Ministry of Higher Education and Scientific

College of Education for Pure Sciences

Physics Department

Mechanics

First Stage

Lecturers: -

Dr .Odai Falah Ameen

Ms. Asmaa Zaki Khalel

Chapter 1

Physical Quantity ,Units and Vectors

Physical Quantity

Physics is an empirical study. Everything we know that the physical world and about the principles that govern its behavior has been learned through observations of the phenomena of nature. The ultimate test of any physical theory is its agreement with observations and measurements of physical phenomena. Thus physics is inherently a science of measurement.

Any number or set of numbers used for a quantitative description of a physical phenomenon is called a physical quantity. To define a physical quantity we must either specify a procedure for measuring the quantity or specify a way to calculate the quantity from other quantities that can be measured.

Units

The quantitative measure of a physical quantity is a number which expresses the ratio of the magnitude of the quantity to the magnitude of an arbitrarily chosen standard amount of the same kind is called unit of the physical quantity.

A complete description of the a physical quantity therefore, requires:-

1) The choice of a unit in which the quantity is to be measured.

2) A number which states how many times, this unit, the quantity in questions contains.

3) The measure number, as it is called, depends upon the size of the unit chosen.

For example, the duration of a day is 24 when expressed in hours. It is (24×60) when expressed in minutes, it is $\left(\frac{1}{365}\right)$ when expressed in years. Therefore:-

$$n \propto \frac{1}{u} \qquad \Longrightarrow \quad nu = Constant = Q$$

Or $$n_1 u_1 = n_2 u_2$$

Where, (n) measure number; (u) units; (Q) physical quantity. Since, physical quantities are related each to other, therefore, they selected the units a limited number, and we can find the units of the other quantities from the relations existing between them.

The few quantities selected for this purpose are called the basic or fundamental units. The units of the other quantities can be derived, and hence are called derived units.

The basic units listed in table.

N	Quantity	Unit	Symbol
1	**Length**	**meter**	**m**
2	**Mass**	**Kilogram**	**kg**
3	**Time**	**Second**	**s**
4	**Electric Current**	**Ampere**	**A**
5	**Temperature**	**Kelvin**	**K**
6	**Amount of substance**	**Mole**	**mol**
7	**Luminous intensity**	**candela**	**cd**

Some derived units usually uses in mechanics.

N	Quantity	Unit	Symbol
1	**Force**	**Newton**	**N**
2	**Energy**	**Joule**	**J**
3	**Power**	**Watt**	**W**
4	**Velocity**	**v**	**m/sec**
5	**Pressure**	**Newton/area**	**N/m^2**
6	**Torque**	**Newton-meter**	**N.m**
7	**momentum**	**$Kg.m.s^{-1}$**	**$Kg.m.s^{-1}$**

Systems of Units

The basic systems are:-

1- The British system of F.P.S

Where: - (F) Foot; unit of length.

(P) Pound; unit of mass.

(S) Second; unit of time.

2- The metric system, the systems have its origin in France in form of C.G.S

Where: - (C) Centimeter; unit of length (cm).

(G) Gram (gm); unit of mass.

(S) Second (s); unit of time.

3- In 1960, the eleventh general conference of weights and measures, on units, proposed revised metric system called the system international of unit in French (abbreviated, SI) which uses the meter (m) for length, the kilogram (kg) for mass, and the second (sec) for time.

Length:- (meter; m) The length equal to 1, 650, 763, 73 wavelengths in vacuum of the radiation corresponding to the transition between the energy levels $2p_{10}$ and 5ds of the krypton-86.

Mass:- kilogram (kg) A certain platinum-iridium cylinder, shall hence forth be considered to be the unit of mass, (diameter equal to its height).

Time:- second (s) The duration of 9.192.631.770 periods of the radiation corresponding to transition between the two hyperfine levels of the ground state of the cesium – 133 atom (1964).

Dimensions and Dimensional equations

The dimensions of physical quantity are the powers to which the fundamental units of length, mass, and time are raised to the unit of the given quantity. The equation:-

$$[Q] = L^a\, M^b\, T^c$$

Which states the relation between the unit of a given quantity and the basic units is called a dimensional equation.

The dimensional equations can be derived from the equations representing the between physical quantity, for examples:-

1) Area of rectangle = length × breadth

$[Area] = [L]\,[L] = L^2$

2) Volume of a cube = Length × breadth × height

$[Volume] = [L] \times [L] \times [L] = L^3$

3) Velocity or speed = $\frac{[distance]}{[time]} = \frac{L}{T} = L\,T^{-1}$

4) Acceleration = $\frac{[velocity]}{[time]} = \frac{[L\,T^{-1}]}{[T]} = L\,T^{-2}$

5) Force = Mass × Acceleration

$[F] = [M]\,[LT^{-2}] = L\,M\,T^{-2}$

6- Work = Force × Distance

$[W] = [\,L\,M\,T^{-2}\,]\,[\,L\,] = L^2\,M\,T^{-2}$

7- Kinetic Energy = $\frac{1}{2}\, mass \times (velocity)^2$

$$[K.E] = [Mass][velocity]^2 = [M]\,[L\,T^{-1}]^2 = M\,L^2\,T^{-2}$$

8- Power = Work / Time

$$[P] = [W] / [T] = \frac{[L^2 M\, T^{-2}]}{[T]} = L^2\ M\ T^{-3}$$

9- Pressure = Force / Area

$$[P] = [F] / [A] = \frac{[L\ M\ T^{-2}]}{[L^2]} = L^{-1} M\ T^{-2}$$

10- [Stress] = [Force / Area]

$$[S] = L^{-1}\ M\ T^{-2}$$

11- Volume Strain = $\frac{change\ in\ volume}{original\ volume} = \frac{\Delta V}{V} = L^0\ M^0\ T^0$

12- Frequency = $\frac{Cycle}{Time} = \frac{Cycle}{Second} = L^0\ M^0\ T^{-1} = T^{-1}$

Uses of Dimensional Equations

The following examples are the main uses of dimensional equations:-

1) Physical equation must be dimensionally homogeneous; for example; $V^2 - V_o^2 = 2\ a\ x$

The dimension of each term, we have:-

$$[V^2] = L^2\ T^{-2} \quad ; \quad [V_o^2] = L^2\ T^{-2}$$

And $[a\ x] = L\ T^{-2} \times L = L^2\ T^{-2}$

Thus we find that all the terms have identical dimensional formula.

2) To check the accuracies of physical equations; for example:-

$$t = 2\ \pi \sqrt{\frac{K^2 + L^2}{L\ g}}$$

The relation can be rewritten in the form

$$t^2 = 4\,\pi^2\,\left[\frac{K^2}{L\,g} + \frac{L^2}{L\,g}\right]$$

Now

$$[t^2] = T^2 \quad ; \quad \left[\frac{K^2}{L\,g}\right] = \frac{L^2}{L \times L\,T^{-2}} = T^2$$

And

$$\left[\frac{L}{g}\right] = \frac{L}{L\,T^{-2}} = T^2$$

Hence, the equation is correct.

To change from one system of unit to another; it has been shown that:-

$$n_1[u_1] = n_2[u_2]$$

Where ***(n_1)*** and ***(n_2)*** are the measure number of given physical quantities in terms of the absolute units (***u_1)*** and ***(u_2)*** respectively.

If ($L^a\,M^b\,T^c$) is the dimensional formula of the quantity, ($L_1\,M_1\,T_1$) and ($L_2\,M_2\,T_2$), basic units of a two system.

Then; $[u_1] = (L_1^a\,M_1^b\,T_1^c)$ and $[u_2] = (L_2^a\,M_2^b\,T_2^c)$

And $n_1\,[L_1^a\,M_1^b\,T_1^c] = n_2\,[L_2^a\,M_2^b\,T_2^c]$

Which gives:-

$$n_2 = n_1\left(\frac{L_1}{L_2}\right)^a \left(\frac{M_1}{M_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

Where ***(n₂)*** can be found out, if ***(n₁)*** is given, thus the knowledge of dimensions of physical quantity enables us to convert the measure number from one system of units to that in another.

Example (1)

If a given force be 1 pound. Calculate its measure in dynes.

$[F] = L\,M\,T^{-2}$

a = 1 ; b = 1 ; and c = - 2

$$n_2 = 1\left(\frac{ft}{cm}\right)^1\left(\frac{Ib}{gm}\right)^1\left(\frac{s}{s}\right)^{-2}$$

$$n_2 = 1\,(30.48)^1(453.6)^1\left(\frac{1}{1}\right)^{-2} = 13803.048\ (dynes)$$

Example (2)

The value of acceleration due to gravity is ($32\ ft/\ sec^2$). What is its value if the unit of the length is the mile and that of time is minute.

Solution:-

We know that [acceleration] = $LM^o\,T^{-2}$

$$n_2 = n_1\left(\frac{L_1}{L_2}\right)^a\left(\frac{M_1}{M_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

$$n_2 = 32\left(\frac{foot}{mile}\right)^1\left(\frac{1\ b}{gm}\right)^1\left(\frac{s}{min}\right)^{-2}$$

$$n_2 = 32\left(\frac{1}{5280}\right)^1\left(\frac{1}{60}\right)^{-2} = \frac{32\times 60\times 60}{5280} = 21.82\ (mile.min^{-2})$$

Example (3)

Find the number of watts in one horse power. (given that 1 Ib =453.6 gm, and 1 ft=30.48 cm, and g=32 ft/sec^2).

Solution:-

[power] = $L^2 M T^{-3}$

and n_1=1 (horse power)

n_1=550 × 32 (foot pounds) $n_2 = 550 \times 32 \left[\frac{ft}{cm}\right]^2 \left[\frac{1\,b}{gm}\right]^1 \left[\frac{s}{s}\right]^{-3} \quad (\frac{erg\,s}{\mathrm{sec}})$

$$n_2 = 550 \times 32\,(30.48)^2 (453.6)$$
$$n_2 = 746.4 \times 10^7 \left(\frac{ergs}{sec}\right) = 746.4\ (Watts)$$

Example (4)

The velocity with which a transverse wave travels along a stretched string may depend upon; (i) stretching force; (ii) the mass of the string (M); (iii) its length (L).

Solution:-

Let be write:- $\nu \propto F^a M^b L^c$

Or $\nu = k\, F^a M^b L^c$ (k; constant)

From dimensional equation, we get:-

$$L^1 M^0 T^{-1} = (L\, M\, T^{-2})^a M^b L^c$$

Or

$$L^1 M^0 T^{-1} = L^{a+c} M^{a+b} T^{-2a}$$

Equating the powers *L*, *M*, *and T* on two sides, we get:-

$$a + c = 1 \quad ; \quad a + b = 0 \quad ; and \ -2a = -1$$

$$a = \frac{1}{2} \quad ; \quad c = \frac{1}{2} \quad ; \quad b = -\frac{1}{2}$$

Hence, $\quad \nu = k\,F^{\frac{1}{2}}\,M^{-\frac{1}{2}}\,L^{\frac{1}{2}} \qquad$ Or $\qquad \nu = k\sqrt{\frac{F\,L}{M}}$

Home work (H.W)

Q1/ In engineering work found that the volume (V) of water which passes any point of a canal during (t) second is connected with the cross-section (A) of the canal and velocity (ν) of the water by the relation: $(V = k\,\nu\,A\,t)$. Test by the method of dimensions if the relation is correct or not.

Q2/ If kilowatt, kilowatt-hour, and mega-newton be chosen as the fundamental units, what would be the units of length, mass, and time?

(Ans. :- L = 3.6 (m) , M = 3.6×10^{12} (kg) , T= 3600 (sec))

Q3/ Calculate:- (i) number of dynes in a newton?

(ii) number of ergs in a Joule?

(iii) number of watts in a horse power?

(Ans.:- 10^5 , 10^7 , 746)

Q4/ Convert 4.2 Joules into foot-pounds.

(Ans,:- 1 (J) = 0.737 foot-pond)

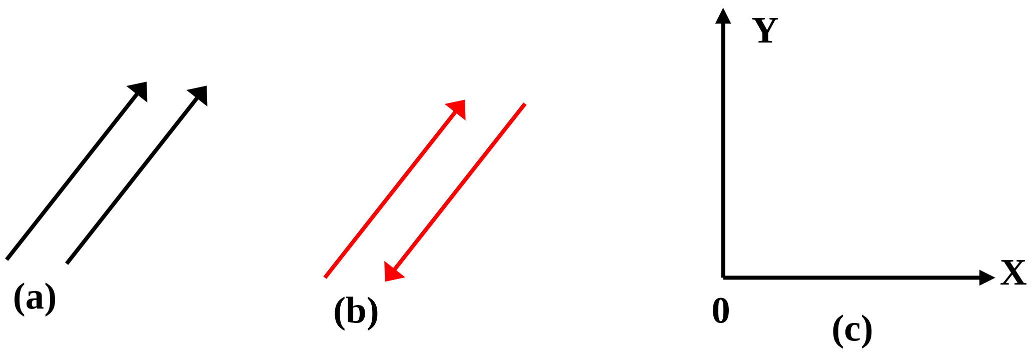
Q5/ Prove dimensionally that the velocity acquired by a body after a free fall through a vertical height (h) is given by,

$$V^2 = k\,g\,h$$

Vectors

Concept of direction

When we are given a straight line, we can move a long, t in two opposite senses; these are distinguished by assigning to each a sign, plus or minus, we say that the line is oriented and call it an axis. The coordinate axis X and Y and oriented lines in which the positive senses are as indicated in figure:-



An oriented line or axis defines a direction.

a) Parallel direction.

b) Antiparallel direction.

c) Oriented coordinate axis.

Vector and Scalar Quantity

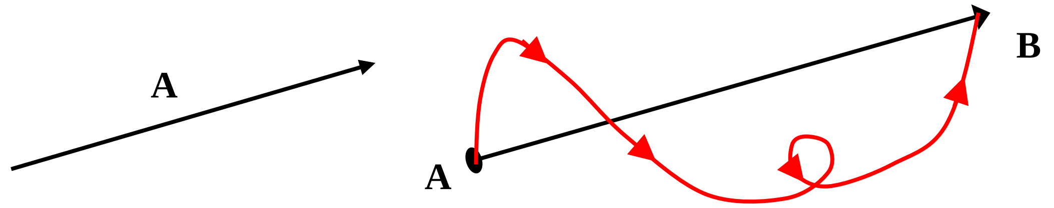
In our study of physics, we often need to work with physical quantities that have both numerical and directional properties.

A **scalar quantity** is completely specified by a single value with an appropriate unit and has no direction, for examples (temperature, volume, mass, speed, work, energy, and time intervals).

A **vector quantity** is completely specified by a number and appropriate units plus direction, for example (velocity, acceleration, force, and torque).

Representative of Vectors

If a particle moves from point A to point B a long straight path as shown in Figure. We represent this displacement by drawing an arrow from A to B.



The direction of the arrowhead represents the direction of the displacement, and the length of the arrow represent the magnitude of the displacement or displacement depends only on the initial and final positions, so the displacement vector is independent of the path taken by the particle between these two points. Also, we can use a boldface letter with an arrow over the letter. Such as, ($\vec{A}$) to represent vector *(A)*.

The magnitude of the vector ($\vec{A}$) is written either *(A)* or $|\vec{A}|$; The magnitude of a vector has physical units, such as meters for displacement. The magnitude of a vector is always a positive number.

Some properties of vector:-

a) If two vectors ($\vec{A}\ and\ \vec{B}$) are equal in magenitudes and in same direction and parallel, we say that:- $\vec{A} = \vec{B}$

b) If two vectors are equal in magnitude and in opposite direction with each to other or :- $\vec{A} = -\vec{B} \quad \Longrightarrow \quad \vec{B} = -\vec{A}$

c) If two vectors in parallel with each to other, and not equals in magnitude then :- $\vec{A} = \lambda\,\vec{B}$; (λ: ration between $\vec{A}$ & $\vec{B}$)

Addition of Vectors

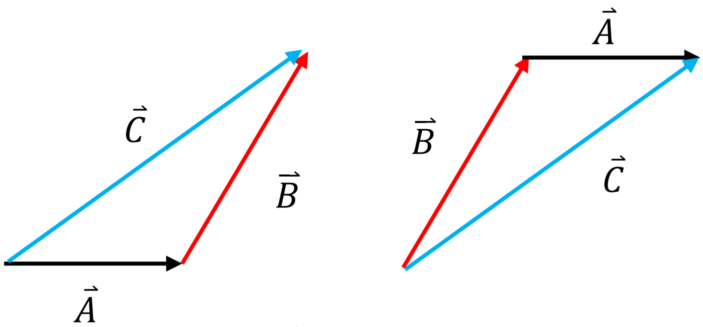
To find sun of two vectors ($\vec{A}$ & $\vec{B}$) added; the graphical method used. Therefore:-

1) Draw an arrow to represent vector ($\vec{A}$). The arrow points in the direction of the vector ($\vec{A}$). The value of a vector is not changed by moving it; as long as its direction and magnitude is not changed.

2) Draw the second vector arrow starting where the first ends; or (tail of the second arrow at the tip of the first).

3) Draw an arrow starting from the tail of the first and ending at the tip of the second. This arrow represents the sum of the two vectors as in figures:-

$\vec{C} = \vec{A} + \vec{B}$ ($\vec{C}$: sum of two vectors $\vec{A}$ *and* $\vec{B}$)



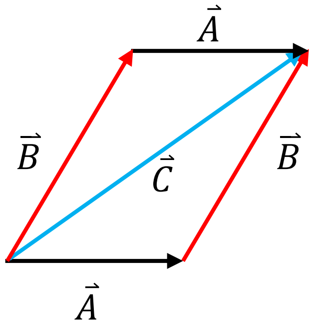
The result being the same if the order in which the vectors ($\vec{B}$ & $\vec{A}$) are added is reversed, or

$$\vec{C} = \vec{B} + \vec{A}$$

Figure shows that the vector ($\vec{C}$) is the sum of vectors $\vec{A}$ and $\vec{B}$, then:-

$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$

Thus, vector addition is commutative.



Example (1)

A man walks (320 m) due east. He then continues walking along a straight line but in a different direction, and stop. Exactly (200 m) north east of his starting point. How far did he walk during the second position of the trip and in what direction?

