBA} = +30 \text{ kNm}$$

$$M_{FBC} = \frac{-30 \times 4^2}{12} = -40 \text{ kNm}, \; M_{FCB} = +40 \text{ kNm}$$

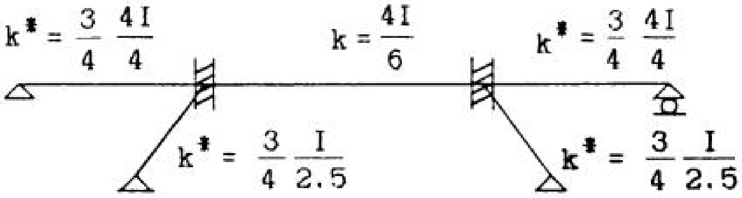
| Joint | A | B | | C |
|---|---|---|---|---|
| Members | AB | BA | BC | CB |
| DF | 0 | 0.47 | 0.53 | 1 |
| FEMS | −30.00 | +30.00 | −40.00 | +40.00 |
| Bal | – | – | – | −40.00 |
| Co | – | – | −20.00 | |
| Total | −30.00 | +30.00 | −60.00 | 0.00 |
| Bal | – | +14.10 | +15.90 | – |
| Co | +7.05 | – | – | – |
| Final | −22.95 | +44.10 | −44.10 | 0.00 |

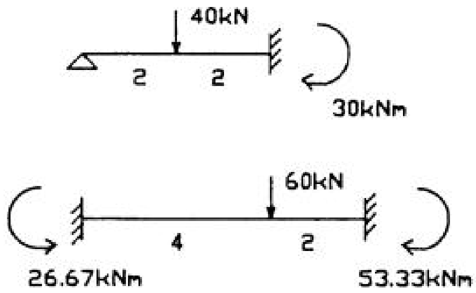
**Example:** by using moment distribution Calculate the moment at the end of members

**E constant throughout. values I for beams 4I; columns I.**



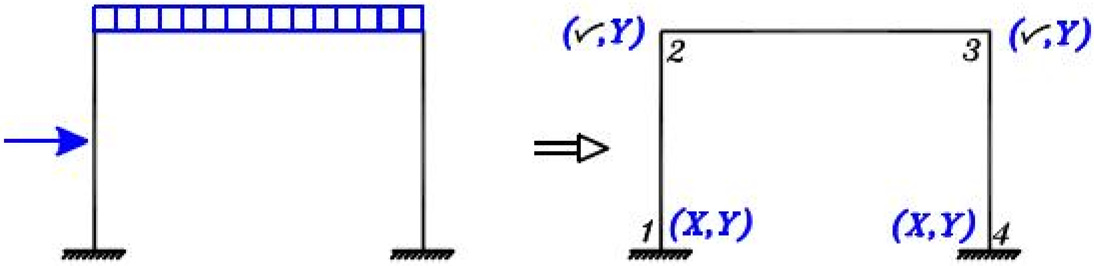
at first calculate distribution factors and fixed end moments from

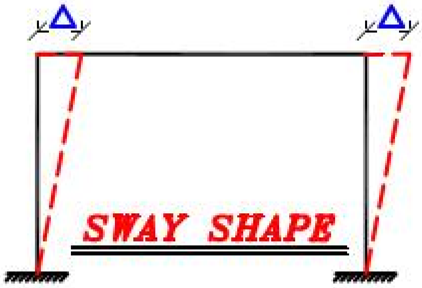


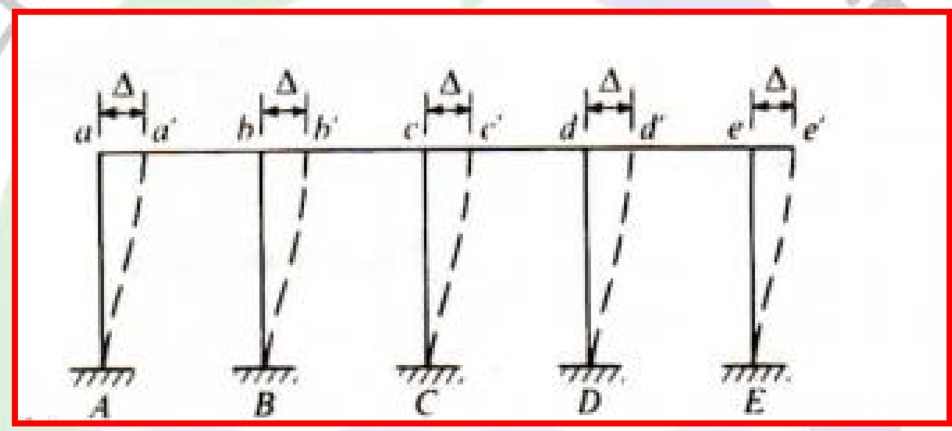




| 21 | 25 | 23 | 32 | 36 | 34 | |
|---|---|---|---|---|---|---|
| 0.44 | 0.17 | 0.39 | 0.39 | 0.17 | 0.44 | Distribution Factors |
| −30 | | 26.67 | −53.33 | | | Fixed End Moments |
| 1.47 | 0.56 | 1.30 | 0.65 | | | |
| | | 10.28 | 20.55 | 8.96 | 23.18 | |
| −4.52 | −1.75 | −4.01 | −2.00 | | | |
| | | 0.39 | 0.78 | 0.34 | 0.88 | |
| −0.17 | −0.07 | −0.15 | −0.08 | | | |
| | | . . . | +0.03 | 0.01 | 0.04 | |
| −33.20 | −1.26 | 34.48 | −33.40 | 9.30 | 24.10 | Final End Moments (kN m) |

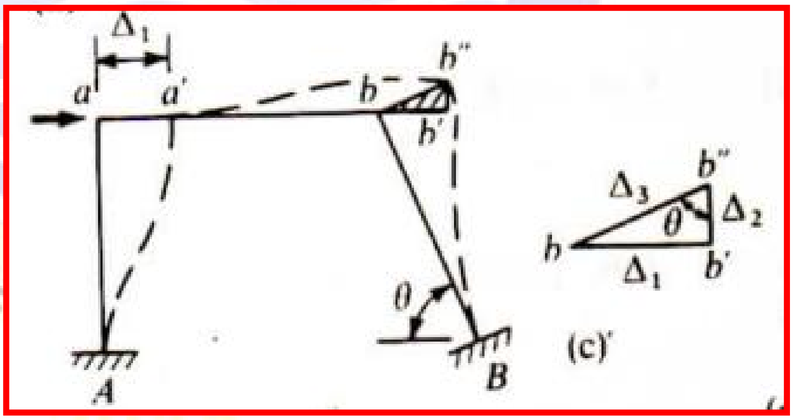
## MOMENT DISTRIBUTION WITH JOINT TRANSLATION (SIDE SWAY)

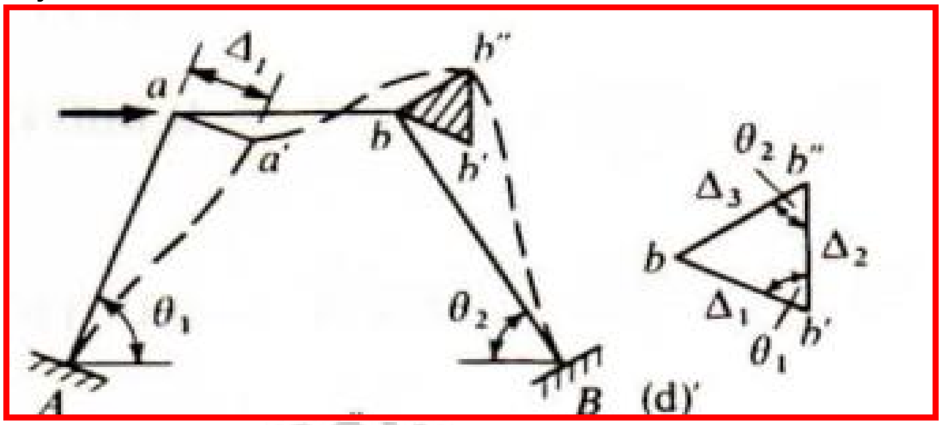



$$M = (6EI/L^2)\,\Delta$$






## Analysis of sway frames is done in the following way.

(1) The moment distribution was carried out by assuming that the joints do not get displaced. The moments obtained from the moment distribution table are called nonsway moments.

(2) The horizontal reactions at the base of the columns are found out which helps in finding the net out of balance force. A prop is assumed to act opposite to the direction of the above force.

(3) Allow the frame to sway in the direction of sway force which is equal and opposite to the prop force (which acts along the axis of the beam level). Let the actual prop force be X.

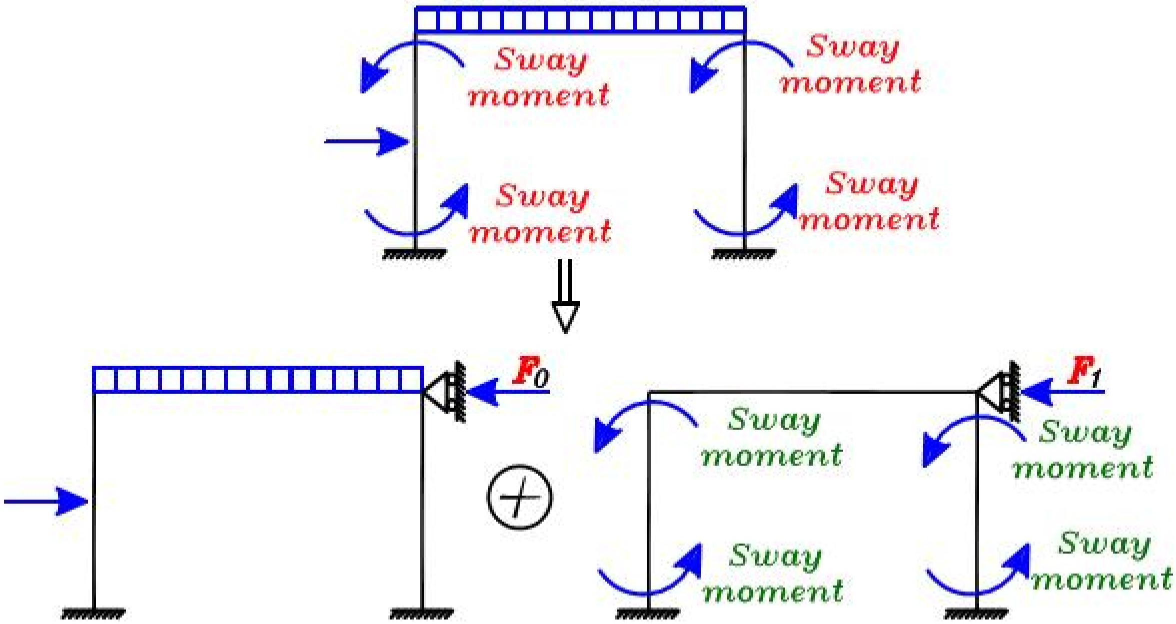
(4) Perform the sway moment distribution, by assuming arbitrary moments as per the joint moment ratios.

Calculate the displacement force (Y) from the sway moment distribution.

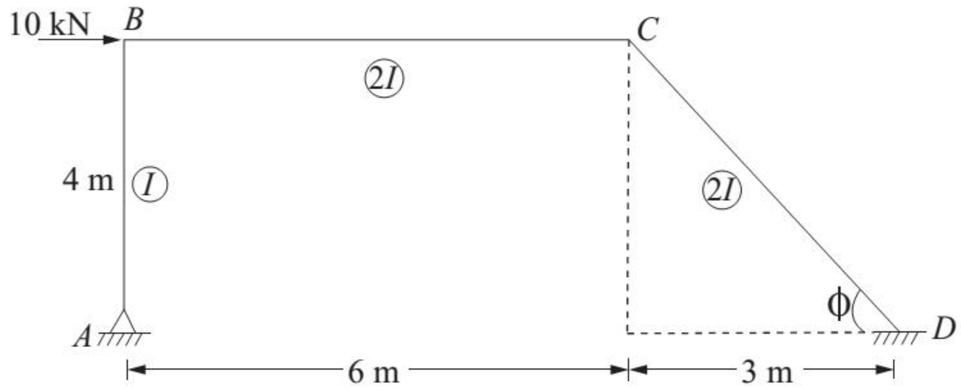
(6) The correction factor/sway factor k = X/Y. The correction factor gives the direction of sway of the frame. If the value of k is positive, then the frame sways in the direction of the sway solution. If the value of k is negative, then the frame sways in the direction opposite to that of assumed sway.