To compute the magnitude of $\vec{C}$

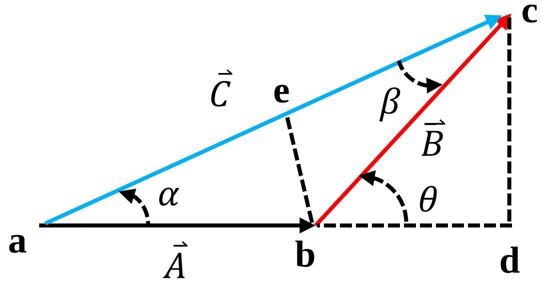
From the triangle (a d c)

$(a\ c)^2 = (a\ d)^2 + (d\ c)^2$

But;

$a\ d = a\ b + b\ d$

$a\ d = \vec{A} + \vec{B}\ cos\theta$



And

$d\ c = \vec{B}\ sin\theta$

Therefore:-

$$\vec{C}^2 = (\vec{A} + B\ cos\theta)^2 + (\vec{B}\ sin\theta)^2$$

$$\vec{C}^2 = \vec{A}^2 + 2\ A\ B\ cos\theta + \ B^2 cos^2\theta + \vec{B}^2\ sin^2\theta$$

$$\vec{C}^2 = \vec{A}^2 + 2\ A\ B\ cos\theta + \ B^2\ (cos^2\theta + sin^2\theta)$$

$$\vec{C}^2 = \vec{A}^2 + 2\ A\ B\ cos\theta + \ B^2$$

$$\vec{C} = \sqrt{\vec{A}^2 + \ B^2 + 2\ A\ B\ cos\theta}$$

To determine the direction of $(\vec{C})$, we need only find the angle (α).

From the figure:- the triangle (a b c)

$sin\alpha = \frac{c\ d}{a\ c} = \frac{c\ d}{\vec{C}} \qquad \Longrightarrow \qquad cd = \vec{C}\ sin\alpha$ …………………… (1)

And in triangle (b d c)

$sin\theta = \frac{c\ d}{b\ c} = \frac{c\ d}{\vec{B}} \qquad \Longrightarrow \qquad cd = \vec{B}\ sin\theta$ ……………………..(2)

Therefore:- (from eq. (1) and eq. (2))

$$\vec{C}\ sin\alpha = \ \vec{B}\ sin\theta$$

Or; $= \frac{\vec{C}}{sin\theta} = \frac{\vec{B}}{sin\alpha}$ …………….…………… (3)

Similarly:- in triangle (a e b)

$$sin\alpha = \frac{b\ e}{a\ b} = \frac{b\ e}{\vec{A}}$$

$b\ e = \overrightarrow{A}\, sin\alpha$ ……………………………….(4)

And in triangle (b e c)

$$sin\beta = \frac{b\ e}{b\ c} = \frac{b\ e}{\vec{B}}$$

Or, $b\ e = \vec{B}\ sin\beta$ …………….……..…………(5)

From eq. (4) and eq. (5), find:-

$$\vec{A}\ sin\alpha = \vec{B}\ sin\beta$$

Or; $\frac{\vec{B}}{sin\alpha} = \frac{\vec{A}}{sin\beta}$ ……………………………(6)

Combining both equations (3 and 6), we get:-

$\frac{\vec{C}}{sin\theta} = \frac{\vec{A}}{sin\beta} = \frac{\vec{B}}{sin\alpha}$ Law of sine

In special case when $(\vec{A})$ and $(\vec{B})$ are perpendicular $(\theta = 90^o)$ the following relations hold:-

$$\vec{C} = \sqrt{\vec{A}^2 + \vec{B}^2}$$

And, $tan\alpha = \frac{\vec{B}}{\vec{A}}$

$$\alpha = tan^{-1} \frac{\vec{B}}{\vec{A}}$$

Vector Subtracted (D)

The difference between two vectors $(\vec{A}\ and\ \vec{B})$ is obtained by adding to the first the negative (or opposite) of the second:- or

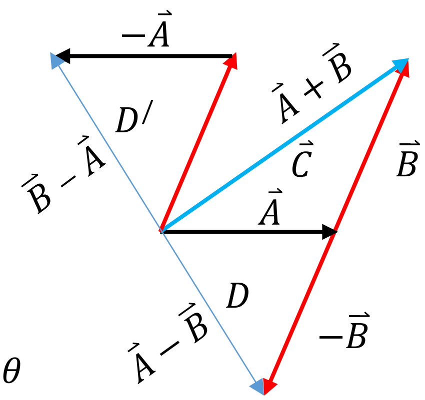
$$D = \vec{A} - \vec{B}$$

And

$$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$$

$D \neq D^/$

As shown in Figure;



Therefore:-

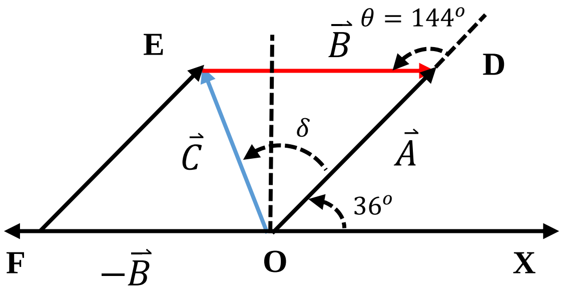
Vector difference is anticommutative.

The magnitude of the difference (D) is:-

$$D = \sqrt{\vec{A}^2 + \vec{B}^2 - 2\,\vec{A}\,\vec{B}\,cos\theta}$$

Example (2)

Given two vectors:- $\vec{A}$ is 6 units long and makes an angle of (36°) with the positive (x-axis); $\vec{B}$ is 7 units long and is in the direction of the negative (x-axis); find:- (a) the sum of the two vectors, (b) the difference between the two vectors.



a) In triangle (ODE)

$$\theta = 180^o - 36^o = 144^o$$

$$\vec{C} = \sqrt{36 + 49 + 2\,(6)(7)cos144^o} = 4.128 \quad (units)$$

To find the angle between $\vec{C}$ and $\vec{A}$, then:-

$$\frac{\vec{C}}{sin\theta} = \frac{\vec{B}}{sin\delta} \qquad \Longrightarrow \qquad \frac{4.128}{sin144} = \frac{7}{sin\delta}$$

$$sin\delta = 0.996 \quad \Longrightarrow \quad \delta = 85^o$$

A direction $(36^o + 85 = 121^o)$ with + x,

b) To find the difference

$$D = \vec{A} - \vec{B}$$

$$\because D = \sqrt{36 + 49 - 2\,(6)(7)cos144^o} = 12.31\,(units)$$

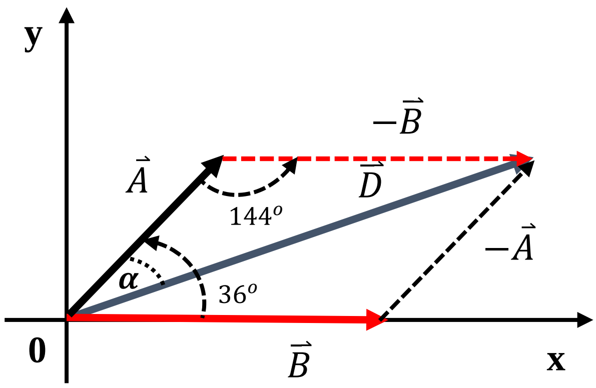
To find the direction of (D).

$$\frac{D}{sin144^o} = \frac{\vec{B}}{sin\alpha}$$

$$\frac{12.31}{sin144^o} = \frac{7}{sin\alpha}$$

$$sin\alpha = 0.334$$

$$\alpha = 19.5^o$$



Direction of is $\vec{D}$ is $(36^o - 19.5^o) = 16.5^o$ in + X

Example (3)

Vector $\vec{A} = 15\ (units)$ long in direction of North, vector $\vec{B} = 5\ (units)$ in direction of $(S\ 70^o\ E)$. Find sum of $(\vec{C} = \vec{A} + \vec{B}\ )$

$$\theta = 180^o - 70^o = 110^o$$

$$\vec{C} = \sqrt{(15)^2 + (5)^2 + 2\ (15)(5)\ cos110^o} = 14.1^o\ (units)$$

To obtain direction of $\vec{C}$.

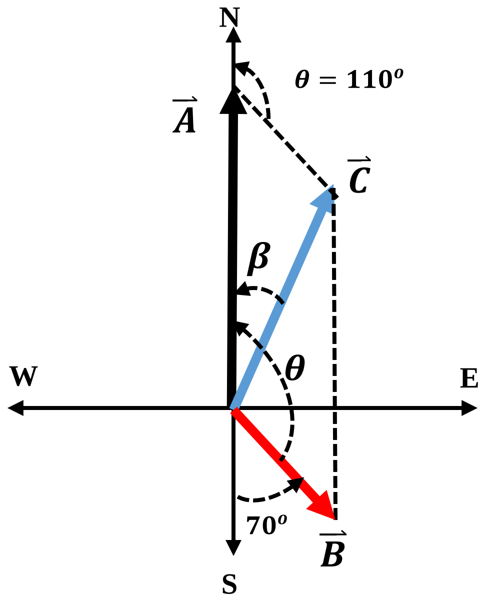
$$\frac{\vec{C}}{sin\theta} = \frac{\vec{B}}{sin\beta} \qquad \Rightarrow \quad \frac{14.1}{sin110} = \frac{5}{sin\beta}$$

$$sin\beta = \frac{5 \times 0.93969}{14.1} = 0.333$$

$$\beta = sin^{-1}0.333 = 19.4^o$$

Thus, the resultant motion is in the direction

$(\,N\ 19.4^o\ E\quad ; or\quad 90 - 19.4 = 70.6\quad \Rightarrow\quad E\ 70.6\ N)$



Example (4)

Two vectors ($\vec{A} = 6\,units$; $\vec{B} = 9\,units$), from an angle of ($a - 0^o$), ($b - 60^o$) , ($c - 90^o$) , ($d - 150^o$) , and ($e - 180^o$). Find the magnitude and the direction of their result.

a) $\theta = 0^o$

$\vec{C} = \vec{A} + \vec{B} = 6 + 9 = 15$ in direction of $\vec{A}$ and $\vec{B}$

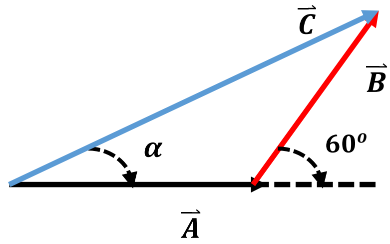
b) $\theta = 60^o$

$$\vec{C} = \sqrt{36 + 81 + 2\,(6)(9)cos60} = 13.07\,(units)$$

$$\frac{13.07}{sin60} = \frac{9}{sin\alpha}$$

$$sin\alpha = \frac{9 \times 0.866}{13.07} = 0.596$$

$$\alpha = 36.608^o$$

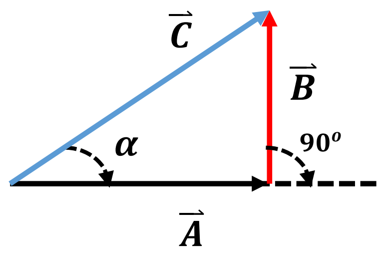


c) $\theta = 90^o$

$$|\vec{C}| = \sqrt{6^2 + 9^2} = \sqrt{117} = 10.81$$

$$\theta = tan^{-1}\frac{9}{6}$$

$$\theta = 56.309^o$$



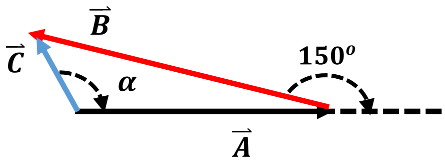
d) $\theta = 150^o$

$$\vec{C} = \sqrt{6^2 + 9^2 + 2\,(6)(9)cos150} = 4.8445\,(units)$$

$$\frac{4.8445}{sin150} = \frac{9}{sin\alpha}$$

$$sin\alpha = \frac{9 \times 0.866}{4.8445} = 0.9288$$

$$\alpha = 68.2^o$$



Direction of ($\vec{C}$) equal (111.72°)

e) $\theta = 180^o$

$$\vec{C} = \vec{A} - \vec{B} = 6 - 9 = -3 \quad (units)$$

In direction of vector ($\vec{B}$).

Example(5): A Vector ($\vec{A} = 20\,units$) due North and vector ($\vec{B} = 35\,units$) in direction (60^o) West of North, find the magnitude and direction of resultant two vectors.

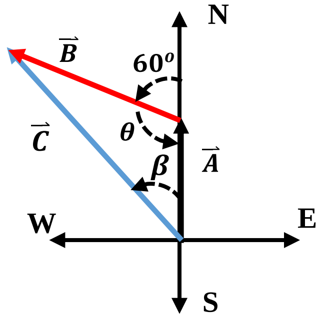
$\vec{C} = \vec{A} + \vec{B}$

The magnitude of ($\vec{C}$) can be obtained from the law Cosine

$$\vec{C} = \sqrt{\vec{A}^2 + \vec{B}^2 + 2\,A\,B\,cos\theta}$$

$$\vec{C} = \sqrt{(20)^2 + (35)^2 + 2\,(20)(35)cos60} = 48.2\ \ (units)$$

Using the law of sine's to find direction of ($\vec{C}$).



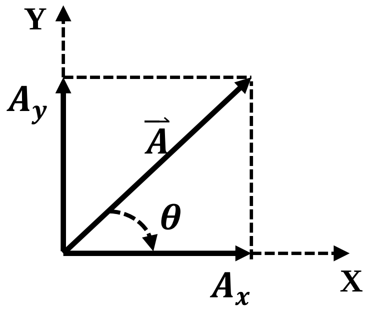
$$\frac{sin\beta}{\vec{B}} = \frac{sin\theta}{\vec{C}}$$

$sin\beta = 0.629$

$\beta = 38.9^o \quad (West\ of\ North)$

Component of Vectors

To define components we use a rectangular (Cartesian) coordinate axis system as in Fig.



A vector ($\vec{A}$) lying in the (XY) plane can be represented as the sum of two perpendicular vectors are labeled ($\boldsymbol{A_x}$, $\boldsymbol{A_y}$) the $\vec{\boldsymbol{A}} = \boldsymbol{A_x} + \boldsymbol{A_y}$

Where:-

$\boldsymbol{A_x}$ and $\boldsymbol{A_y}$ are called the components of a vector $\vec{\boldsymbol{A}}$.

From the definitions of the trigonometric function:-

$$\boldsymbol{A_x = A\,cos\theta} \quad ; \quad \boldsymbol{A_y = A\,sin\theta}$$

The magnitude and direction of vector ($\vec{\boldsymbol{A}}$) may be found as:-

$$\vec{\boldsymbol{A}} = \sqrt{A_x^2 + A_y^2}$$

And

$$tan\theta = \frac{\boldsymbol{A_y}}{\boldsymbol{A_x}}$$

$$\theta = \arctan\frac{\boldsymbol{A_y}}{\boldsymbol{A_x}}$$

Unit vector:- is a vector having a magnitude of unity. Its only purpose is describe a direction in space:- $u(A) = \frac{\vec{A}}{|A|}$

by using unit vector, the vector ($\vec{\mathrm{A}}$) is written in terms of its components as; $\vec{\boldsymbol{A}} = \boldsymbol{A_x\hat{\imath} + A_y\hat{\jmath}}$

Can be expressed using unit vectors as follows:- for two vectors

$$\vec{\boldsymbol{A}} = \boldsymbol{A_x\hat{\imath} + A_y\hat{\jmath}}$$

And $\vec{\boldsymbol{B}} = \boldsymbol{B_x\hat{\imath} + B_y\hat{\jmath}}$

Then $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \boldsymbol{C_x\hat{\imath} + C_y\hat{\jmath}}$

$$\vec{\boldsymbol{C}} = (\boldsymbol{A_x\hat{\imath} + A_y\hat{\jmath}}) + (\boldsymbol{B_x\hat{\imath} + B_y\hat{\jmath}})$$

$\vec{\boldsymbol{C}} = (\boldsymbol{A_x + B_x})\boldsymbol{\hat{\imath}} + (\boldsymbol{A_y + B_y})\boldsymbol{\hat{\jmath}}$

$\vec{\boldsymbol{C}} = \boldsymbol{C_x\hat{\imath} + C_y\hat{\jmath}}$

$$\left|\vec{C}\right| = \sqrt{\boldsymbol{C_x^2 + C_y^2}}$$

$$\theta = tan^{-1}\frac{C_y}{C_x}$$

My be expressed by vector position (r)

$$\vec{r} = \boldsymbol{x_x \hat{\imath} + x_y \hat{\jmath}}$$

$$\vec{r} = \sqrt{\boldsymbol{x^2 + y^2}}$$

$$\boldsymbol{\theta} = tan^{-1}\frac{y}{x}$$

If a vector $(\vec{A})$ in a space, there are three rectangular components given by:- A_x , A_y , and A_z , as in Figure.:-

Therefore:- $\vec{A} = A_x + A_y + A_z$

Rewrite the components in term unit vector:-

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

Where:-

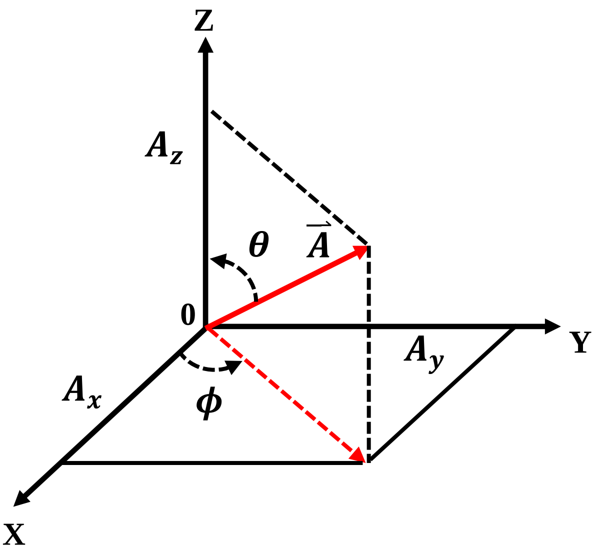
$$A_x = A\ sin\theta\ cos\phi$$

$$A_y = A\ sin\theta\ sin\phi$$

$$A_z = A\ cos\theta$$

The magnitude $(\vec{A})$ is given by:-

$$A = \sqrt{\boldsymbol{A_x^2 + A_y^2 + A_z^2}}$$

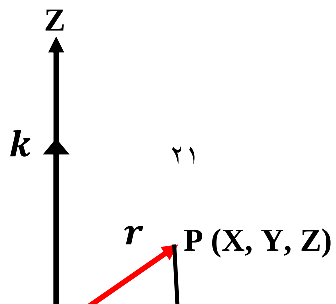


An important case of a three-dimensional vector is the position vector $(\vec{r})$ of a point (P) having coordinates (x, y, z) as in Fig.:-