(7) The final end moments are obtained as:

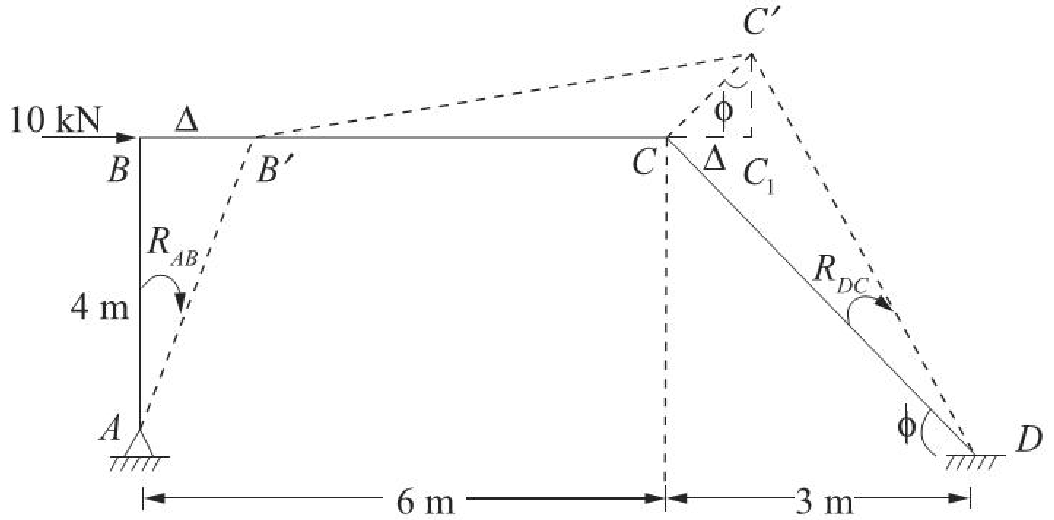
**Final end moments = Nonsway moments +'Correction factor' × Sway moments**




**Example** : Analyze the rigid frame shown in figure by the moment distribution method and draw the BM diagram Support A is hinged and support D is fixed support.




Determination of sway (Δ) of members




Sway of $AB = \Delta_1 = \Delta_{AB} = BB' = +\Delta$

Sway of $BC = \Delta_2 = \Delta_{BC} = -C_1C' = -0.75\Delta$

Sway of $CD = \Delta_3 = \Delta_{CD} = +CC' = +1.25\Delta$

$M_{FAB} = 0$

$M_{FAB} = -3EI_1\Delta_1/l_1^2 = -3EI\Delta/4^2 = -\dfrac{3}{16}EI\Delta$

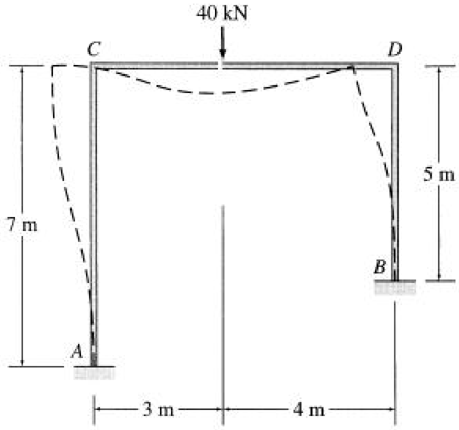
$M_{FBC} = M_{FCB} = -6EI_2\Delta_2/l_2^2 = -6E(2I)(-0.75\Delta)/6^2 = \dfrac{EI\Delta}{4}$

$M_{FCD} = M_{FDC} = -6EI_3\Delta_3/l_3^2 = -6E(2I)(-0.25\Delta)/5^2 = \dfrac{3EI\Delta}{5}$

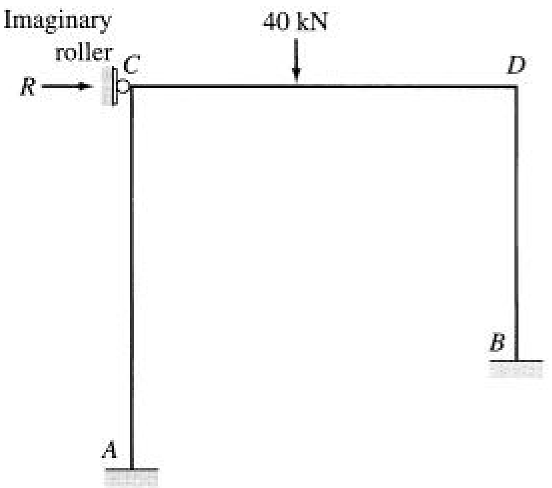
| Joint | Members | Relative Stiffness Values I/l | Σk | DF = k/Σk |
|---|---|---|---|---|
| | BA | $\frac{3}{4}\left(\frac{I}{4}\right) = \frac{3}{16}I$ | | 0.36 |
| B | | = 0.1875I | = 0.5175I | |
| | BC | $\frac{2I}{6} = 0.33I$ | | 0.64 |
| | CB | $\frac{2I}{6} = 0.33I$ | | 0.45 |
| C | | | 0.73I | |
| | CD | $\frac{2I}{5} = 0.4I$ | | 0.55 |

| Joint | A | B | | C | | D |
|---|---|---|---|---|---|---|
| Members | AB | BA | BC ⟷ | CB | CD ⟶ | DC |
| DF | 1 | 0.36 | 0.64 | 0.45 | 0.55 | 0 |
| FEMS | | −15.00 | +20.00 | +20.00 | −48.00 | −48.00 |
| Bal | | −1.80 | −3.20 | +12.60 | +15.40 | |
| CO | | | +6.30 | −1.60 | | +7.70 |
| Bal | | −2.27 | −4.03 | +0.72 | +0.88 | |
| CO | | | +0.36 | −2.02 | | +0.44 |
| Bal | | +0.13 | −0.23 | +0.91 | +1.11 | |
| CO | | | +0.46 | −0.12 | | +0.56 |
| Bal | | −0.17 | −0.29 | +0.05 | +0.07 | |
| CO | | | +0.03 | −0.15 | | +0.04 |
| Bal | | −0.01 | −0.02 | +0.07 | +0.08 | |
| CO | | | +0.04 | −0.01 | | +0.04 |
| Bal | | −0.01 | −0.03 | +0.005 | +0.005 | |
| | | −19.39 | 19.39 | 30.46 | −30.46 | −39.22 |

**Example**: Determine the member end moments for the frame shown by using the moment-distribution method




*EI* = constant
(a) Frame



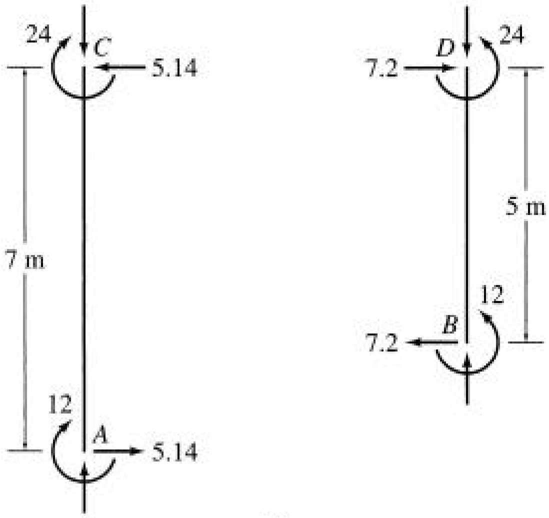

(b) Frame with Sidesway Prevented

$$FEM_{CD} = +39.2 \text{ kN}\cdot\text{m} \qquad FEM_{DC} = -29.4 \text{ kN}\cdot\text{m}$$