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

Magnitude of $(\vec{r})$ is:-

$$\vec{r} = \sqrt{x^2 + y^2 + z^2}$$



Vector position also uses to find distance between two points in space, as in Fig. :-

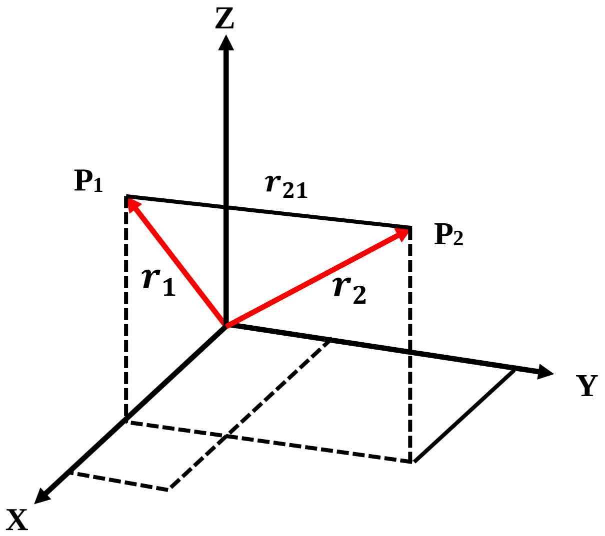
$$\overrightarrow{r_{21}} = \overrightarrow{P_2P_1}$$

$$\overrightarrow{r_{21}} = \overrightarrow{r_2} - \overrightarrow{r_1}$$

$$\overrightarrow{r_{21}} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$

And Magnitude of ($\vec{r}$).

$$|\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



Addition of Several Vectors

To add several vectors (A_1, A_2, A_3, ….) by using the method of components. For simplicity let us consider the case where all vectors are in one plane so that we need to use only two components. Then

$$\vec{A} = (A_{1x}\hat{\imath} + A_{1y}\hat{\jmath}) + (A_{2x}\hat{\imath} + A_{2y}\hat{\jmath}) + (A_{3x}\hat{\imath} + A_{3y}\hat{\jmath}) + \cdots + (A_{nx}\hat{\imath} + A_{ny}\hat{\jmath})$$

$$\vec{A} = (A_{1x} + A_{2x} + A_{3x} + \cdots + A_{nx})\hat{\imath} + (A_{1y} + A_{2y} + A_{3y} + \cdots + A_{ny})\hat{\jmath}$$

$$\vec{A} = \left(\sum_{i=1}^{n} A_{ix}\right)\hat{\imath} + \left(\sum_{i=1}^{n} A_{iy}\right)\hat{\jmath}$$

$$\vec{A} = \left(\sum_{i=1}^{n} A\, cos\theta_n\right)\hat{\imath} + \left(\sum_{i=1}^{n} A\, sin\theta_n\right)\hat{\jmath}$$

Where:-

$$A_{1x} + A_{2x} + A_{3x} + \cdots + A_{nx} = \textstyle\sum_i A_{ix}$$

$$A_{1y} + A_{2y} + A_{3y} + \cdots + A_{ny} = \sum_i A_{iy}$$

The magnitude of ($\vec{A}$) is:-

$$\vec{A} = \sqrt{(A_{ix})^2 + (A_{iy})^2}$$

And direction of ($\vec{A}$) is:-

$$\theta = tan^{-1}\frac{A_{iy}}{A_{ix}}$$

Example (6)

1- If the vector ($\vec{A} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$). Find $|\hat{u}(A)|$; $\hat{u}(A)$

$$|A| = \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{29}$$

$$\hat{u}(A) = \frac{\vec{A}}{|A|} = \frac{2i + 3j - 4k}{\sqrt{29}}$$

$$|\hat{u}(A)| = \left[\left(\frac{2}{\sqrt{29}}\right)^2 + \left(\frac{3}{\sqrt{29}}\right)^2 + \left(\frac{-4}{\sqrt{29}}\right)^2\right]^{\frac{1}{2}}$$

$$|\hat{u}(A)| = \left[\frac{4}{29} + \frac{9}{29} + \frac{16}{29}\right]^{\frac{1}{2}} = 1$$

2- If ($\vec{B} = 4\hat{\imath} - 3\hat{\jmath}$). Find $|\vec{B}|$; $\hat{u}(B)$; $|\hat{u}(B)|$

$$|B| = \sqrt{(4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$\hat{u}(B) = \frac{4\hat{\imath} - 3\hat{\jmath}}{5}$$

$$|\hat{u}(B)| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{-3}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = \sqrt{\frac{25}{25}} = 1$$

3- If $\vec{A}_1 = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{A}_2 = 2\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

$\vec{A}_3 = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

Find magnitude of $\vec{A}_3,\ \vec{A}_2,\ and\ \vec{A}_1$

$|A_3| = \sqrt{(-1)^2 + (2)^2 + (2)^2} = 3 \quad (units)$

$|A_2| = \sqrt{(2)^2 + (-4)^2 + (-3)^2} = \sqrt{29} \quad (units)$

$|A_1| = \sqrt{(3)^2 + (-2)^2 + (1)^2} = \sqrt{14} \quad (units)$

Find $|A_1 + A_2 + A_3|$

$$A_1 + A_2 + A_3 = (3 + 2 - 1)\hat{\imath} + (-2 - 4 + 2)\hat{\jmath} + (1 - 3 + 2)\hat{k}$$

$$A_1 + A_2 + A_3 = 4\hat{\imath} - 4\hat{\jmath}$$

$|A_1 + A_2 + A_3| = \sqrt{(4)^2 + (4)^2} = 5.65\ (units)$

Find $2\overrightarrow{A_1} - 3\overrightarrow{A_2} - 5\,\overrightarrow{A_3} = 5i - 2j + k$

Find $|2\overrightarrow{A_1} - 3\overrightarrow{A_2} - 5\,\overrightarrow{A_3}| = \sqrt{(5)^2 + (-2)^2 + (1)^2} = \sqrt{30} = 5.47\ (units)$

4- If $\vec{A} = -i + 2j - 2k \quad ; \vec{B} = 3i + 4j - 12k$

Find :- $|\vec{A}| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = 3 \quad (units)$

Find :- $\hat{u}(A) = \frac{-i+2j-2k}{3}$

Direction of cosines of $(\vec{A})$ are $\left(\frac{-1}{3}\ ;\ \frac{2}{3}\ ;\ \frac{-2}{3}\right)$

Find $|B| = \sqrt{9 + 16 + 144} = \sqrt{169} = 13 \quad (units)$

$$\hat{u}(B) = \frac{3i + 4j - 12k}{13}$$

Direction of $(\vec{B})$ are $\left(\frac{3}{13}\ ;\ \frac{4}{13}\ ;\ \frac{-12}{13}\right)$

$$cos\theta = \left[-\frac{1}{3} \times \frac{3}{13} + \frac{2}{3} \times \frac{4}{13} + \frac{-2}{3} \times \frac{-12}{13}\right] = \frac{29}{39}$$

5- If $\vec{A}_1 = \hat{\imath}(4) + \hat{\jmath}(-3)$

$\vec{A}_2 = \hat{\imath}(-3) + \hat{\jmath}(3)$

$\vec{A}_3 = \hat{\imath}(11) + \hat{\jmath}(-6)$

$\vec{A}_4 = \hat{\imath}(7) + \hat{\jmath}(-8)$

Find

a) $\vec{A} = \hat{\imath}\sum_i A_{ix} + \hat{\jmath}\sum_i A_{iy}$

$\vec{A} = \hat{\imath}(19) - \hat{\jmath}(14)$

b) $|\vec{A}| = \sqrt{(19)^2 + (-14)^2} = 23.6 \quad (units)$

c) $tan\theta = \frac{\sum_i A_y}{\sum_i A_x} = \frac{-14}{19} = -0.737 \quad \Rightarrow\ \theta = tan^{-1}(-0.737) = -36.4^o$

6- Find the distance between the two points with coordinates $P_1(6,8,10)$ and $P_2(4, 4, 10)$.

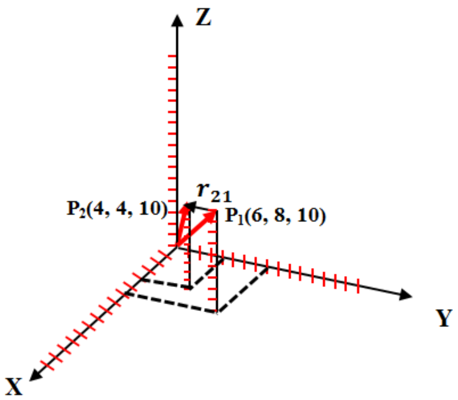
$$\vec{r}_{21} = (4-6)\hat{\imath} + (4-8)\hat{\jmath} + (10-10)\hat{k}$$

$$\vec{r}_{21} = (-2)\hat{\imath} + (-4)\hat{\jmath} + (0)\hat{k}$$

$$\vec{r}_{21} = -2\hat{\imath} - 4\hat{\jmath}$$

$$|\vec{r}_{21}| = \sqrt{(-2)^2 + (-4)^2} = \sqrt{20} = 4.47\ (units)$$

$$\theta = tan^{-1}\frac{-4}{-2} = 63.4^o$$



7- H.W.:- A vector ($\vec{A} = 4\,units$) in direction (-45^o) with (+X) positive axis; and vector ($\vec{B} = 2\,units$) in direction (120^o) with positive axis (+X); find ($\vec{A} + \vec{B}$) and ($\vec{A} - \vec{B}$).

$$\theta = 120^o + 45^o = 165^o$$

$$\vec{C} = \sqrt{(4)^2 + (2)^2 + 2 \times 4 \times 2 \times cos165}$$

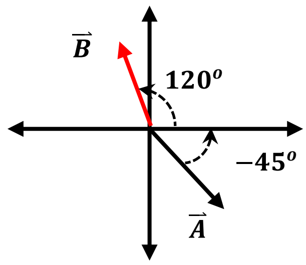
$$\vec{C} = \sqrt{20 - 15.45} = 2.14$$

$$\frac{\vec{C}}{sin165} = \frac{\vec{B}}{sin\alpha} \quad \Longrightarrow \quad \frac{2.14}{0.2588} = \frac{2}{sin\alpha}$$

$$sin\alpha = \frac{2 \times 0.2588}{2.14} = 0.24$$

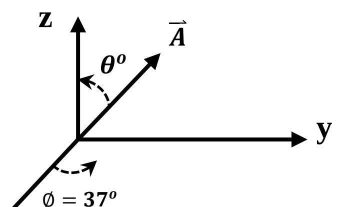
$$\alpha = sin^{-1}0.24 = 14^o$$

$\therefore$ Direction of $\vec{C} = -45 + 14 = -31^o$



8- Find the components of the vector that is (13 units) long and makes an angle of (22.6^o) with the (z-axis). And whose projection in the xy-plane makes an angle (ϕ) of (37^o) with the (+x-axis).

$\because\ A_x = A\ sin\theta\ cos\phi$

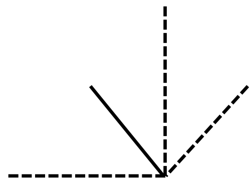


$A_y = A\ sin\theta\ sin\phi$

$A_z = A\ cos\theta$

$\because\ \theta = 22.6^o$

$\therefore\ cos\theta = cos22.6 = 0.923$



To find the z-component of $(\vec{A})$

$A_z = A\ cos\theta = 13\ \times 0.923 = 12\ (units)$

The projection of $(\vec{A})$ onto the (xy) plane

$$A_{xy} = A\ sin\phi = 13\ sin22.6 = 13 \times 0.384 = 4.992\ (units)$$

$$A_x = A_{xy}\ \ cos\phi$$

$$A_x = A\ sin\theta\ cos\phi = A\ sin22.6\ cos37$$

$$A_x = 13\ (0.384)(0.8)$$

$$A_x = 4 \quad (units)$$

$A_y = A_{xy}\ sin\phi$

$A_y = A\ sin\theta\ sin\phi = \ A\ sin22.6\ sin37$

$A_y = 13\ (0.384)(0.6) = 3 \quad (units)$

The vector $\vec{A} = 4\ \hat{\imath}\ + 3\ \hat{\jmath}\ \ + 12\ \hat{k}$

Example (7)

Find the sum of two vectors $(\vec{A})$ and $(\vec{B})$ lying in the (xy) plane and given by; $\vec{A} = 2\ \hat{\imath} + 2\ \hat{\jmath}$ and $\vec{B} = 2\ \hat{\imath} - 4\ \hat{\jmath}$

Sol. :-

$\vec{A} = A_x\ \hat{\imath} + A_y\ \hat{\jmath}$; $A_x = 2$; $A_y = 2$

$\vec{B} = B_x\ \hat{\imath} + B_y\ \hat{\jmath}$; $B_x = 2$; $B_y = -4$

$\vec{R} = \vec{A} + \vec{B} = \ (A_x + B_x)\ \hat{\imath} + \left(A_y + B_y\right)\hat{\jmath} = (2 + 2)\ \hat{\imath} + (2 - 4)\hat{\jmath}$

$\vec{R} = R_x\ \hat{\imath} + R_y\ \hat{\jmath}$

$\vec{R} = 4\ \hat{\imath} - 2\ \hat{\jmath}$; $R_x = 4$; $R_y = -2$;

$$|\vec{R}| = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(4)^2 + (-2)^2} = \sqrt{20} = 4.5 \quad (units)$$

$$tan\theta = \frac{R_y}{R_x} \frac{-2}{4} = -0.5$$

$$\theta = tan^{-1}(-0.5) = -27^o$$

Example (8)

A particle undergoes three consecutive displacements

$\Delta\overrightarrow{r_1} = (15\,\hat{\imath} + 30\,\hat{\jmath} + 12\hat{k})$; $\Delta\overrightarrow{r_2} = (23\,\hat{\imath} - 14\,\hat{\jmath} - 5\hat{k})$;
$\Delta\overrightarrow{r_3} = (-13\,\hat{\imath} + 15\,\hat{\jmath})$; find the components of the resultant displacement and its magnitudes resultant displacement.

$$\Delta r = \Delta\overrightarrow{r_1} + \Delta\overrightarrow{r_2} + \Delta\overrightarrow{r_3}$$

$$\Delta r = (15 + 23 - 13)\hat{\imath} + (30 - 14 + 15)\hat{\jmath} + (12 - 5 + 0)\hat{k}$$

$$\Delta r = 25\hat{\imath} + 31\,\hat{\jmath} + 7\hat{k}$$

$$R = \sqrt{(25)^2 + (31)^2 + (7)^2} = 40 \ (units)$$

Vector Multiplication

Two kinds of product:-

1- Scalar product (or dot product)
2- Vector product (or cross product)

Scalar product:- the scalar product of two vectors $(\vec{A})$ and $(\vec{B})$, denoted by $\vec{A}.\vec{B}$ (read $\vec{A}$ dot $\vec{B}$) is defined as the scalar quantity obtained or:-

$$\vec{A}.\vec{B} = A\ B\ cos\theta$$

$$\vec{A}.\vec{B} = |A|\ |B|\ cos\theta$$

$$cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|}$$

- If $\vec{A}\ //\ \vec{B}$ or $\theta = 0 \Rightarrow cos0 = 1\ (\vec{A}\,.\vec{B}\ \ the\ largest)$
  $\therefore\ \vec{A}\,.\vec{B} = A\ B \Rightarrow If\ \vec{A} = \vec{B}$ then
  $\vec{A}\,.\vec{B} = A^2 = B^2$
- If $\vec{A}\ \perp \vec{B}$ or $\theta = 90^o \Rightarrow cos90 = 0\ (\vec{A}\,.\vec{B}\ \ the\ smallest)$

$\therefore\ \vec{A}\,.\vec{B} = 0 \quad ;$

$\theta = acute angle\ \left(\vec{A}\,.\vec{B}\ positive\right);\ \theta = obtuse angle\ \left(\vec{A}\,.\vec{B}\ negative\right)$

- $\vec{A}\,.\vec{B} = \vec{B}\,.\,\vec{A}$ is commutative.

Example:-

If a vector $\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$ and vector $\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$; find the dot product of $(\vec{A}\,.\vec{B}\,)$.

$\vec{A}\,.\vec{B} = \left(A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}\right).\left(B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}\right)$

$\vec{A}\,.\vec{B} = A_xB_x(\hat{\imath}.\hat{\imath}) + A_xB_y(\hat{\imath}.\hat{\jmath}) + A_xB_z\left(\hat{\imath}.\hat{k}\right) + A_yB_x(\hat{\jmath}.\hat{\imath}) + A_yB_y(\hat{\jmath}.\hat{\jmath}) + A_yB_z\left(\hat{\jmath}.\hat{k}\right) + A_zB_x\left(\hat{k}.\hat{\imath}\right) + A_zB_y\left(\hat{k}.\hat{\jmath}\right) + A_zB_z\left(\hat{k}.\hat{k}\right)$

$\because\ \hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1\ ;\ \ \theta = 0 \quad \Longrightarrow \quad cos0 = 1$

And $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0\ \ ; \theta = 90^o \quad \Longrightarrow \quad cos90 = 0$

$$\vec{A}\,.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

To find the angle between vector $\left(\vec{A}\ and\ \vec{B}\right)$.

$$cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{\left|\vec{A}\right|\left|\vec{B}\right|}$$

Example:-

Find the angle between the vector $(\vec{A})$ and vector $(\vec{B})$. Where;

$$\vec{A} = 2\,\hat{\imath} + 3\,\hat{\jmath} - \hat{k} \quad ; \qquad \vec{B} = -\,\hat{\imath} + \hat{\jmath} + 2\hat{k}$$

$$\because\ cos\theta = \frac{\vec{A}\,.\vec{B}}{\left|\vec{A}\right|\left|\vec{B}\right|}$$

$$\vec{A}\,.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A}\,.\vec{B} = (2)(-1) + (3)(1) + (-1)(2) = -1$$

$$\left|\vec{A}\right| = \sqrt{4 + 9 + 1} = 3.74\ \ (units)$$

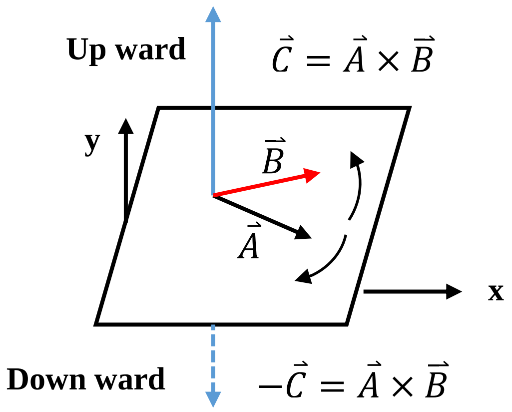
$$\left|\vec{B}\right| = \sqrt{1 + 1 + 4} = 2.45\ \ (units)$$

$$cos\theta = \frac{-1}{(3.74).(2.45)}$$

$$\theta = cos^{-1}(-0.109) = 96.3^o$$

Vector Product

- ❖ The vector product of two vectors $(\vec{A}\ and\ \vec{B}\ )$ is denoted by $(\vec{A} \times \vec{B})$ (also called the cross product); the two vectors then lie in a plane.
- ❖ The vector products is defined as a vector quantity with a direction perpendicular to this plane (to both $\vec{A}\ and\ \vec{B}\ $) and a magnitude given by $(A\ B\ sin\theta)$ or :-
  If $\vec{C} = \vec{A} \times \vec{B}$ then $\vec{C} = A\ B\ sin\theta \quad \hat{u}$
- ❖ The product of two vectors is a vector the direction determined by the "right hand rule". The curl fingers of the right hand a round this perpendicular line so that the thumb then gives the direction of the vector product, as in Figure:-



The right hand product is $\vec{A} \times \vec{B} = A\ B\ sin\theta\ \ \hat{u}$

Where $(\hat{u})$ is unit vector , which is indicate to direction.