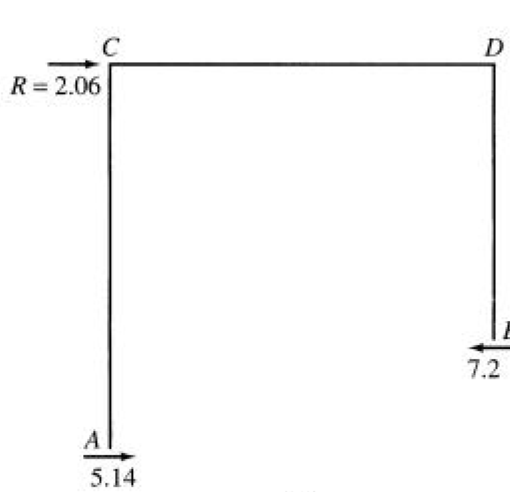
$$FEM_{AC} = FEM_{CA} = FEM_{BD} = FEM_{DB} = 0$$

| *AC* | *CA* | *CD* | *DC* | *DB* | *BD* |
|---|---|---|---|---|---|
| | 0.5 | 0.5 | 0.417 | 0.583 | |
| | | +39.2 | −29.4 | | |
| | −19.6 | −19.6 | +12.3 | +17.1 | |
| − 9.8 | | + 6.2 | − 9.8 | | + 8.6 |
| | − 3.1 | − 3.1 | + 4.1 | + 5.7 | |
| − 1.6 | | + 2.1 | − 1.6 | | + 2.9 |
| | − 1.1 | − 1.1 | + 0.7 | + 0.9 | |
| − 0.6 | | + 0.4 | − 0.6 | | + 0.5 |
| | − 0.2 | − 0.2 | + 0.3 | + 0.3 | |
| −12 | −24 | +23.9 | −24 | +24 | +12 |

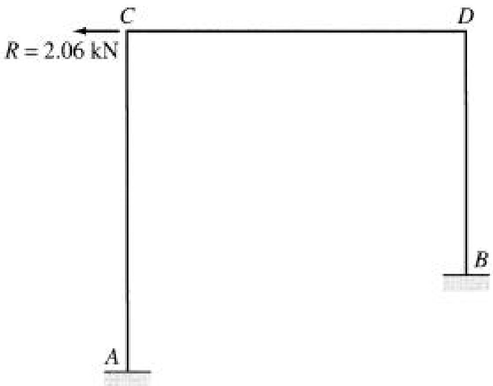
(c) Member End Moments for Frame with Sidesway Prevented — $M_O$ Moments



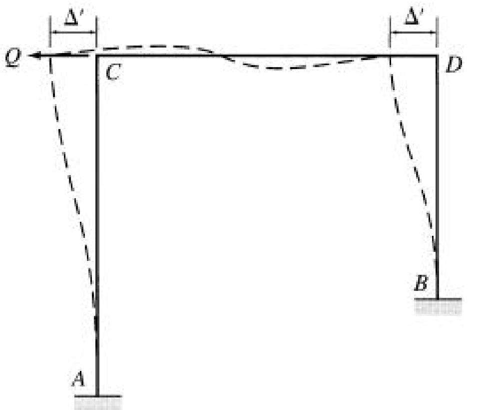

(d)



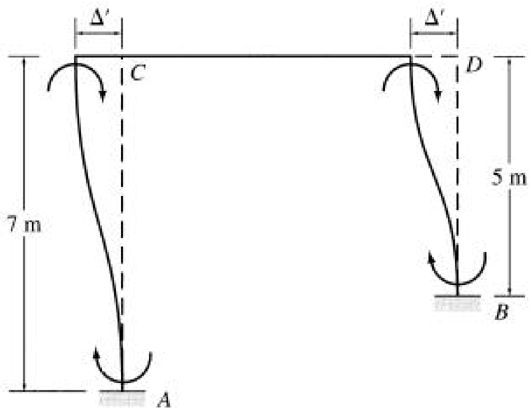

(e)




(f) Frame Subjected to $R = 2.06$ kN — $M_R$ Moments




(g) Frame Subjected to an Arbitrary Translation $\Delta'$ — $M_Q$ Moments




(h) Fixed-End Moments Due to Known Translation $\Delta'$

$$\text{FEM}_{AC} = \text{FEM}_{CA} = -\frac{6EI\Delta'}{(7)^2} = -\frac{6EI\Delta'}{49}$$

$$\text{FEM}_{BD} = \text{FEM}_{DB} = -\frac{6EI\Delta'}{(5)^2} = -\frac{6EI\Delta'}{25}$$

$$\text{FEM}_{CD} = \text{FEM}_{DC} = 0$$

assume the fixed-end moment $FEM_{AC}$ to be50 kN.m; that is,

$$\text{FEM}_{AC} = \text{FEM}_{CA} = -\frac{6EI\Delta'}{49} = -50 \text{ kN} \cdot \text{m}$$

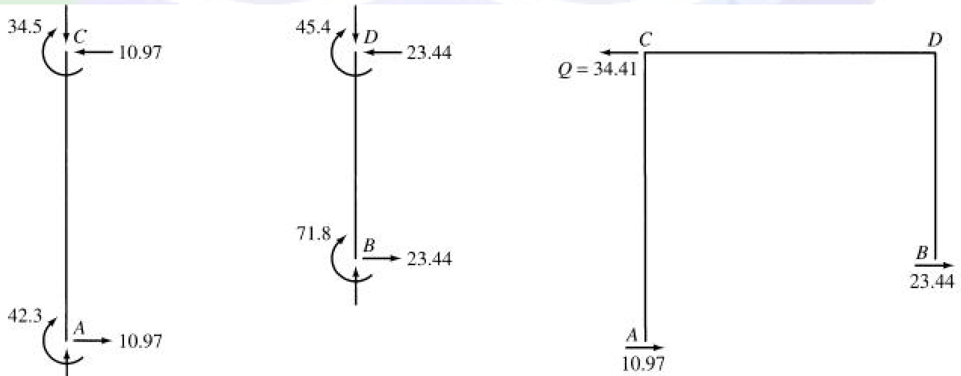
By solving for $\Delta'$, we obtain

$$\Delta' = \frac{408.33}{EI}$$

$$\text{FEM}_{BD} = \text{FEM}_{DB} = -\frac{6(408.33)}{25} = -98 \text{ kN} \cdot \text{m}$$