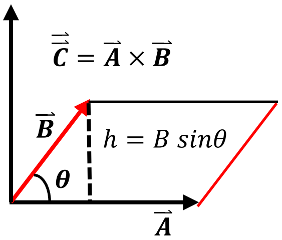
From the Figure:- that

- ❖ $\vec{A} \times \vec{B} \neq -\vec{B} \times \vec{A}$ (vector product is no commutative)

- ❖ If $\vec{A} = \vec{B}$, If $\vec{A} \parallel \vec{B}$ or $\theta = 0$
  Or if $(\vec{A}\ )$ parallel to $(\vec{B})$ then $(sin\theta = 0)$nd $(\vec{A} \times \vec{B} = 0)$
  $(\vec{A} \times \vec{B}\ the\ smallest)$

- ❖ If $\vec{A} \perp \vec{B}$ or $\theta = 90$, $sin90 = 1$; $(\vec{C} = \vec{A} \times \vec{B})$ $(\vec{A} \times \vec{B}\ the\ largest)$

❖ Vector product also uses to find the area of a parallelogram with sides ($\vec{A}$) and ($\vec{B}$) or $|\vec{A} \times \vec{B}|$= the area of a parallelogram.
From the Figure:



That:

$$\vec{A} \times \vec{B} = A\ B\ sin\theta \quad \hat{u}$$

The magnitude is $\quad |\vec{A} \times \vec{B}| = A\ B\ sin\theta$

$$|\vec{A} \times \vec{B}| = A\ h = Area$$

$$h = B\ sin\theta$$

The component vector product of ($\vec{A}$) and ($\vec{B}$) are known:-

$$\vec{A} \times \vec{B} = \left(A_x\ \hat{\imath} + A_y\ \hat{\jmath} + A_z\ \hat{k}\right) \times (B_x\ \hat{\imath} + B_y\ \hat{\jmath} + B_z\ \hat{k})$$

$$\vec{A} \times \vec{B} = A_x\ \hat{\imath} \times B_x \hat{\imath} + A_x \hat{\imath} \times B_y\ \hat{\jmath} + A_x \hat{\imath} \times B_z\ \hat{k} + A_y \hat{\jmath} \times B_x\ \hat{\imath} + A_y\ \hat{\jmath} \times B_y\ \hat{\jmath} + A_y \hat{\jmath} \times B_z \hat{k} + A_z \hat{k} \times B_x \hat{\imath} + A_z \hat{k} \times B_y \hat{\jmath} + A_z \hat{k} \times B_z \hat{k}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z\ B_y)\ \hat{\imath} + (A_z B_x - A_x B_z)\ \hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$

Where:-

$$\hat{\imath} \times j = k \quad \Longrightarrow \quad j \times \hat{\imath} = -k$$

$$\hat{\jmath} \times k = i \quad \Longrightarrow \quad k \times \hat{\jmath} = -i$$

$$k \times i = j \quad \Longrightarrow \quad i \times k = -j$$

And

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

If ($\vec{C} = \vec{A} \times \vec{B}$) ; the component of ($\vec{C}$) is:-

$$C_x = A_y B_z - A_z B_y \quad ; \quad C_y = A_z B_x - A_x B_z \ ; \quad C_z = A_x B_y - A_y B_x$$

The same result obtained by using determinant from:-

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = +\hat{\imath}\left(A_y B_z - A_z B_y\right) - j\,(A_x B_z - A_z B_x) +$$
$$k\,(A_x B_y - A_y B_x)$$

Example:- If $(\vec{\boldsymbol{A}} = \mathbf{3}\,\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}} + \mathbf{2}\,\hat{\boldsymbol{k}})$ **and** $\vec{B} = 2\,\hat{\boldsymbol{\imath}} + \mathbf{3}\,\hat{\boldsymbol{\jmath}} - \hat{\boldsymbol{k}}$ **; Find:-**

1- $\vec{A} \times \vec{B}$

$$\left|\vec{A} \times \vec{B}\right| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 2 \\ 2 & 3 & -1 \end{vmatrix} = i\begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} - j\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = -5\,\hat{\imath} + 7\,\hat{\jmath} + 11\,\hat{k}$$

2- The Angle between $(\vec{A})$ and $(\vec{B})$

$$\boldsymbol{cos\theta} = \frac{\boldsymbol{A_x B_x + A_y B_y + A_z B_z}}{|\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}|}$$

$$|\vec{\boldsymbol{A}}| = \sqrt{(3)^2 + (-1)^2 + (2)^2} = \sqrt{14}$$

$$|\vec{\boldsymbol{B}}| = \sqrt{(2)^2 + (3)^2 + (-1)^2} = \sqrt{14}$$

$$cos\theta = \frac{6-3-2}{(\sqrt{14})(\sqrt{14})} = \frac{1}{14} = 0.072$$

$$\theta = cos^{-1}(0.072) = 85.87^o$$

Example:-

Given the two vectors $(\vec{\boldsymbol{A}} = \mathbf{2}\,\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} - \hat{\boldsymbol{k}})$ **;** $(\vec{\boldsymbol{B}} = \hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}} + \mathbf{2}\,\hat{\boldsymbol{k}})$

Find: $(\vec{\boldsymbol{A}} - \mathbf{2}\vec{\boldsymbol{B}})$ **;** $(\vec{\boldsymbol{A}}.\vec{\boldsymbol{B}})$; $(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}})$, unit vector along $(\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}})$.

i. $\vec{A} - 2\vec{B} = 3\,\hat{j} - 5\,\hat{k}$

ii. $\vec{A}.\vec{B} = -1$

iii. $\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = i - 5\,j - 3\,k$

iv. Let $\vec{C} = \vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = -i + 5\,j + 3\,k$

$$\hat{\boldsymbol{u}}\,(\boldsymbol{C}) = \frac{\vec{\boldsymbol{C}}}{|\vec{\boldsymbol{C}}|} = \frac{-i + 5\,j + 3\,k}{\sqrt{35}}$$

H.W.:-

1- Find the area of a triangle with a sides $\left(\vec{\boldsymbol{A}} = \boldsymbol{2}\,\boldsymbol{i} + j - k\right)$; $(\vec{\boldsymbol{B}} = \boldsymbol{i} - j + 2\,k)$ and find the angle between $(\vec{\boldsymbol{A}})$ **and** $(\vec{\boldsymbol{B}})$.

(ans. : 2.95 units ; 99.6^o)

2- Vector $\left(\vec{\boldsymbol{A}} = 25\,units\right)$ in direction south east $(\vec{\boldsymbol{B}} = 40\,units)$ in direction $(\boldsymbol{60^o}\ \boldsymbol{North\ of\ East})$; find component of each vectors. (ans.:- $37.7\,\hat{\imath} + 16.9\,\hat{\jmath}$)

Physical Quantities Used in Mechanics

Quantity (الكمية)	(المعنى) *Meaning*	(الرمز) *symbol*	*Unit* (الوحدة)

Length	الطول	L	*meter*
height	الارتفاع	h	m
Mass	الكتلة	M	kg
Time	الزمن	T	sec
Electric current	التيار الكهربائي	I	*Ampere*
Temperature	درجة الحرارة	t	*kelvin*
Amount of substance	كمية المادة	n	*mole*
Force	القوة	F	$\text{Newton} = \text{kg.m}/sec^2$
Energy	الطاقة	E	*Joule*
Power	القدرة	P	*Watt*
Distance	المسافة	d	m
Displacement	الازاحة	x	m
Velocity	السرعة	v	m/sec
Pressure	الضغط	p	$pa = N/m^2$
Torque	عزم الدوران	τ	$N.m$
omentum*M*	الزخم	p	$kg.m.sec^{-1}$
Work	الشغل	W	*Joule*
Acceleration	التعجيل	a	m/sec^2
Density	الكثافة	ρ	kg/m^3
Volume	الحجم	V	m^3
Area	المساحة	A	m^2
Kinetic Energy	الطاقة الحركية	$K.E$	*Joule*
Potential Energy	الطاقة الكامنة	$P.E$	*Joule*
Frequency	التردد	f	Hz
Average velocity	متوسط السرعة	$V_{avg} = \frac{\Delta x}{\Delta t}$	m/sec
Instantaneous velocity	السرعة اللحظية (الأنية)	$v = \frac{dx}{dt}$	m/sec
Average acceleration	متوسط التعجيل	$a_{avg} = \frac{\Delta v}{\Delta t}$	m/sec^2

Instantaneous acceleration	التعجيل اللحظي (الأني)	$a = \frac{dv}{dt}$	m/sec^2
Ground acceleration	التعجيل الارضي	g	$9.8N\ or\ 980Dyne$
Angle or corner	الزاوية	θ	$Dgree$
Angular velocity	السرعة الزاوية	$\omega\ or\ \Omega$	Rad/sec
Angular accelration	التعجيل الزاوية	α	Rad/sec^2
Radius	نصف القطر	R	m
Central Force	القوة المركزية	F_N	N
Central acceleration	التعجيل المركزي	a_N	m/sec^2

Trigonometric Functions

الزاوية(θ^o)	sin (الجيب)	cos (الجيب تمام)	tan (الظل)
0^o	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sin 0}{\cos 0} = 0$
30^o	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45^o	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60^o	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\sqrt{3}$
90^o	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$	∞

Chapter 2

Motion on a Straight Line

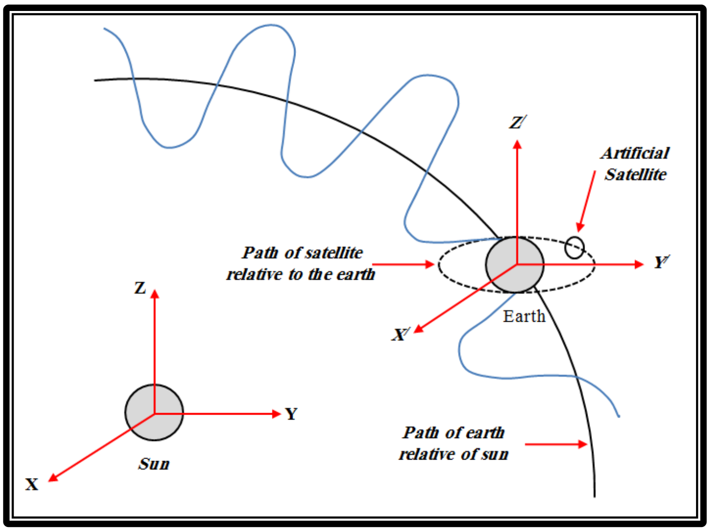
(Motion in One Dimension)

Introduction

Mechanics which is the science of motion and also science of momentum, force, and energy. The description of motion using quantities (position, velocity, acceleration) is called kinematics, and perfect motion based on forces and Newton's law called Dynamics.

Rest and Motion:

An object is in motion relative to another when its position, measured relative to the second body, is changing with time. On the other hand if this relative position does not change with time, the object is at relative rest. Both rest and Motion are relative concept. A tree and a hous are at rest relative to the earth, but in motion relative to the sun.



To describe motion, therefore, must define a frame of reference, for example; let us consider tow observers, one on the sun and the other on the earth as in fig. Both observers studying the motion of an artifical satellite of the earth; for earth observer, the satellite appears circular path around the earth; while to the solar observer the satellite orbite appear as a wavy line.

As a first step in studying classical mechanics to describe the motion of an object. In physics the motion categorize into three types: translational, rotational, and vibrational, in this chapter we concerned to translational motion of object as a particle motion, therefore: motion may be defined as a continuous change of position, the simplest motion to describe motion of a point along a straight line.

Position, Velocity and Speed

The motion of a particales is completely known if the particales position in space is known at all times, a particle position is the location of the particale with respect to a chosen reference point (origin of a coordinate system).

The change in position of the particale for various time intervals as a displacement. Or displacement is defined as the change in position in some time interval (As the particle moves from an initial position $(x_i)$to final position (x_f) the displacement is given by

$$\Delta\vec{x} = \vec{x}_f - \vec{x}_i \quad (\ \vec{x}\ in\ meter)$$

And distance is the length of a path followed by a particle.

The average velocity denoted by $(\vec{v}_{avg})$ is define as the particles displacement $(\Delta\vec{x})$ divided by the time interval Δt or:-

$$\vec{v}_{avg} = \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i} = \frac{\Delta\vec{x}}{\Delta t}$$

To determine the instantaneous velocity at a point, we make Δt (time interval) small or no change in the state of motion occur, in mathematical language, we must compute the limiting value, this written as:

$$\vec{v} = \lim_{\Delta t \to 0} \vec{v}_{avg} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

Or

$$\vec{v} = \frac{d\vec{x}}{dt} \qquad \Longrightarrow \qquad d\vec{x} = \vec{v}dt \qquad (by\ integration)$$

$$\int_{\vec{x}_o}^{\vec{x}} dx = \int_{t_o}^{t} \vec{v} dt$$

$$[change\ displacement] = \vec{x} - \vec{x}_o = \int_{t_o}^{t} \vec{v} dt = \vec{v}_1 dt + \vec{v}_2 dt + \cdots$$

Change in velocity per unit time during the time interval is defined as average acceleration: $(\vec{v}_{avg})$

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta\vec{t}}$$

The instantaneous acceleration is the limiting value of the average acceleration when the time interval (Δt) Become very small, that is

$$\vec{a} = \lim_{\Delta t \to 0} \vec{a}_{avg} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Or

$$\vec{\mathrm{a}} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{d^2\vec{x}}{dt^2}$$

$$\int_{v_0}^{v} d\vec{v} = \int_{t_0}^{t} a dt$$

$$\vec{v} - \vec{v}_o = \int_{t_0}^{t} a dt$$

$change\ in\ velocity = \vec{v} - \vec{v}_o = \vec{a}_1 dt_1 + \vec{a}_2 dt_2 + \cdots$

Example (1)

A particle move along x axis, its position varies with time according the expression:

$(\vec{x} = -4t + 2t^2)$

1) Determine the displacement of the particle in t =1 , t=3 (sec).

$$\vec{x}_1 = -4(1) + 2(1)^2 = -4 + 2 = -2\ (m)$$

$$\vec{x}_2 = -4(3) + 2(3)^2 = -12 + 18 = +6\ (m)$$

2) Find $\vec{v}_{avg} = \frac{6-(-2)}{2} = \frac{8}{2} = 4\left(\frac{m}{sec}\right)$

3) Find instantaneous velocity $(\vec{v})$

$$(\vec{v}) = \frac{d\vec{x}}{dt} = \frac{d}{dt}\,(-4t + 2t^2)$$

$$\vec{v} = -4 + 4t \quad \Longrightarrow \quad t = 3\ (sec)$$

$$\vec{v} = -4 + 4(3) = 8\ (sec)$$

Example (2)

A particle moves along the x-axis position given by $(\vec{x} = 5t^2 + 1)$

1) Find average velocity in the time interval between (2 sec and 3 sec)?

$$\vec{x}_1 = 5(2)^2 + 1 = 5(4) + 1 = 21(m) \Longrightarrow t = 2\ (sec)$$

$$\vec{x}_2 = 5(3)^2 + 1 = 5(9) + 1 = 46\ (m)$$

$$\Delta\vec{x} = \vec{x}_2 - \vec{x}_1 = 46 - 21 = 25\ (m)$$

$$\vec{v}_{avg} = \vec{v} = \frac{\Delta\vec{x}}{\Delta t} = \frac{25}{1} = 25\left(\frac{m}{sec}\right)$$

2) Find instantaneous velocity at (t =2 sec)?

$$\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}(5t^2 + 1) = 10\ t = 20\left(\frac{m}{sec}\right)$$

Example (3)

A body moves along x-axis according to the law ($\vec{x} = 2t^3 + 5t^2 + 5$)

1) Find the velocity and acceleration at any time?
2) Find the position, velocity and acceleration at (t=2 sec) and (t=3 sec)?
3) Find the average velocity and acceleration between (t=2 sec) and (t=3 sec)?

Ans.:

1) $\vec{v} = \frac{d\vec{x}}{dt} = 6t^2 + 10t$

$$\vec{a} = \frac{d\vec{v}}{dt} = 12t + 10$$

2) At t=2 (sec)

$$\vec{x}_1 = 41(m)\ ;\quad \vec{v}_1 = 44\left(\frac{m}{sec}\right),\quad \vec{a}_1 = 34\left(\frac{m}{sec^2}\right)$$

Similarly for t =3 (sec)

$$\vec{x}_2 = 104\ (m)\ ;\quad \vec{v}_2 = 84\left(\frac{m}{sec}\right)\ ;\quad \vec{a}_2 = 46\left(\frac{m}{sec^2}\right)$$

3) $\vec{v}_{avg} = \frac{\Delta\vec{x}}{\Delta t} = \frac{63}{1} = 63\left(\frac{m}{sec}\right)$

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{40}{1} = 40\left(\frac{m}{sec^2}\right)$$

Example (4)

The acceleration of a body moving along a straight line is given by ($\vec{a} = 4 - t^2$); Where a is in ($m.sec^{-2}$) and t is in seconds.

Find the expressions for the velocity and displacement as functions of times ,given that when t =3 (sec) ,$\vec{v} = 2\ (m.sec^{-1})\ and\ \vec{x} = 9\ (m)$.