| *AC* | *CA* | *CD* | *DC* | *DB* | *BD* |
|---|---|---|---|---|---|
| | 0.5 | 0.5 | 0.417 | 0.583 | |
| −50 | −50 | | | −98 | −98 |
| | +25 | +25 | +40.9 | +57.1 | |
| +12.5 | | +20.5 | +12.5 | | +28.6 |
| | −10.3 | −10.3 | − 5.2 | − 7.3 | |
| − 5.2 | | − 2.6 | − 5.2 | | − 3.7 |
| | + 1.3 | + 1.3 | + 2.2 | + 3 | |
| + 0.7 | | + 1.1 | + 0.7 | | + 1.5 |
| | − 0.6 | − 0.6 | − 0.3 | − 0.4 | |
| − 0.3 | | − 0.2 | − 0.3 | | − 0.2 |
| | + 0.1 | + 0.1 | + 0.1 | + 0.2 | |
| −42.3 | −34.5 | +34.3 | +45.4 | −45.4 | −71.8 |

(i) Member End Moments Due to Known Translation $\Delta'$ — $M_Q$ Moments



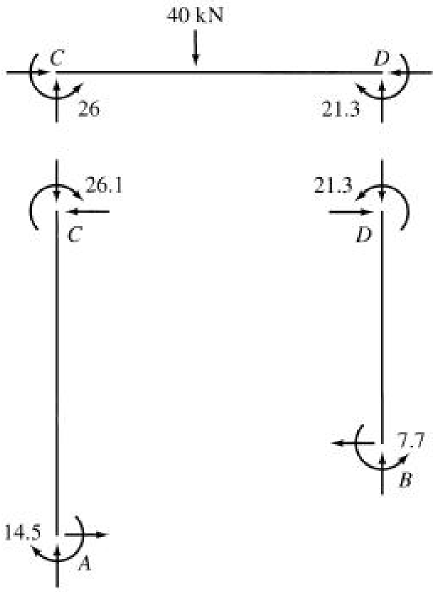

(j) Evaluation of $Q$

$$+ \rightarrow \sum F_X = 0$$

$$-Q + 10.97 + 23.44 = 0$$

$$Q = 34.41 \text{ kN} \leftarrow$$

ratio R/Q=2.06/34:41.




(k) Actual Member End Moments (kN · m)

$$M_{AC} = -12 + \left(\frac{2.06}{34.41}\right)(-42.3) = -14.5 \text{ kN} \cdot \text{m}$$

$$M_{CA} = -24 + \left(\frac{2.06}{34.41}\right)(-34.5) = -26.1 \text{ kN} \cdot \text{m}$$

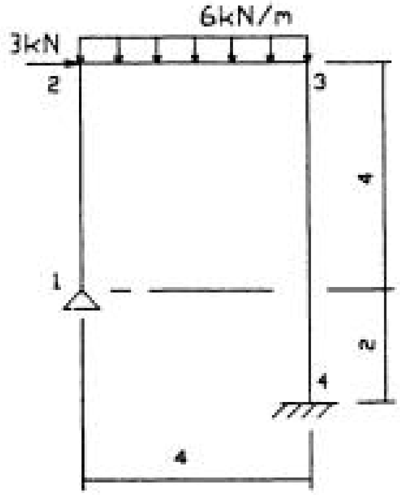
$$M_{CD} = 23.9 + \left(\frac{2.06}{34.41}\right)(34.3) = 26 \text{ kN} \cdot \text{m}$$

$$M_{DC} = -24 + \left(\frac{2.06}{34.41}\right)(45.4) = -21.3 \text{ kN} \cdot \text{m}$$

$$M_{DB} = 24 + \left(\frac{2.06}{34.41}\right)(-45.4) = 21.3 \text{ kN} \cdot \text{m}$$

$$M_{BD} = 12 + \left(\frac{2.06}{34.41}\right)(-71.8) = 7.7 \text{ kN} \cdot \text{m}$$

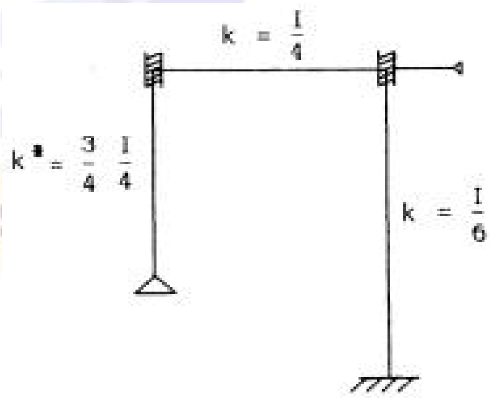
**Example**: Determine the member end moments for the frame shown by using the moment-distribution method



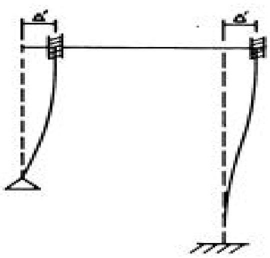

find D.F.for members then solve for no sway moment and determine the reaction of roller suport

Part A—No-sway Moment Distribution

| 21 | 23 | 32 | 34 | 43 |
|---|---|---|---|---|
| 0.43 | 0.57 | 0.60 | 0.40 | — |
| | +8.00 | −8.00 | | |
| −3.44 | −4.56 | −2.28 | | |
| | 3.09 | 6.17 | 4.11 | 2.06 |
| −1.33 | −1.76 | −0.88 | | |
| | 0.27 | 0.53 | 0.35 | 0.18 |
| −0.11 | −0.15 | −0.08 | | |
| | . . . | 0.05 | 0.03 | 0.01 |
| −4.88 | 4.89 | −4.49 | 4.49 | 2.25 |




reaction force = -3.10 KN, solve for sway moment

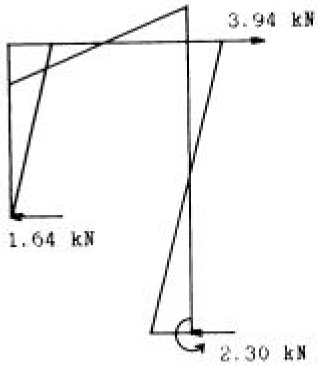


Select $\Delta'$ such that

$$\frac{3EI}{16}\Delta' = 10 \qquad \therefore \quad \Delta' = \frac{160}{3EI}$$

then $m_{21} = 10$; $m_{34} = m_{43} = 8.89$

| 21 | 23 | 32 | 34 | 43 | |
|---|---|---|---|---|---|
| 0.43 | 0.57 | 0.60 | 0.40 | — | |
| 10.00 | | | 8.89 | 8.89 | |
| −4.30 | −5.70 | −2.85 | | | |
| | −1.81 | −3.62 | −2.42 | −1.21 | |
| 0.78 | 1.03 | 0.52 | | | |
| | −0.16 | −0.31 | −0.21 | −0.10 | |
| 0.07 | 0.09 | 0.05 | | | |
| | . . . | −0.03 | −0.02 | . . . | |
| 6.55 | −6.55 | −6.24 | 6.24 | 7.58 | Arbitrary sway moments |
| 5.17 | −5.17 | −4.93 | 4.93 | 5.99 | Sway moments |
| −4.88 | 4.88 | −4.49 | 4.49 | 2.25 | No-sway moments |
| 0.29 | −0.29 | −9.42 | 9.42 | 8.24 | Final moments |




$F_{11} = 3.94$ kN

Equilibrium equation:

$$F_{10} + \frac{\Delta}{\Delta'}F_{11} = 0$$

$$\therefore \quad \frac{\Delta}{\Delta'} = \frac{3.10}{3.94}$$

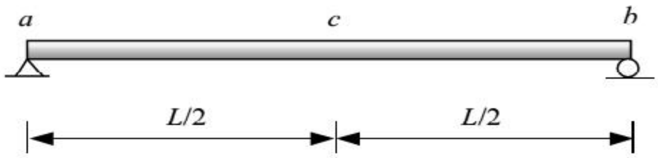
$$= 0.79$$

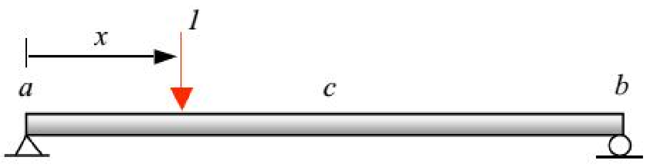
Adjust arbitrary sway moments by 0.79

# INFLUENCE LINES

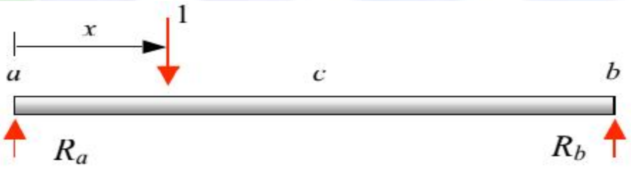
An influence line is a diagram showing the variation of a particular force (i.e., reaction, shear, bending moment at a section, stress at a point or other direct function) due to a unit load moving across the structure.

**Example**: Consider the beam shown below. determine the influence lines for Rb, Vc and Mc.



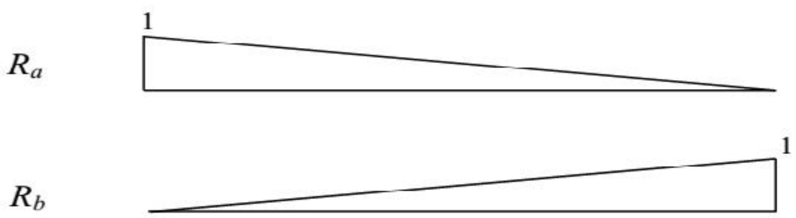




*Location of the unit load is the variable.*



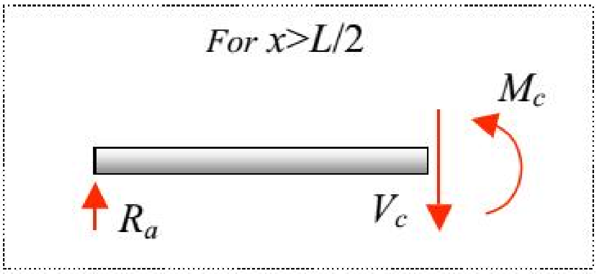

*FBD to solve for $R_a$ and $R_b$.*

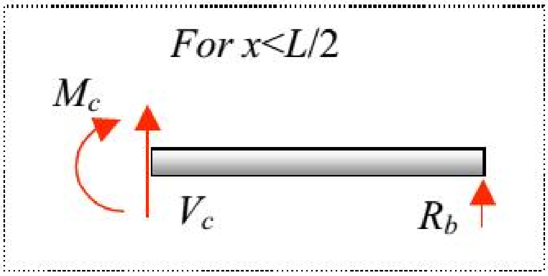
$$\Sigma M_b = 0 \quad \Rightarrow \quad R_a = (L-x)/L \qquad \Sigma M_a = 0 \quad \Rightarrow \quad R_b = x/L$$




*Influence lines for $R_a$ and $R_b$.*

For Vc and Mc, we need to solve for them using appropriate free-body-diagrams.

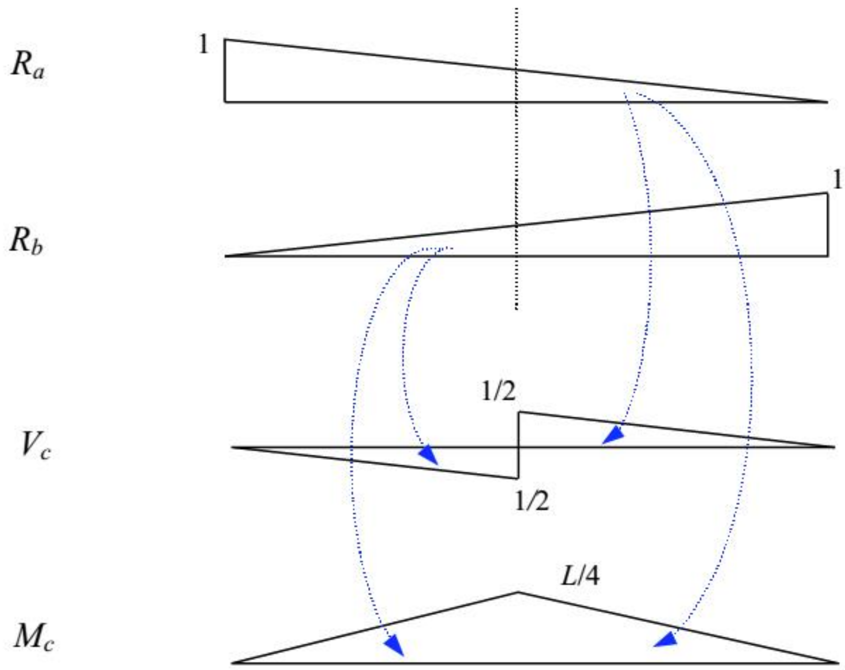



The above FBDs are selected so that we do not have to include the unit load in the equilibrium equations. Consequently, the left FBD is valid for the unit load being located to the right of section c(x>L/2 ) and the right FBD is for the unit load located to the left of the section(x<L/2 ). From each FBD, we can obtain the expressions of Vc and Mc as functions of Ra and Rb.