Ans:

$$\vec{a} = 4 - t^2$$

$$\int d\vec{v} = \int \vec{a}dt = \int (4 - t^2)dt$$

$$\vec{v} = 4t - \frac{1}{3}t^3 + c_1$$

$$2 = 4(3) - \frac{1}{3}(3)^3 + c_1$$

$$c_1 = -1$$

$$\vec{v} = 4t - \frac{t^3}{3} - 1$$

$$\int d\vec{x} = \int \vec{v}\, dt = \int \left(4t - \frac{t^3}{3} - 1\right) dt$$

$$\vec{x} = 2t^2 - \frac{t^4}{12} - t + c_2$$

$$9 = 2(3)^2 - \frac{81}{12} - 3 + c_2$$

$$c_2 = \frac{3}{4}$$

$$\vec{x} = 2t^2 - \frac{t^4}{12} - t + \frac{3}{4}$$

(Equations of motion)

1-Motion at a constant velocity:

$\vec{v}$ = constant $\Longrightarrow \vec{a} = \frac{d\vec{v}}{dt} = 0$

Or $\quad \vec{a} = 0 \Longrightarrow$ no acceleration

Average velocity = $\bar{\vec{v}} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}-\vec{x}_0}{t-t_0}$

$$\vec{x} - \vec{x}_o = \vec{v}(t - t_o)$$

If $\quad \vec{x}_o = 0;\ and\ t_o = 0\ (the\ motion\ from\ origin\ point)$

Or $\quad [\vec{x} = \vec{v}t]$

And $\quad \bar{\vec{v}} = \frac{\vec{v}_0+\vec{v}}{2}$

2- The particle under constant Acceleration (uniformly accelerated motion)

Using definition of average acceleration get:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_o}{t - t_o}\ ; (t_o = 0)$$

$$\vec{v} - \vec{v}_o = \vec{a}t$$

$$[\vec{v} = \vec{v}_o + \vec{a}t] \qquad \dots\dots\dots\dots\dots (1)$$

And

$$\bar{\vec{v}} = \frac{\Delta \vec{x}}{\Delta t} \quad \Longrightarrow \quad \vec{x} - \vec{x}_o = \bar{\vec{v}}\, t\ \ (t_o = 0)$$

$$\vec{x} = \vec{x}_o + \left(\frac{\vec{v}_0 + \vec{v}}{2}\right)t$$

$$\vec{x} = \vec{x}_0 + \frac{1}{2}\vec{v}_o\, t + \frac{1}{2}(\vec{v}_o + \vec{a}t)t$$

$$\vec{x} = \vec{x}_o + \frac{1}{2}\vec{v}_o t + \frac{1}{2}\vec{v}_o t + \frac{1}{2}\vec{a}t^2$$

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2 \quad , \quad if \quad \vec{x}_o = 0 \qquad then$$

$$\left[\vec{x} = \vec{v}_o\, t + \frac{1}{2}\vec{a}t^2\right] \qquad \dots\dots\dots\dots\dots (2)$$

And also:

$$\vec{x} = \vec{v}\, t$$

$$= \frac{1}{2}(\vec{v}_o + \vec{v})\left(\frac{\vec{v} - \vec{v}_o}{\vec{a}}\right)$$

$$\vec{x} = \frac{1}{2\vec{a}}\left(\vec{v}^2 - \vec{v}_o{}^2\right)$$

Or $\quad 2\vec{a}\vec{x} = \vec{v}^2 - \vec{v}_0{}^2$

Rearranging terms, we get:

$$\left[\vec{v}^2 = \vec{v}_0{}^2 + 2\vec{a}\vec{x}\right] \qquad \dots\dots\dots\dots\dots (3)$$

Mathematical method also uses to get equation of motion under constant acceleration that:- $\quad \vec{a} = \frac{d\vec{v}}{dt} \quad ; \quad d\vec{v} = \vec{a}\ dt \quad (\,by\ integration)$

$$\int d\vec{v} = \int \vec{a}dt$$

$$\vec{v} = \vec{a}t + c_1$$

$c_1\ is\ constant\ ; \quad at \quad t = 0 \implies \vec{v} = \vec{v}_o \quad (condition\ of\ motion)$

$$\vec{v}_o = 0 + c_1 \implies c_1 = \vec{v}_o$$

$rewrite\ the\ above\ equation$:

$$[\vec{v} = \vec{v}_o + \vec{a}t] \qquad \dots\dots\dots\dots\dots (1)$$

$$\vec{v} = \frac{d\vec{x}}{dt} = \vec{v}_o + \vec{a}t$$

$$\int d\vec{x} = \int (\vec{v}_o + \vec{a}t)\, dt \qquad (\, integrated\ eq.)$$

$$\vec{x} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2 + c_2$$

Where: $c_2 = constant$; $\quad and\ when \quad t = 0 \quad \Longrightarrow \quad \vec{x} = 0$

We get $c_2 = 0$

The equations become:

$$\left[\vec{x} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2\right] \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

The third equation may find using definition of ($\vec{a}$):

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \vec{v}\frac{d\vec{v}}{d\vec{x}}$$

$$\int \vec{v}\, d\vec{v} = \int \vec{a}\, d\vec{x}$$

$$\frac{1}{2}\vec{v}^2 = \vec{a}\,\vec{x} + c_3$$

Motion condition that: $\vec{x} = 0 \;\; \Longrightarrow \;\; \vec{v} = \vec{v}_o$

We find: $c_3 = \frac{1}{2}\vec{v}_o^{\,2}$

Above equation become:

$$\frac{1}{2}\vec{v}^2 = \frac{1}{2}\vec{v}_o^{\,2} + \vec{a}\vec{x}$$

Or $\quad \vec{v}^2 = \vec{v}_o^{\,2} + 2\vec{a}\vec{x} \qquad \ldots\ldots\ldots\ldots (3)$

Example: -

A body moves along the x-axis with constant acceleration $\left(\vec{a} = 4\ \frac{m}{sec^2}\right)$ $at\ time\ (t = 0\,\text{sec})\ it\ is\ at\ (\vec{x} = 5m) and\ \left(\vec{v} = 3\frac{m}{sec}\right) find$:

(a) The position and velocity at t=2 sec?

(b) Where is the body when its velocity is $(5\frac{m}{sec})$?

Ans.:

(a) $\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$

$$\vec{x} = (5) + (3)(2) + \frac{1}{2}(4)(2)^2 = 19\ (m)$$

$$\vec{v} = \vec{v}_o + \vec{a}t = 3 + 4(2) = 11\left(\frac{m}{sec}\right)$$

(b) $\vec{v}^2 = \vec{v}_o{}^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$

$$(5)^2 = (3)^2 + (2)(4)(x - 5)$$

$$x = 7\ (m)$$

(H.w.)

A car starts from rest and has a constant acceleration of $\left(1.2\ \frac{m}{sec^2}\right)$ for (12 sec).

a) How far has the car traveled at the end of the (12 sec).

b) What is the speed of the car at the seventh second, and calculate distance in seventh second?

Freely Falling Bodies

(Motion due to gravity)

The most common example of motion with constant acceleration is that of a falling body near the earth's surface, if we neglect air resistance, we find all bodies, regardless of their size, shape or composition, fall with the same acceleration. Which is denoted by (g) is called acceleration due to gravity.

Near the earth's surface the magnitude of (g=9.8)

The direction of the free-fall acceleration of any point down-ward.

Although we speak of falling bodies, bodies in upward motion experience the same free-fall acceleration (magnitude and direction).

That is, no matter whether a particle is moving up or down, the direction of its acceleration under the influence of the earth gravity is always down.

The equations describing freely falling body are:

$$\vec{v} = \vec{v}_o - gt \qquad \dots (1)$$

$$\vec{y} = \vec{v}_o t - \frac{1}{2} g t^2 \qquad \dots (2)$$

$$\vec{v}^2 = {\vec{v}_o}^2 - 2gy \qquad \dots (3)$$

Example (1)

A stone throw straight upward on a bridge. The stone hits a stream (44.1 m) below the point at which it release it, (4 sec) later find:

a) What is the velocity of the stone just after it leaves your hand?

b) What is the velocity of the stone just before it hits the water?

Ans.:

a) $y = \vec{v}_o t + \frac{1}{2} g t^2$

$$44.1 = \vec{v}_o (4) + \frac{1}{2}(9.8)(4)^2$$

$$\vec{v}_o = -8.6 \left(\frac{m}{sec}\right)$$

b) $\vec{v} = \vec{v}_o + gt$

$$\vec{v} = -8.6 + 39.2$$

$$\vec{v} = 30.6 \left(\frac{m}{sec}\right)$$

Example (2)

A body is dropped from rest and falls freely. Determine the position and velocity of the body after (1, 2, 3, and 4 sec) have elapsed?

Ans:

$$y = \vec{v}_o t - \frac{1}{2} g t^2$$

$$y = -\frac{1}{2} g t^2$$

$$y = -\frac{1}{2}(9.8)(1)^2 = -4.9\ (m)$$

$$\vec{v} = \vec{v}_o - gt = -gt = -9.8 \times 1 = -9.8 \left(\frac{m}{sec}\right)$$

Example (3)

A ball is thrown vertically upward from the ground with speed of (25.2 m/sec).

(a) How long does it take to reach its highest point?

(b) How high does it rise?

(c) At what times will it be (27 m) above the ground?

Ans.:

(a) $t = \frac{\vec{v}_o - \vec{v}_y}{g} = \frac{25.2-0}{9.8} = 2.57\ (sec)$

(b) $y = \vec{v}_o t - \frac{1}{2}gt^2 = (25.2)(2.57) - \frac{1}{2}(9.8)(2.5)^2 = 32.4\ (m)$

(c) $\frac{1}{2}gt^2 - \vec{v}_o t + y = 0$

$(4.9)t^2 - (25.2)t + 27 = 0$

Solving find that

$t = 1.52\ (\text{sec})\ \ and\ \ t = 3.62\ (\text{sec})\ at\ t = 1.52\ (\text{sec})$

$velocity\ of\ ball\ is\ \vec{v}_y = \vec{v}_0 - gt = 25.2 - 9.8 = 10.3\ \left(\frac{m}{sec}\right)$

$at\ t = 3.62\ \text{sec} \qquad \overrightarrow{v} = \vec{v} - gt = 25.2 - 9.8 \times 3.62 = -10.3$

Tow velocities identical magnitude

Example (4)

A stone is dropped from the top of a tower, (50 m) high. At the same time another stone is thrown upwards from the foot of the tower with velocity of (25m/sec), when and where the tow stones cross each other?

Ans.:

First stone: $y = \vec{v}_o t + \frac{1}{2} g t^2 = \frac{1}{2} g t^2 \ldots (1)$

Second stone: $50 - y = 25t - \frac{1}{2} g t^2 \ldots (2)$

Adding (1) and (2)

$50 = 25\, t \quad \Longrightarrow \quad t = 2\ (sec)$

Where the stones cross each other

$$y = \vec{v}_o t + \frac{1}{2} g t^2$$

$$y = 0 + \frac{1}{2} \times 9.8 \times (2)^2 = 19.6\ (m)$$

Example (5)

A ball is thrown vertically up-ward from the ground with a speed of (25.2 m/sec).

(a) How long does it take to reach its highest point?

(b) How high does it rise?

(c) At what times will it be (27 m) in above the ground?

Ans:

(a) $\vec{v} = \vec{v}_o - g t \quad \Longrightarrow \quad t = 2.57\ (sec)$

(b) $y = \vec{v}_o t - \frac{1}{2} g t^2 \Longrightarrow y = (25.2)(2.57) - \frac{1}{2}(9.8)(2.27)$

$$y = 32.4\ (m)$$

(c) $y = \vec{v}_o t - \frac{1}{2} g t^2 \ or \ \frac{1}{2} g t^2 - \vec{v}_o t + y = 0$

$$(4.9)t^2 - (25.2)t + 27 = 0$$

Solving this quadratic equation, we find t =1.52 (sec) and t = 3.62 (sec) at t = 1.52 (sec)

$$\vec{v}_y = \vec{v}_o - gt = 25.2 - 9.8 \times 1.52 = 10.3 \left(\frac{m}{sec}\right) \ and \ at \ t = 3.62\ (sec)$$

$$\vec{v}_y = -10.3$$

The two velocities have identical magnitude but opposite direction.

Example (6)

A stone was thrown vertically upwards, from the ground, with velocity (49 m/sec) after (2 sec) another stone was thrown vertically upwards from the same place. If both the stones strike the ground at the same time, find the velocity with which the second stone was thrown?

Ans.:

$$\vec{v} = \vec{v}_o - 9.8t \Longrightarrow 0 = 49 - 9.8\, t \ \Longrightarrow t = 5\ (sec)$$

Total time of flight =5+5=10 (sec)

For second stone $\Longrightarrow time = 10 - 2 = 8\ (sec)$

Time take to reach maximum height =4 (sec)

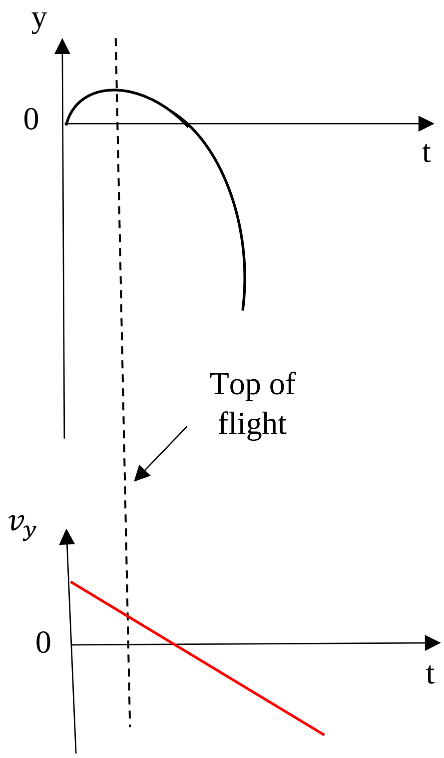
$$\vec{v}_o = 9.8 \times (4) = 39.2 \left(\frac{m}{sec}\right)$$

Example (1)

A stone throw straight upward on a bridge. The stone hits a stream $(44.1\ m)$ below the point at which it release it, $(4\ sec)$ later find:

a) What is the velocity of the stone just after it leaves your hand?
b) What is the velocity of the stone just before it hits the water?

Ans:



a) $\vec{y} = \vec{v}_o t - \frac{1}{2} g t^2$

$$-44.1 = \vec{v}_o(4) - \frac{1}{2}(9.8)(4)^2$$
$$-44.1 = 4v_0 - 78.4$$
$$-44.1 + 78.4 = 4\vec{v}_o$$
$$34.3 = 4\,\vec{v}_o \quad \Longrightarrow \vec{v}_o = \frac{34.3}{4} = 8.6\ \ (m/sec)$$
$$\vec{v}_o = 8.6\ (\frac{m}{sec})$$

The initial velocity is 8.6 m/sec upward.

b) $\vec{v} = \vec{v}_o - gt$

$$\vec{v} = 8.6 - 39.2 = -30.6\ (m/sec)$$
$$\vec{v} = -30.6\left(\frac{m}{sec}\right)$$

The Final velocity is 30.6 m/sec downward.

Example (2)

A body is dropped from rest and falls freely. Determine the position and velocity of the body after (1. 2. 3 and 4 sec) have elapsed?

Ans:

At t=1 (sec)

$$\vec{y} = \vec{v}_o t + \frac{1}{2} g t^2 = \frac{1}{2} g t^2$$
$$\vec{y} = \frac{1}{2}(9.8)(1)^2 = 4.9\ \ (m)$$

$$\vec{v} = \vec{v}_o + gt = gt = 9.8 \times 1 = 9.8\left(\frac{m}{sec}\right)$$

At t=2sec

$$\vec{y} = \frac{1}{2}(9.8)(2)^2 = 19.6 \quad (m)$$

$$\vec{v} = \vec{v}_o + gt = gt = 9.8 \times 2 = 19.6\left(\frac{m}{sec}\right)$$

At t=3sec

$$\vec{y} = \frac{1}{2}(9.8)(3)^2 = 44.1 \quad (m)$$

$$\vec{v} = \vec{v}_o + gt = gt = 9.8 \times 3 = 29.4 \quad \left(\frac{m}{sec}\right)$$

At t=4sec

$$\vec{y} = \frac{1}{2}(9.8)(4)^2 = 78.4 \quad (m)$$

$$\vec{v} = \vec{v}_o + gt = gt = 9.8 \times 4 = 39.2 \quad \left(\frac{m}{sec}\right)$$

Example (3)

A ball is thrown vertically upward from the ground with speed of $(25.2 \ m/sec)$.

(a) How long does it take to reach its highest point?

(b) How high does it rise?

(c) At what times will it be $(27m)$ above the ground?

Ans:

a) $\vec{v} = \vec{v}_o - gt \quad \Longrightarrow \quad t = \frac{\vec{v}_o - \vec{v}_y}{g} \quad \Longrightarrow \ 0 = 25.2 - 9.8 \times t$

$$t = \frac{25.2 - 0}{9.8} = 2.57\ (sec)$$

b) $\vec{y} = \vec{v}_o t - \frac{1}{2}gt^2 = (25.2)(2.57) - \frac{1}{2}(9.8)(2.57)^2 = 32.4 \ (m)$

c) $\vec{y} = \vec{v}_o t - \frac{1}{2}gt^2 \quad or \quad \frac{1}{2}gt^2 - \vec{v}_o t + \vec{y} = 0$

$$(4.9)t^2 - (25.2)t + 27 = 0 \qquad \Rightarrow \qquad (t = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a})$$

Solving this quadratic equation, we find that:

$t = 1.52\ (\text{sec})\ and\ \ t = 3.62\ \ (\text{sec})$

$$At \quad t = 1.52sec \quad \Longrightarrow \quad \vec{v}_y = \vec{v}_o - gt = 25.2 - 9.8 \times 1.52 = 10.3\ (\frac{m}{sec})$$

$$At \quad t = 3.62\,\text{sec} \quad \Longrightarrow \quad \vec{v}_y = \vec{v}_o - gt = 25.2 - 9.8 \times 3.62$$

$$= -10.3\ (\frac{m}{sec})$$

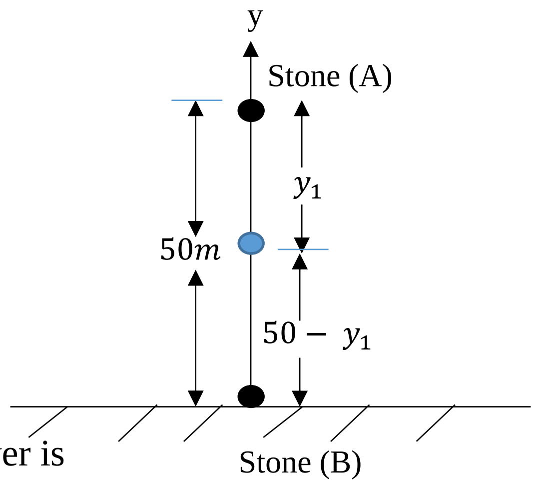
The tow velocities have identical magnitude but opposite direction.