| Left FBD: Valid for $x>L/2$ | Right FBD: Valid For $x<L/2$ |
|---|---|
| $V_c = R_a$ | $V_c = -R_b$ |
| $M_c = R_a L/2$ | $M_c = R_b L/2$ |

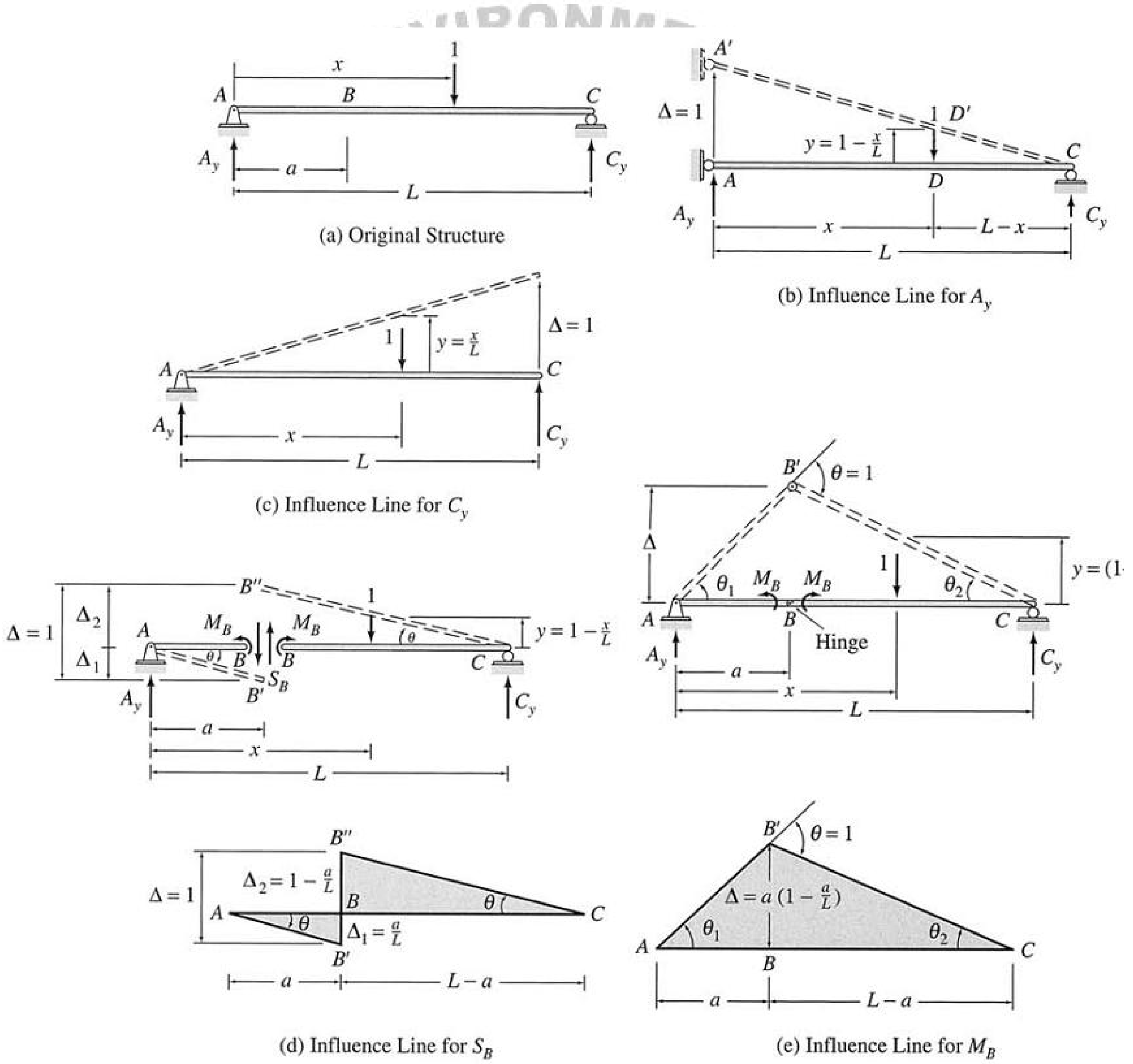
Using the influence lines of Ra and Rb we can construct the influence lines of Vc and Mc by cut-and-paste and adjusting for the factors L/2 and the negative sign.




## INFLUENCE LINE BY VIRTUAL WORK METHOD (MULLER METHOD)

The influence line for a force (or moment) response function is given by the deflected shape of the released structure obtained by removing the restraint corresponding to the response function from the original structure and by giving the released structure a unit displacement (or rotation) at the location and in the direction of the response function, so that only the response function and the unit load perform external work.