Example (4)

A stone is dropped from the top of a tower, $(50m)$ high. At the same time another stone is thrown upwards from the foot of the tower with velocity of $(25m/sec)$, when and where the tow stones cross each other?

Ans:

First stone: distance travelled by stone (A) from the top of tower is:-



$$y = \vec{v}_o t + \frac{1}{2} g t^2$$

$$y_1 = (0)t + \frac{1}{2} g t^2$$

$$y_1 = \frac{1}{2} g t^2 \qquad \dots (1)$$

For same stone (A), distance from foot of tower is

$$y = 50 - y_1$$

$$y = 50 - \frac{1}{2} g t^2 \qquad \dots (2)$$

Second stone: Now for stone (B), distance from foot of tower is

$$y = \vec{v}_o t - \frac{1}{2} g t^2$$

$$y = 25t - \frac{1}{2}gt^2 \dots (3)$$

Stone (A) and stone (B) will meet at same distance from foot of the tower (eq(2) and eq(3) are equal)

$$50 - \frac{1}{2}gt^2 = 25t - \frac{1}{2}gt^2$$

$$50 - \cancel{\frac{1}{2}gt^2} + \cancel{\frac{1}{2}gt^2} = 25\,t$$

$$50 = 25t \rightarrow t = \frac{50}{25} = 2\,(\text{sec}) \qquad \textit{Substitution in eq.}(2)$$

$$y = 50 - \frac{1}{2}(9.8)(2)^2 = 50 - 30.4 = 19.6 \quad (m)$$

The tow stones meet at the height of $(19.6m)$ from foot of tower and takes (2sec)

Example (5)

A stone was thrown vertically upwards, from the ground, with velocity $(49m/sec)$ after (2sec) another stone was thrown vertically upwards from the same place. If both the stones strike the ground at the same time, find the velocity with which the second stone was thrown?

Ans:

$$\vec{v} = \vec{v}_o - gt$$

$0 = 49 - 9.8t \implies t = 5\,(\text{sec})$ زمن الصعود

Total time of flight =5+5=10 (sec)

For second stone $\implies time\ (t) = 10 - 2 = 8\ (\text{sec})$

Time take to reach maximum height $(t) = \frac{8}{2} = 4\,(sec)$

$$\vec{v} = \vec{v}_o - gt$$

$$0 = \vec{v}_o - 9.8(4) \implies 0 = v_0 - 39.2$$

$$\vec{v}_o = 39.2\,\left(\frac{m}{sec}\right)$$

متى تكون السرعة صفرا في حالة السقوط الحر

جسم يسقط من السكون	جسم قذف بسرعة لاسفل	جسم قذف بسرعة لاعلى
$v_i = 0m/sec$	لها قيمة واتجاهها لاسفل $= v_i$	لها قيمة واتجاهها لاعلى $= v_i$
لها قيمة واتجاهها لاسفل $= v_f$	لها قيمة واتجاهها لاسفل $= v_f$	$v_f = 0m/sec$
$g = +9.8m/sec^2$	$g = +9.8m/sec^2$	$g = -9.8m/sec^2$

زمن الصعود = زمن النزول

Chapter 3

Motion in a plane
(Motion in a tow dimension)

Motion in a plane

(Motion in a tow dimension)

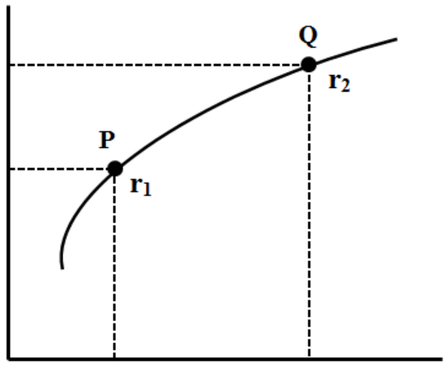
Examples (baseball, projectiles, revolve of earth around sun, satellite around earth)

A particle moves along a curve of x-y plane

P:- position of particle at (t_1)

Q:- position of particle at (t_2)

And every point represents by (r)



$\overrightarrow{r_1} = \overrightarrow{x_1} i + \overrightarrow{y_1} j$; $\overrightarrow{x_2} = \vec{x}_2 i + \vec{y}_2 j$

$\Delta\vec{r} = i\,\vec{r}_x + j\,\vec{r}_y$

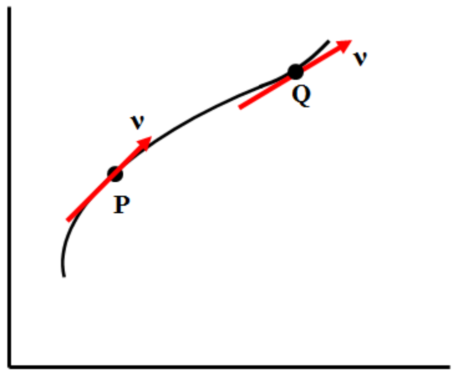
And velocity average: -

$$v_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

$$v_{ins} = \frac{dr}{dt}$$

$\vec{v} = i\frac{d\vec{x}}{dt} + j\frac{d\vec{y}}{dt}$ component of $(\vec{v})$

$$\vec{v} = i\,\overrightarrow{v_x} + j\,\overrightarrow{v_y}$$



Value of:

$$\vec{v} = \sqrt{(v_x)^2 + (v_y)^2}$$

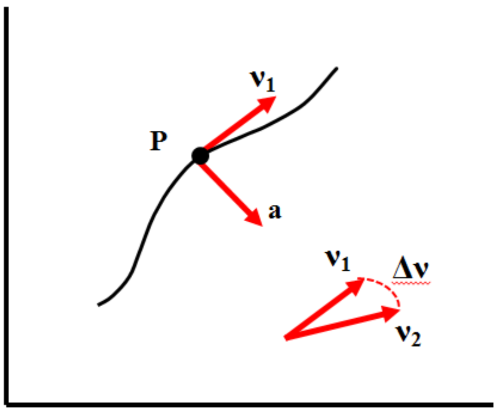
$$\theta = \tan^{-1}\frac{v_y}{v_x}$$

And the value of $(\vec{a})$

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

$$\vec{a}_{ins} = \lim_{\Delta t \to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = i\,\frac{d\vec{v}_x}{dt} + j\,\frac{d\vec{v}_y}{dt}$$



$$\vec{a} = i\frac{d^2\vec{x}}{dt^2} + j\frac{d^2\vec{y}}{dt^2}$$

$$\vec{a} = i\vec{a}_x + j\vec{a}_y \qquad (\,component\ of\ a)$$

$$\vec{a} = \sqrt{\vec{a}_x{}^2 + \vec{a}_y{}^2}$$

In actual motion the acceleration of particle in a curve analysis to tow components ($a_{//}$) and ($a_{\perp}$) with direction is varies with the $a_{\parallel}$ varies with varies of value $(a\ //\ v)$ of $\left(\frac{dv}{dt}\right)$

$(a\ \perp\ v\,)$ $(a_{\perp}\,)$ Varies with direction.

Example: A particle moves over a path such that the components of its position with respect to an origin of coordinates are given as a function of time by:

$x = -t^2 + 12t + 5$

$y = -2t^2 + 16t + 10$

Where *(t)* is in seconds and *x* and *y* are in meters.

(a) Find the particle's position vector $\vec{r}$ as a function of time, and find its magnitude and direction at *t* = 6 sec.

(b) Find the particle's velocity vector $\vec{v}$ as a function of time, and find its magnitude
and direction at *t* = 6 sec.

(c) Find the particle's acceleration vector $\vec{a}$ as a function of time, and find its magnitude and direction at *t* = 6 sec.

Solution:

(a) The position vector is given at time *t* by:

$$\vec{r} = \vec{x_\imath} + \vec{y_\jmath} = (-t^2 + 12t + 5)\vec{\imath} + (-2t^2 + 16t + 10)\vec{\jmath}$$

$$\vec{r} = 41\vec{\imath} + 34\vec{\jmath}$$

The magnitude of $\vec{r}$ is:

$$r = \sqrt{x^2 + y^2} = \sqrt{(41)^2 + (34)^2} = 53.6(m)$$

The angle θ between $\vec{r}$ and the direction of increasing *x* is:

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{34}{41}\right) = \tan^{-1}(0.83) = 39.7^{o}.$$

(b) The velocity components along the *x* and *y* axes are:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-t^2 + 12t + 5) = -2t + 12$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(-2t^2 + 16t + 10) = -4t + 16$$

At $t = 6\,\text{sec}$ $\quad v_x = 0\frac{m}{sec} \qquad v_y = -8\left(\frac{m}{sec}\right)$

$$\vec{v} = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(0)^2 + (-8)^2} = 8\frac{m}{sec}$$

The angle θ between $\vec{r}$ and the direction of increasing *x* is:

$$\theta = \tan^{-1}\frac{\mathrm{v_y}}{\mathrm{v_x}} = \tan^{-1}\left(\frac{-8}{0}\right) = \tan^{-1}(-\infty) = 270^o$$

(c) The components of the acceleration along the *x* and *y* axes are:

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-2t^2 + 12) = -2\frac{m}{sec^2}$$

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(-4t^2 + 16) = -4\frac{m}{sec^2}$$

$$a = \sqrt{{a_x}^2 + {a_y}^2} = \sqrt{(-2)^2 + (-4)^2} = \sqrt{20} = 4.47\left(\frac{m}{sec^2}\right)$$

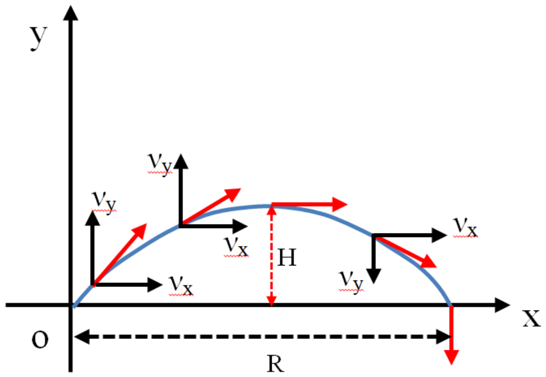
The angle θ between $\vec{a}$ and the direction of increasing *x* is:

$$\theta = \tan^{-1}\frac{\mathrm{a}_y}{\mathrm{a}_x} = \tan^{-1}\left(\frac{-4}{-2}\right) = 180 + \tan^{-1}(2) = 180 + 63.4$$
$$= 243.4^o$$

Motion in a plane

Projectile Motion

A common example of motion in tow dimension is projectile motion near the earth's surface, in which projectile, such as a golf ball (baseball)Figure: shows trajectory of a body projected with an initial velocity ($\overrightarrow{v_0}$)



At an angle of departure (θ) , the distance (R) is the horizontal range, and (H) is maximum height; the particle projected has two components

1-vertical 2- horizontal

The following terms, which used in this term:

1- Trajectory: the path followed by a projectile.

2- The angle of elevation: is the angle of the initial velocity above the horizontal.

3- Time of flight: total time taken by projectile to reach maximum height and to return back to the ground.

4- (R): horizontal range of projectile.

5- (H): maximum height of projectile.

Motion of a Projectile

Consider a particle projectile upwards from a point (O) see fig. At an angle θ with the horizontal, and with an initial velocity (v_0). Resolving this velocity into its vertical and horizontal component

$$\vec{v}_x = \vec{v}_o cos\theta \qquad \& \qquad \vec{v}_y = \vec{v}_0 sin\theta$$

Where: ($v_0 cos\theta$) component will remain constant, since there is no acceleration.

And: $(\vec{v}_0 sin\theta)$ component is subjected to retardation due to gravity.

The particle will reach maximum height, when the vertical component becomes zero. The combined effect of the horizontal and vertical component will be to move the particle along some path.

Equation of the path of a projectile

From equation of motion:

$$\vec{y} = \vec{v}_{0y} - \frac{1}{2} g\, t^2 \;\; ; \quad \vec{v}_{0y} = \vec{v}_0 \, sin\theta \;\; ; \quad \vec{x} = (\vec{v}_0 cos\theta)\, t$$

$$\vec{y} = (\vec{v}_0 sin\theta)\, \frac{\vec{x}}{\vec{v}_0 cos\theta} - \frac{1}{2} g (\frac{\vec{x}}{\vec{v}_0 cos\theta})^2$$

$$\vec{y} = (tan\theta)\vec{x} - (\frac{g}{2\vec{v}_0{}^2 cos^2\theta})\vec{x}^2$$

By comparing with equation of a parabola $(y = ax - bx^2)$

Since this is equation of path of a parabola, thus the path of a projectile is a parabola.

Time of flight a projectile

$$\because \overrightarrow{y} = (\vec{v}_0 sin\theta)t - \frac{1}{2} g t^2$$

$\because \overrightarrow{y} = 0$ When a particle strike the ground

$$(\vec{v}_0 sin\theta)\, t = \frac{1}{2} g\, t^2$$

$$t = \frac{2\, \vec{v}_0 \, sin\theta}{g}$$

And the time required for projectile to reach its highest point.

$\because$ Velocity of the projectile at the point of greatest height $(v_y = 0)$.

$$\therefore \;\; \vec{v}_y = \vec{v}_0 \, sin\theta - g\, t$$

$$0 = \vec{v}_0 sin\theta - gt$$

$$t = \frac{\vec{v}_0 \, sin\theta}{g}$$

Horizontal range of a projectile

$\because R =$ Horizontal velocity $\times$ T$ime\ of\ flight$

$$R = (\overrightarrow{v_0}\ cos\theta)\left(\frac{2\overrightarrow{v_0}sin\theta}{g}\right)$$

$$R = \frac{2\ (v_0)^2\ cos\theta\ sin\theta}{g} \qquad ; \qquad [2\cos\theta\sin\theta = \sin 2\theta]$$

$$R = \frac{(\overrightarrow{v_0})^2\ \sin 2\theta}{g}$$

The range will be maximum

$\sin 2\theta = 1 \qquad \Longrightarrow \qquad 2\theta = 90 \qquad \Longrightarrow \qquad \theta = 45^o$

$$R_{max} = \frac{(\overrightarrow{v_0})^2}{g}$$

Maximum height of a projectile

$$H\ =\ Average\ vertical\ velocity\ \times\ Time\ of\ flight$$

$$H = \frac{\vec{v}_0\ sin\theta + 0}{2} \times \frac{\vec{v}_0\ sin\theta}{g}$$

$$H = \frac{(\vec{v}_0)^2 sin^2\theta}{2g}$$

Example (1)

If a particle is projected inside a horizontal tunnel which (5 m) height with velocity of (60 m/sec). Find:- ($\boldsymbol{\theta\ and\ R}$) ?

Ans.:-

$$H = \frac{(60)^2 sin^2\theta}{2 \times 9.8} = 5 \qquad \Longrightarrow \qquad sin\theta = 0.165 \qquad \Longrightarrow \qquad \theta = 9.49^o$$

$$R = \frac{(60)^2 \sin(2 \times 9.49^o)}{9.8} = \frac{(60)^2 sin 19^o}{9.8}$$

$R = 119.61\ (m) \qquad Answer$

Example (2)

Football is projected horizontally with an angle (45^o) and horizontally range is (32 m). Find:-

a) The maximum height of projectile?

b) The angle of departure (2θ) with same initial velocity of projectile at horizontal range of (20 m)?

Ans.:-

$$R = \frac{(\vec{v}_0)^2 sin2\theta}{g}$$

$$32 = \frac{(\vec{v}_0)^2 sin2 \times 45^o}{9.8} \qquad \Longrightarrow \qquad \vec{v}_0 = 17.7 \left(\frac{m}{sec}\right)$$

$$H = \frac{(\overrightarrow{v_0})^2 sin^2\theta}{2\ g} = \frac{(17.7 \times \frac{1}{\sqrt{2}})^2}{2 \times 9.8} = 7.99 \quad (m)$$

The angle departure

$$20 = \frac{(17.7)^2 sin2\theta}{9.8}$$

$$sin2\theta = 0.625 \qquad \Longrightarrow \qquad 2\theta = 38.7 \qquad \Longrightarrow \qquad \theta = 19.4^o$$

Or $\quad 2\theta = 180 - 38.7 = 141.3$

$$\theta = 70.6^o$$

Example (3)

A projectile is lunched at velocity (600 m/sec) with an angle (60^o). Find:-

a) Horizontal range.

b) The velocity and height of projectile after (30 sec). From lunching.

c) Maximum height reaches of projectile.

Ans.:-

a) $R = \frac{(\overrightarrow{v_0})^2 sin2\theta}{g} = \frac{(600)^2 sin2\times60}{9.8}$

$$R = 31813.2\ (m) = 31.8\ (km)$$

b) $\vec{v}_y = \overrightarrow{v_0} \, sin\theta - gt$

$\vec{v}_y = 600 \times sin60 - 9.8 \times 30$

$\vec{v}_y = 225.6 \left(\frac{m}{sec}\right)$

$\vec{v}_x = v_0 \, cos\theta = 600 \times cos60 = 300 \left(\frac{m}{sec}\right)$

$\vec{v} = \sqrt{(300)^2 + (210)^2} = 366.2 \left(\frac{m}{sec}\right)$

$\vec{y} = (\vec{v}_0 \, sin\theta) \; t - \frac{1}{2} g \, t^2$

$\vec{y} = (600 \times sin60)(30) - \frac{1}{2}(9.8)(30)^2 = 11200 \; (m)$

c) $H = \frac{(\overrightarrow{v_0})^2 sin^2\theta}{2 \, g}$

$$H = \frac{(600)^2 sin60}{2 \times 9.8} = 13775.1 \; (m)$$

Example (4) (H. W.)

A body is projectile at such an angle that the horizontal range is three times the greatest height. Find the angle of projectile. ($\boldsymbol{Ans.}$: $\mathbf{53.8^o}$)

Example (5)

An aero plane, flying horizontally, at height of (1960 m), with velocity (450 km/h). Has aimed to hit tank. Find, at what distance from the tank he should be release the bomb in order to hit the target.