**Example:**




(a) Original Structure

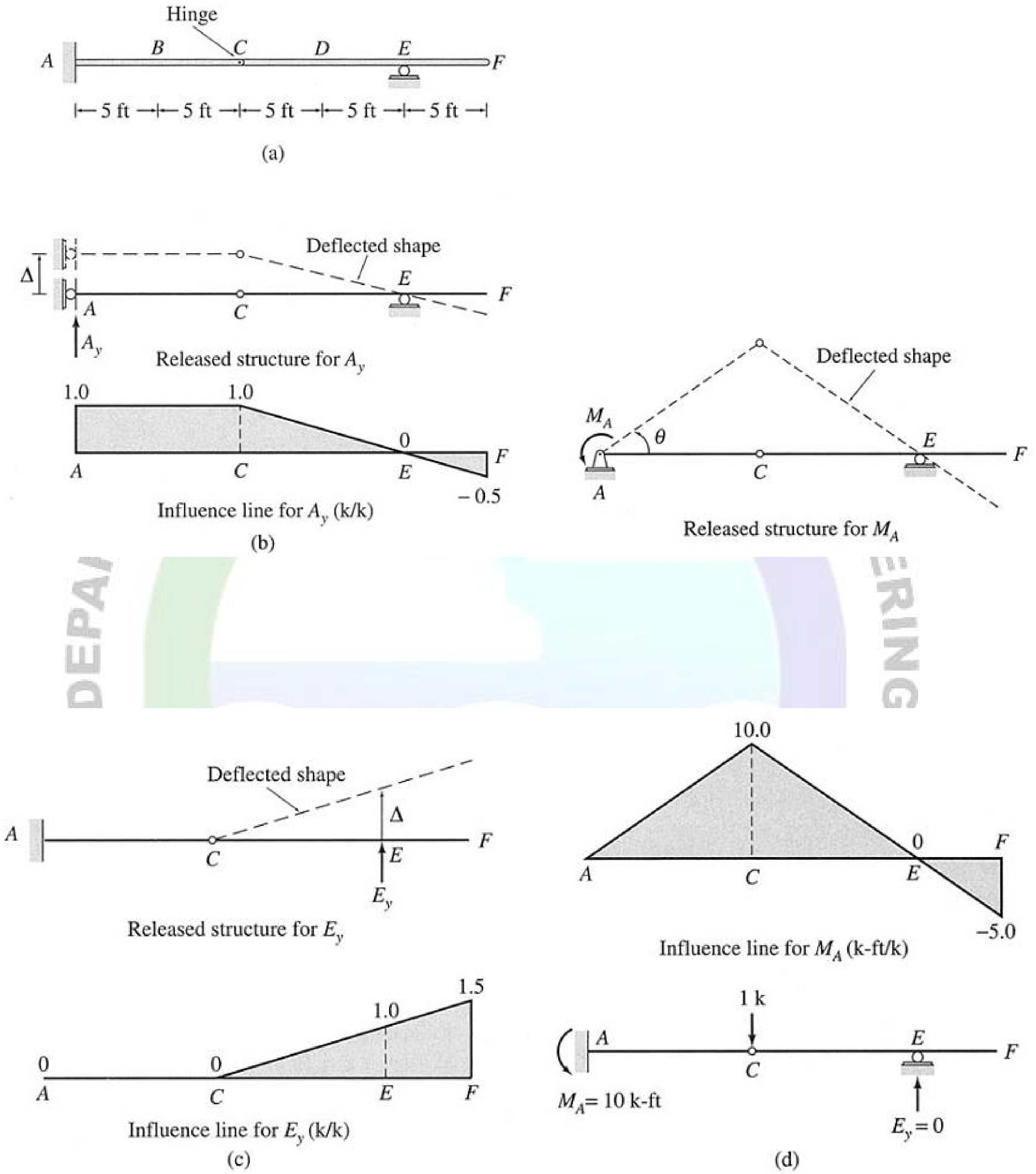
(b) Influence Line for $A_y$

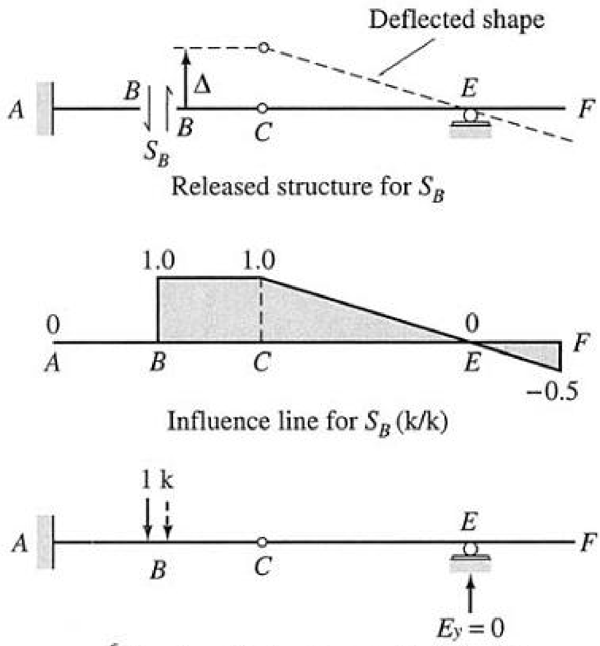
(c) Influence Line for $C_y$

(d) Influence Line for $S_B$

(e) Influence Line for $M_B$

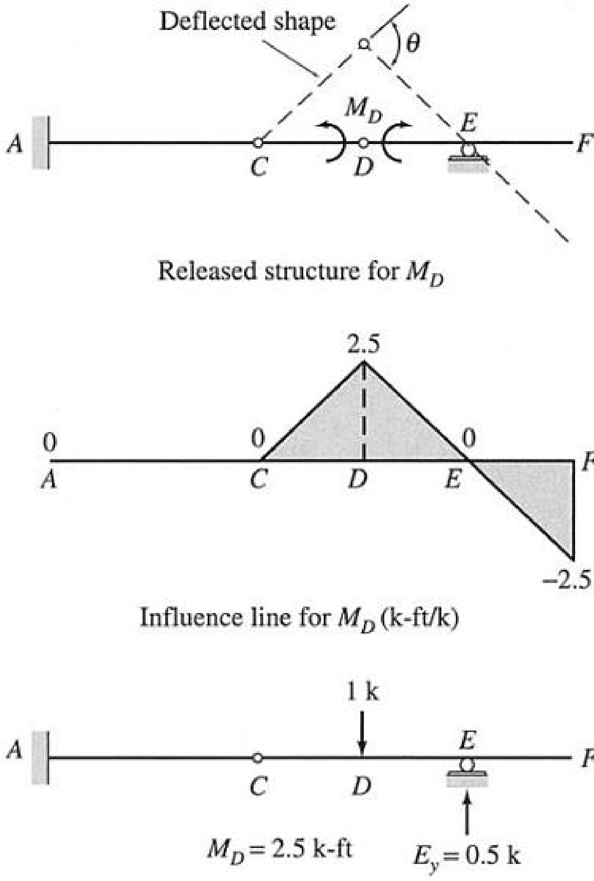
**Example**: Find influence lines for ($R_A$, $M_A$, $R_E$ ,MD,$V_D$


Hinge
A
B
C
D
E
F
5 ft
5 ft
5 ft
5 ft
5 ft
(a)
Deflected shape
Δ
A
C
E
F
$A_y$
Released structure for $A_y$
1.0
1.0
0
−0.5
Influence line for $A_y$ (k/k)
(b)
Deflected shape
$M_A$
θ
Released structure for $M_A$
10.0
0
−5.0
Influence line for $M_A$ (k-ft/k)
Deflected shape
Δ
$E_y$
Released structure for $E_y$
0
0
1.0
1.5
Influence line for $E_y$ (k/k)
(c)
1 k
$M_A$= 10 k-ft
$E_y$ = 0
(d)




$$S_B = \begin{cases} 0 \text{ when 1 k is at just to the left of } B \\ 1\text{ k when 1 k is at just to the right of } B \end{cases}$$

(e)




(f)

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Kassimali, A. , " STRUCTURAL ANALYSIS",3ed ed. Nelson, a division of Thomson Canada Limited. 2005.

Mau, S. T., " Fundamentals of Structural Analysis", The Library of Congress, 2003.