Ans.:-

$\vec{y} = \vec{v}_{0y} \; t + \frac{1}{2} g \, t^2$

$-1960 = 0 + \frac{1}{2}(-9.8)(t)^2$

$1960 = 4.9 \; (t)^2 \quad \Longrightarrow \quad (t)^2 = 400 \quad \Longrightarrow \quad t = 20 \; (sec)$

$\vec{x} = \vec{v}_{ox} t \quad \Longrightarrow \quad \vec{v} = 450 \left(\frac{km}{h}\right) = \frac{450 \times 1000}{60 \times 60} \left(\frac{m}{sec}\right) = 125 \left(\frac{m}{sec}\right)$

$\vec{x} = 125 \times 20 = 2500 \; (m) \; = 2.5 \; (km)$

Example (6)

A ball is projectile horizontally with a velocity (5 m/sec). Find the position and velocity after $\left(\frac{1}{4}\ \ sec\right)$.

Ans. :-

$$\vec{x} = \vec{v}_0\, t = 5 \times \frac{1}{4} = 1.25\ (m)$$

$$\vec{y} = -\frac{1}{2} g\ (t)^2 = -\frac{1}{2}(9.8)\left(\frac{1}{4}\right)^2 = -0.306\ (m)$$

$$\vec{r} = \sqrt{\vec{x}^2 + \vec{y}^2} = \sqrt{(1.25)^2 + (-0.306)^2} = 1.29\ (m)$$

$$\vec{v}_x = \vec{v}_0 = 5\left(\frac{m}{sec}\right)$$

$$\vec{v}_y = -gt = -9.8 \times \frac{1}{4} = -2.45\ \left(\frac{m}{sec}\right)$$

$$\vec{v} = \sqrt{\vec{v}_x^{\ 2} + \vec{v}_y^{\ 2}} = \sqrt{(5)^2 + (-2.45)^2} = \ 5.57\ \ \left(\frac{m}{sec}\right)$$

$$\theta = \tan^{-1}\frac{-2.45}{5} = -26.1^o$$

Example (7)

A man throws a ball with a velocity of (32 m/sec), at an angle of (40^o) with the ground. Find:-

a) The velocity and position of the ball after (3 sec)?

b) The range and time required for the ball to returns to ground?

Ans.:

$$\vec{v}_x = \vec{v}_0\ cos\theta = 24.5\frac{m}{sec}$$

$$\vec{v}_y = \vec{v}_0\ sin\theta - 9.8\ t$$

$$\vec{v}_y = 20.6 - 9.8\ (3) = -8.8\ \left(\frac{m}{sec}\right)$$

$$\vec{v} = \sqrt{(24.5)^2 + (-8.8)^2} = 26.0\ \left(\frac{m}{sec}\right)$$

$$\vec{x} = (\overrightarrow{v_0}\ con\theta)t = 24.5 \times 3 = 73.5\ \ (m)$$

$$\vec{y} = (\overrightarrow{v_0}\, sin\theta)t - \frac{1}{2}gt^2 = (20.6)(3) - \frac{1}{2}(9.8)(3)^2$$

$$y = 17.7\ (m)$$

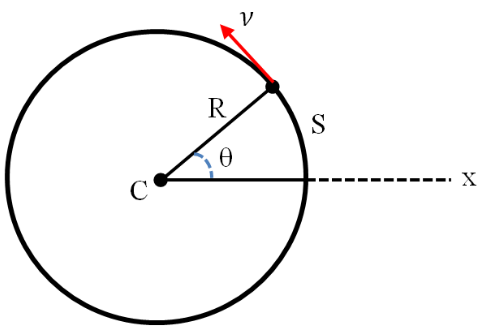
$$\vec{r} = \sqrt{\vec{x}^2 + \vec{y}^2} = 75.6\ (m)$$

$$R = \frac{(\overrightarrow{v_0})^2 sin2\theta}{g} = \frac{(32)^2 sin2 \times 40}{9.8} = 77.65\ (m)$$

$$t = \frac{2(\vec{v}_0)sin\theta}{g} = \frac{2 \times 32 \times sin40}{9.8} = 4.197\ (sec)$$

Circular Motion

Let us consider the motion of a particle along a curved path, such motion called uniform circular motion when the body moving with constant speed, since the velocity is tangent to the circle and perpendicular to the radius (R) with distances measured along the circumference of the circle, then



$$S = R\,\theta \qquad \Longrightarrow \qquad R = constant$$

$$\because\ \vec{v} = \frac{ds}{dt} = R\,\frac{d\vec{\theta}}{dt} = R\,\vec{\omega} \quad ; \quad where \ \Longrightarrow \ \vec{\omega} = \frac{d\vec{\theta}}{dt}$$

The quantity (ω) called angular velocity defined as the time rate of change of the angle and expressed in radians per second (rad/sec); then the angular velocity a vector quantity whose direction is perpendicular to the plane of motion, with a sense given by the thumb of the right hand when the fingers point in the sense of the particle motion.

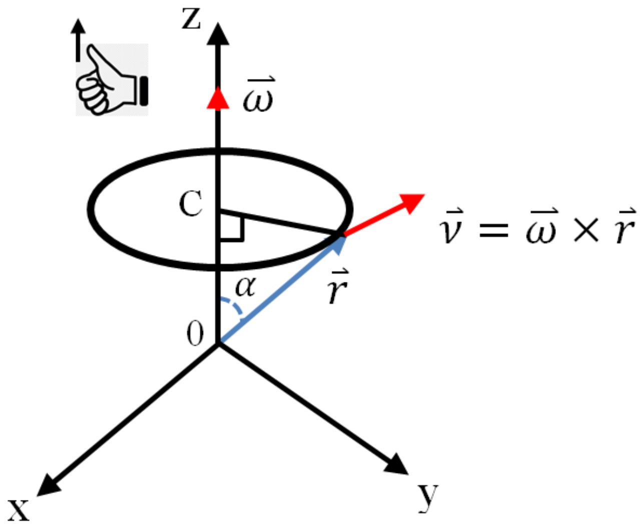
From the Figure:

$$R = r\,sin\alpha \quad ; \quad therefore: -$$

$$v = \omega\, r\, sin\alpha$$

The vector relation holds

$$\vec{v} = \vec{\omega} \times \vec{r}$$



1- Motion with ($\omega = constant$) is uniform circular motion; in this case the particle passes through each point of the

2- Circle at regular intervals of time. The period (τ) is the time required for a complete turn or revolution, and the frequency (f) is the numl of revelution per unit time, then ($\tau = \frac{t}{n}$) and the frequency is($f = \frac{n}{t}$) , both quantities are then related by ($f = \frac{1}{\tau}$). If the angular velocity is constant.

$$\because \vec{\omega} = \frac{d\vec{\theta}}{dt} \quad \Longrightarrow \quad d\vec{\theta} = \vec{\omega}\, dt \quad ; \; integrating$$

$$\vec{\theta} = \vec{\theta}_0 + \vec{\omega}\,(t - t_0) \;\; ; \; when \;\; \vec{\theta}_0 = 0 \;\; and \;\; t_0 = 0$$

$$[\theta = wt]$$

For complete revolution ($\omega = \frac{2\pi}{\tau} = 2\pi f$)

When the angular velocity of a particle changes with time, the angular acceleration is

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

In the particular case of circular motion, the tangential acceleration is related to the angular acceleration by:

$$a_T = \frac{d\vec{v}}{dt} = R\frac{d\vec{\omega}}{dt} = R\vec{\alpha}$$

And normal (or centripetal) acceleration is

$$a_N = \frac{\vec{v}^2}{R} = \vec{\omega}^2 R$$

Note that in uniform circular motion (no angular acceleration) there is no tangential acceleration, but there is still normal centripetal acceleration due to the change in the direction of the velocity.

In this case of uniform circular motion

3- we may compute the acceleration directly by using

$\vec{v} = \vec{\omega} \times \vec{r}$; $then$

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$\vec{a} = \vec{\omega} \times \vec{v}$$

$$\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

And its magnitude is

$$[\vec{a} = \vec{\omega}^2 R]$$

Motion with constant angular acceleration

When:- $\alpha = constant$

$$\frac{d\vec{\omega}}{dt} = \vec{\alpha} \quad . \ by\ integreting$$

$$\int d\vec{\omega} = \int \vec{\alpha}\, dt$$

$$\vec{\omega} = \vec{\alpha}\, t + c_1 \qquad \Longrightarrow \ constant$$

When $t = 0$ $\Longrightarrow$ $\vec{\omega}_0 = c_1$; $then$

$$[\vec{\omega} = \vec{\omega}_0 + \vec{\alpha} t] \qquad \dots\dots\dots\dots\dots (1)$$

Since: $\vec{\omega} = \frac{d\vec{\theta}}{dt}$ or by integrating

$$\int d\vec{\theta} = \int \vec{\omega}_0 dt + \int \vec{\alpha}\, t\, dt$$

$$\vec{\theta} = \vec{\omega}_0\, t + \frac{1}{2}\,\vec{\alpha}\, t^2 + c_2$$

When $t = 0 \qquad \Longrightarrow \qquad c_2 = \theta \quad ; \quad then\, we$

Find $\left[\vec{\theta} = \vec{\theta}_0 + \vec{\omega}_0 t + \frac{1}{2}\vec{\alpha} t^2\right] \qquad \ldots\ldots\ldots\ldots\ldots (2)$

If we write the angular acceleration

$$\vec{\alpha} = \vec{\omega}\,\frac{d\vec{\omega}}{d\vec{\theta}}$$

4- then $\int \vec{\alpha}\, d\vec{\theta} = \int \vec{\omega}\, d\vec{\omega} + c_3$

$$\vec{\alpha}\,\vec{\theta} = \frac{1}{2}\vec{\omega}^2 + c_3$$

When $t = 0 \qquad \Longrightarrow \qquad c_3 = \theta_0 \quad and\ initial\ angular\ velocity\ is\ (\omega_0) then$

$$c_3 = \vec{\alpha}\,\vec{\theta}_0 - \frac{1}{2}(\vec{\omega}_0)^2 \quad and \;\; \theta_0 = 0$$

$$\left[\vec{\omega}^2 = \vec{\omega}_0{}^2 + 2\,\vec{\alpha}\,\vec{\theta}\right] \ldots\ldots\ldots\ldots\ \ldots (3)$$

Example (8)

The angular velocity of a body is $(4\ rad.s^{-1})$ at time (t=0) and its angular acceleration is constant and equal to $\left(2\,\frac{rad}{sec^2}\right)$.

a) What angle does this line make with the horizontal at time (t=3 sec)?
b) What is the angular velocity at this time?

Ans.:

a) $\vec{\theta} = \vec{\theta}_0 + \vec{\omega}_0 t + \frac{1}{2}\ \vec{\alpha}\ t^2$

$$\vec{\theta} = 0 + (4)(3) + \frac{1}{2}(2)(3)^2 = 21\ (rad) = 3.34\ (rev.)$$

$\theta = 57^o$

b) $\vec{\omega} = \vec{\omega}_0 + \vec{\alpha}\ t = 10\ (rad/sec)$

Example (9)

A car traveling at a constant speed of (20 m/sec), rounds a curve of radius (100 m) . What its acceleration?

Ans.:

$$\vec{\omega} = \frac{\vec{v}}{R} = \frac{20}{100} = 0.2\left(\frac{rad}{sec}\right)$$

$$\vec{\alpha} = 0$$

$$\vec{a}_N = \frac{\vec{v}^2}{R} = \frac{(20)^2}{100} = \frac{400}{100} = 4\left(\frac{m}{sec^2}\right)$$

Example (10)

Fly wheel needed 3 sec to rotate (234 rad), and in the end of time the angular velocity is (108 rad/sec). Find its angular acceleration.

Ans.:

$$108 = \vec{\omega}_0 + \vec{\alpha}(3) \qquad \ldots\ldots(1)$$

$$234 = 3\vec{\omega}_0 + \frac{1}{2}\vec{\alpha}\,(3)^2 \qquad \ldots\ldots(2)$$

Solving above Eq. **Find:**

$$144 = 3\vec{\omega}_0 \qquad \Longrightarrow \qquad \vec{\omega}_0 = 48$$

From eq. (1)

$$108 = 48 + 3\,\vec{\alpha}$$

$$\vec{\alpha} = 20\left(\frac{rad}{sec^2}\right)$$

Example (11)

A car travel in a circle of (R=5 m) making one complete circle in (4sec.), what is the acceleration.

Ans.:

$$\vec{v} = \frac{2\pi\, R}{t} = \frac{2\pi\,(5)}{4} = 7.85\left(\frac{m}{sec}\right)$$

$$a_{\perp} = \frac{\vec{v}^2}{R} = \frac{(7.85)^2}{5} = 12.3\left(\frac{m}{sec}\right)$$

Example (12) (H.W.)

A pulley 2m in diameter is keyed to a shaft which makes (240r.p.m.). Find the angular velocities of a particle on the periphery of the pulley (answer: 13rad/sec) (H.W.)

Example (13)

A car exhibits a constant acceleration of ($3\ \frac{m}{sec^2}$) parallel to the roadway. The car passé over a rise in the roadway such that the top of the rise is shaped like a circle of radius (500 m), at the moment the car is at the top of the rise. Its velocity vector is horizontal has magnitude of (6.0 m/sec). what the magnitude and direction of the total acceleration vector for the car at this instant?

Ans.:

$$a = \frac{-v^2}{r} = \frac{-(6.0)^2}{500} = -0.072\frac{m}{sec^2}$$

$$a = \sqrt{{a_N}^2 + {a_T}^2} = \sqrt{(-0.072)^2 + (3)^2} = 0.300\left(\frac{m}{sec^2}\right)$$

$$\varphi = \tan^{-1}\frac{a_N}{a_T} = \frac{-0.072}{3} = -13.5^o$$

Example (14) (H.W.)

What is the centripetal acceleration of the earth as it moves in its orbit around the sun? ($R = 1.496 \times 10^{11} m$).

Chapter 4

Forces

Forces

The force is an important factor in the field of mechanics, which may be broadly defined as an agent which produces or tends to produce destroys or tends to destroy motion.

The forces may be classes as contact forces such as (when coiled spring is pulled) or (when a stationary cart is pulled) or (when a football is kicked), that is they involve physical contact between two objects.

Another classes of forces known as field forces do not involve physical between two objects an example of this class is a gravitational force, which keeps objects bounded to the earth.

Effect of a force

A force may produce the following effects in a body on which it acts:

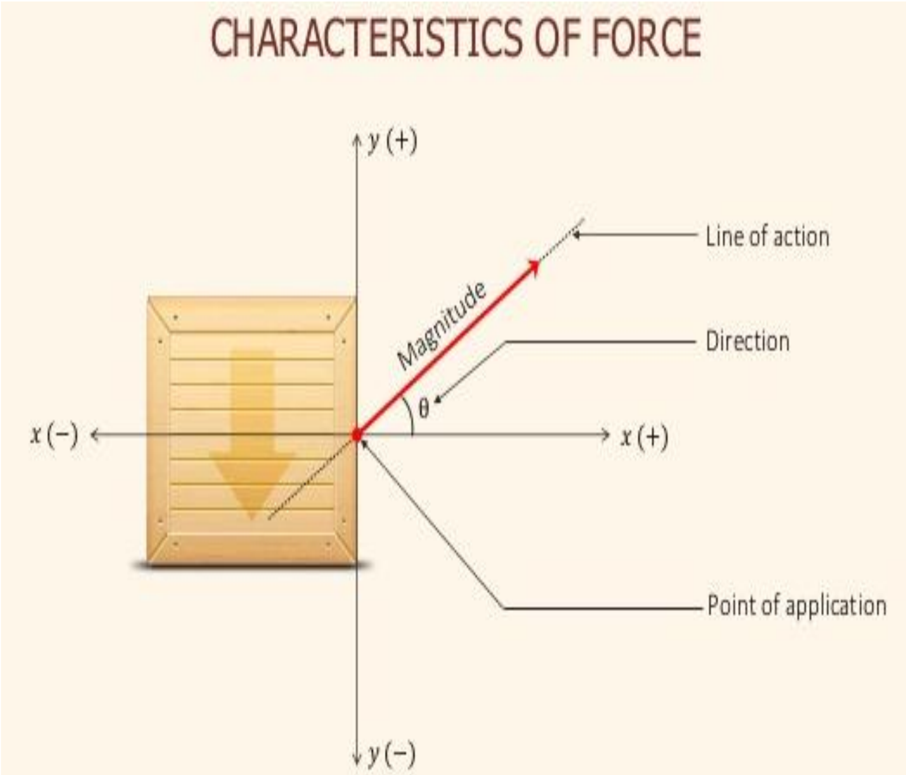
1- It may change the motion of the body; i.e. if a body is at rest, the force may set the body in motion, and if the body is already in motion the force may accelerate it.

2- It may retard the motion of a body.

3- It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.

4- It may give rise to the internal stresses in the body, on which it acts.

Characteristics of force



The following characteristics of force:-

1- The magnitude of the force **(i.e.; 10 N, 20 N …)**.
2- The direction of the force **(i.e. 30º North; or East, etc).**
3- Nature of the force **(i.e. whether the force is pull, push, tension…).**
4- The point at which the force acts on the body.

Resultant force:-(Net force)

If a number of forces $(\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots \dots \dots . ect)$ are acting simultaneously on a particle, it is possible to find out a single force, which could replace them. This single force is called resultant force, and the given forces $(\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots \dots \dots . ect)$ are called component forces.

Methods for finding out the resultant force

There are many methods for finding out the resultant force of a number given forces the following are:-

1- Analytical method

May be found out by:

a) Parallelogram law of force

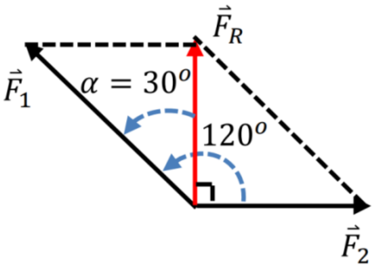
b) Method of resolution

2- Graphical method.

Example:- ***(<u>Parallelogram method</u>)***

Tow forces act an angle of (120^o) the larger force is of $(40\ N)$ and the resultant is perpendicular to the smaller one. Find the smaller force.

Ans.:



$$\boldsymbol{R = \sqrt{\vec{F}_1^2 + \vec{F}_2^2 + 2\,\vec{F}_1\vec{F}_2\ Cos\alpha}}$$

$$\alpha = 120^o - 90^o = 30^o$$

$$\tan\alpha = \frac{\vec{F}_2\ sin\theta}{\vec{F}_1 + \vec{F}_2\ cos\theta}$$

$$\tan\alpha = \frac{\vec{F}_2\ sin120}{40 + \vec{F}_2\ cos120}$$

$$\frac{1}{\sqrt{3}} = \frac{\vec{F}_2 \times \frac{\sqrt{3}}{2}}{40 - \frac{\vec{F}_2}{2}} \quad \Longrightarrow \quad 2\vec{F}_2 = 40 \quad \Longrightarrow \quad \vec{F}_2 = 20\ \ (N)$$

b) Resolution of a force:

A force is generally, resolved along tow perpendicular components. The resultant force, of a given system of forces, may be found out by:

1- Resolve all the forces vertically and find the algebraic sum of all the vertical components$(i.e\ \sum \vec{F}_y)$.

2- Resolve all the forces horizontally and find the algebraic sum $(i.e\ \sum \vec{F}_x)$.

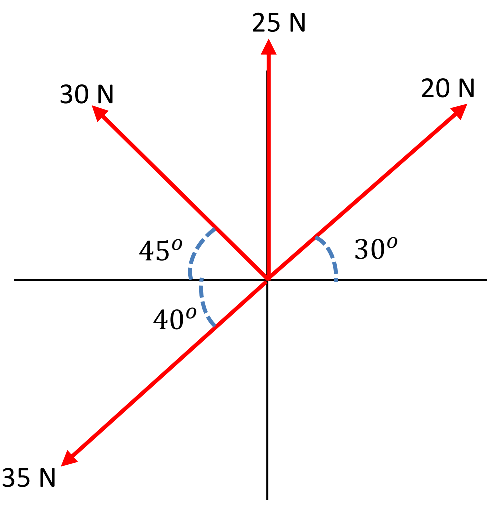
3- The resultant (R) is given as:-

$$R = \sqrt{\left(\sum \vec{F}_x\right)^2 + \left(\sum \vec{F}_y\right)^2} \qquad (magnitude)$$

$$\tan\theta = \frac{\sum \vec{F}_y}{\sum \vec{F}_x} \qquad \Longrightarrow \qquad (direction)$$

Example:-

Find the magnitude and direction of the resultant force for the system of concurrent forces shown below:



Solution:

$$\sum \vec{F}_x = 20\cos 30 - 30\cos 45 - 35\cos 40$$

$$\sum \vec{F}_x = -30.70N$$

$$\sum \vec{F}_y = 20\sin 30 + 25 + 30\sin 45 - 35\sin 40$$

$$\sum \vec{F}_y = 33.72N$$

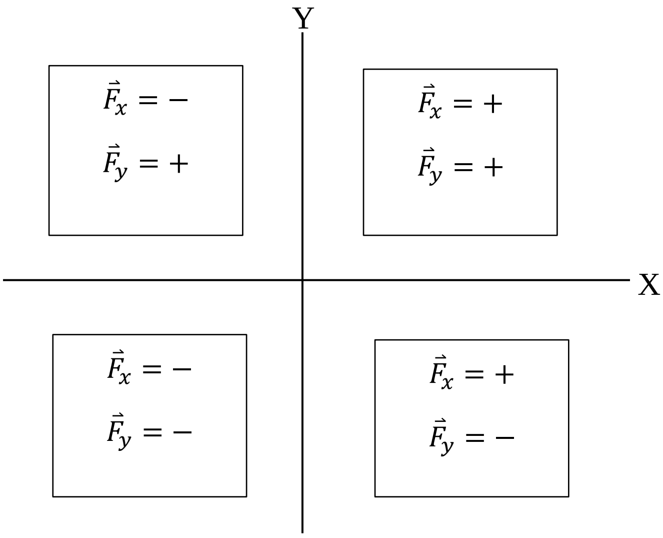
$$R = \sqrt{(\sum \vec{F}_x)^2 + (\sum \vec{F}_y)^2}$$

$$R = \sqrt{(-30.70)^2 + (33.72)^2}$$

$R = 45.60 \quad (N)$

$$\tan\theta = \frac{\sum \vec{F}_y}{\sum \vec{F}_x}$$

$$\theta = \tan^{-1}\frac{\sum \vec{F}_y}{\sum \vec{F}_x} = \tan^{-1}\frac{33.72}{-30.70} = 47.68^o$$



The laws of motion

1-Newton's first law

The first law is a mere statement of Galilean principle of inertia according to this law.

$\vec{a} = 0$ Where $\vec{F} = 0$

(i.e.) The acceleration of a body or a system of bodies is zero, if the resultant external force acting on it is zero. In other words, when a body in complete equilibrium, its linear as well as angular acceleration is zero.

The first law of motion statement reads:

"Everybody continues in its state of rest or of uniform motion in a straight line; unless it is compelled to change that state by forces impressed on it".

The importance of the law will be evident from the following:

(i) The law asserts that in the absence of an external force, the body either remains at rest or moves with a uniform speed in a straight.

(ii) The law defines by implication, an inertial frame of reference.

(iii) The law gives qualitative definition of force, the law, however, does not give an operational definition of force i.e. it does not give the procedure for measuring the force.

(vi) The law presents inertia, the inability of a material body to change by itself its state of rest or of uniform motion in a straight line; the qualitative measure of inertia of a body is its mass.

2-Newton's second law

The second law which gives a quantitative relation between force and inertia. According this law:

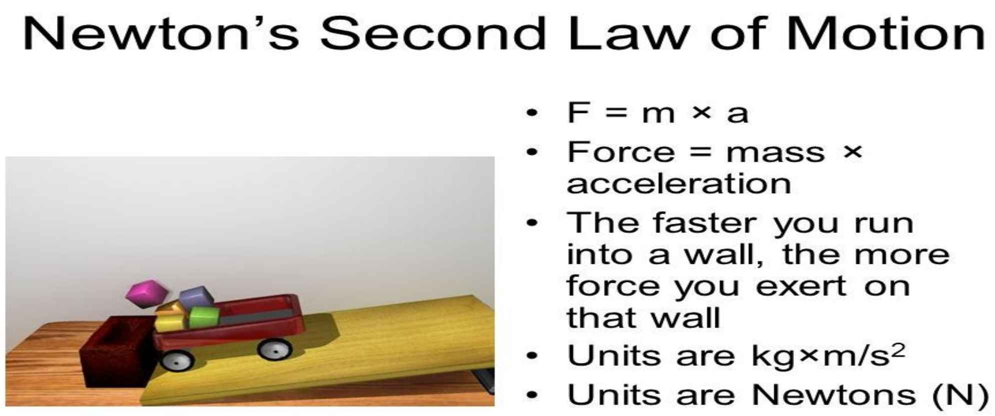
$$\vec{F} = \frac{dp}{dt} \quad ; \quad Where\ \ p\ is\ \ momentum\ of\ \ the\ body$$

$$\vec{F} = \frac{d(m\vec{v})}{dt} \quad ; \quad m \ \ is\ constant$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

$$[\therefore\ \vec{F} = m\vec{a}]$$

The acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to its mass. The second law gives a procedure for measurement of force.



3-Newton's third law

In nature the forces always occur in pairs. The forces always arise as a result of interaction between to objects. This important property of forces was first stated by newton in his third law of motion, whenever two bodies interact, the force $\vec{F}_2$ on the second body is equal and opposite to the force $(\vec{F}_1)$ on the first body **i.e** $\left[\vec{F}_1 = -\vec{F}_2\right]$; and acts along same line, the two forces are often called "action" and "reaction", and acts at same time on two different objects.

The four basic forces

1- Gravitational force

Which holds the earth together, keeps the moon in its orbit around the earth. However, the gravitational interaction becomes important only on large scale.

2- Electromagnetic force

The force between charged bodies, the electromagnetic forces are much stronger than the gravitational force because magnetic influences have to do with charge in relative motion. A charging electric field cannot exist without magnetism. The electric and magnetic fields are two different aspects of one; they are really to be attributed to one field called the electromagnetic field.

3- Nuclear reaction

The energy involved in a nuclear reaction is a million or more times stronger than the energy involved in the molecular reactions, therefore the force holding the nucleus together are very stronger.

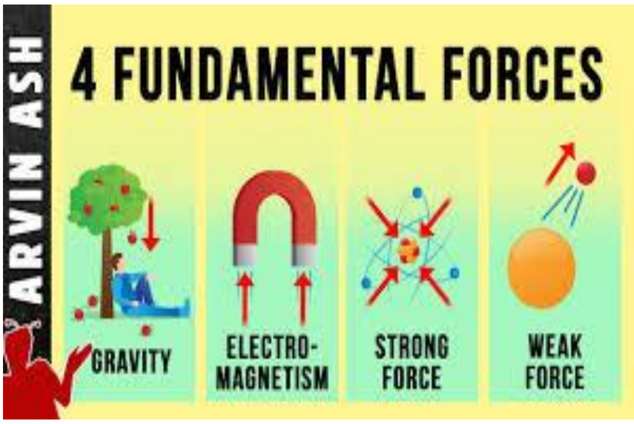
4- Elementary particles (Weak interaction)

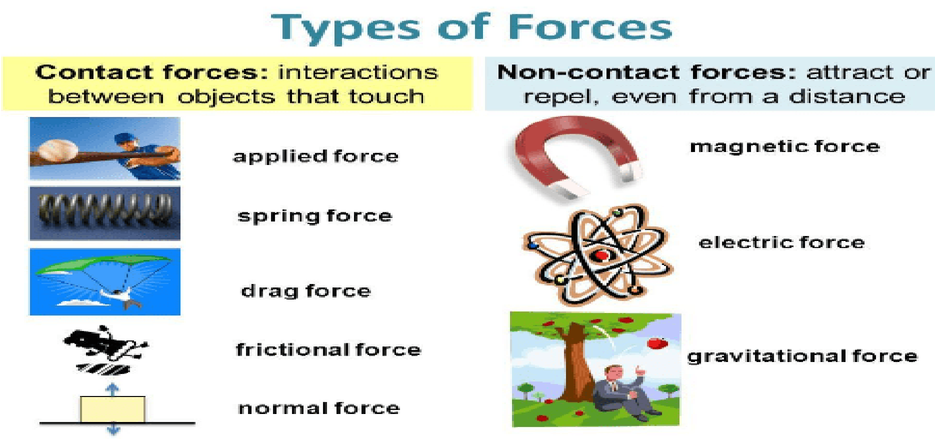
There is another type of nuclear interaction which is weaker than electromagnetic interaction but stronger than the gravitational one, this interaction (leptons, electron, neutrino…), the interaction is $\beta - decay$ of radioactive nuclei is very short range, interaction. This weak interaction cannot from a stable state of matter.

To sum arise, there are four basic interactions which in order of decreasing strengths are:

1- Strong Interaction.
2- Electromagnetic.
3- Weak.
4- Gravitational.

Their relative strengths are: $1:10^{-2}\ :10^{-5}\ :10^{-40}$





Centripetal Force

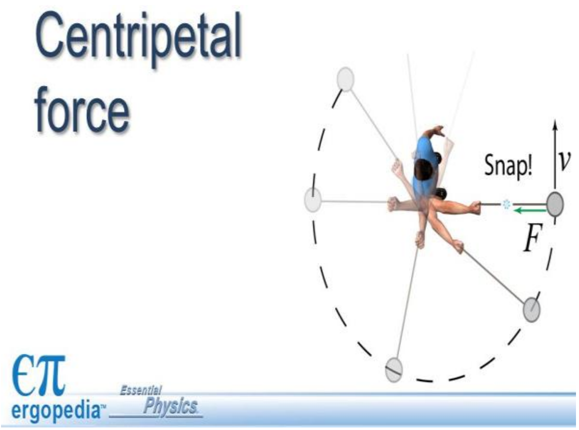
Newton's second law governs circular motion as well as other motion of a particle, the acceleration toward the center of the circle, for the particle in uniform circular motion, must be caused by a force also directed toward the center. Since the magnitude of the radial acceleration $\vec{a}_{\perp}$ given as:

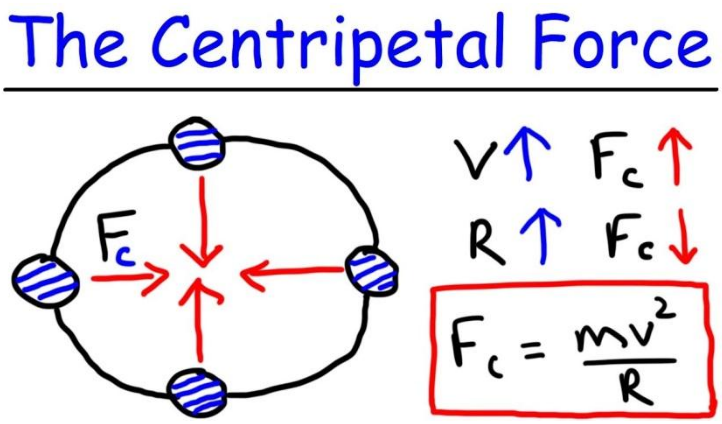
$$\vec{a}_{\perp} = \frac{\vec{v}^2}{R}$$

And its direction is toward the center, the magnitude of the radial force on a particle of mass m is given:

$$\vec{F} = m\,\vec{a}_{\perp} = \frac{m\,\vec{v}^2}{R}$$

The force ($\vec{F}$) is sometimes called centripetal force denoted by ($\vec{F}_c$).





Example:-

A small body of mass (0.2 kg) revolves uniformly in a circle on a horizontal surface attached by a cord (0.2 m) long to a pin set in the surface. If the body makes two complete revolutions per second, find the force exerted on it by the cord.

Ans.: The circumference of the circle is (s)

$$s = 2\pi\, R = 2\pi\,(0.2) = 0.4\pi \quad \Longrightarrow \quad \tau = 0.5\ (sec)$$

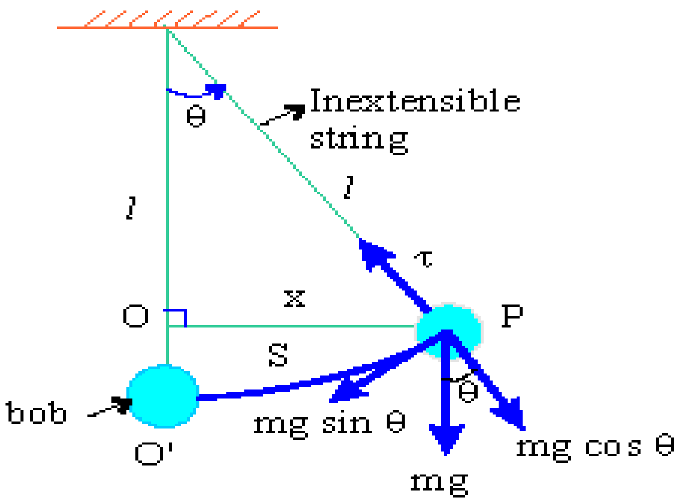
$$\because\ \vec{a} = \frac{\vec{v}^2}{R} = \frac{(0.8\pi)^2}{0.2} = 31.6 \quad ; \quad (where: - \quad \vec{v} = \frac{0.4\pi^2}{0.5} \quad \Longrightarrow \quad \vec{v} = 0.8\pi\ )$$

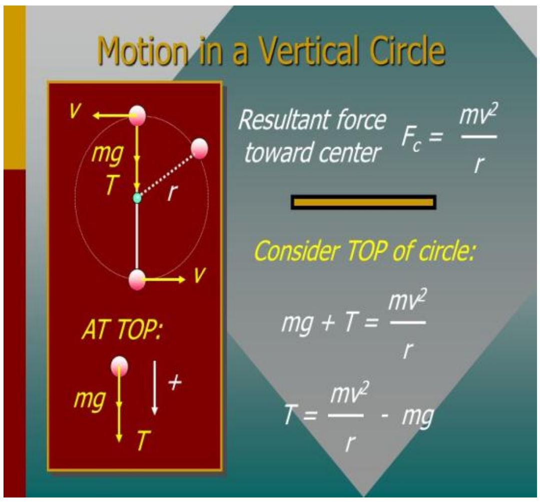
$$\because\ \vec{F_c} = m\,\vec{a}$$

$$\vec{F_c} = (0.2)(31.6)$$

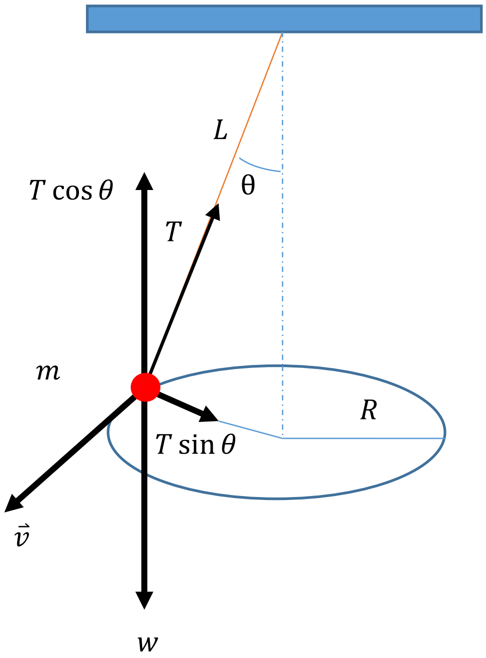
$$\vec{F_c} = 6.32 \quad (N)$$

Motion in a horizontal circle





If a small body of mass (m) revolves in a horizontal circle with velocity $(\vec{v})$ of constant magnitude at the end cord of length (l) as the body swings around its path, the cord sweeps over the surface of a cone. The cord makes an angle of (θ) with vertical.



$(\boldsymbol{Conical\ pandulum})$

Solution:-

$\because$ Radius of circle ; $\sin\theta = \frac{R}{L}$

$$R = L\sin\theta$$

and magnitude of velocity ; $\vec{v} = \frac{2\pi R}{\tau} = \frac{\text{محيط الدائرة}}{\text{زمن ذبذبة كاملة}}$

$$\vec{v} = \frac{2\pi\, L\sin\theta}{\tau}$$

And (T) resolved by:

$$\vec{F}_r = T\ sin\theta \quad , \quad \vec{F}_r = m\,\vec{a}$$

$$T_x = T\sin\theta = m\,\frac{\vec{v}^2}{R} \qquad \dots\dots\dots\dots\dots (1)$$

$$T_y = T\cos\theta = w \qquad \dots\dots\dots\dots\dots (2)$$

The eq. (1) divided by the eq.(2) and $(\,w = mg)$, the result is:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{m\vec{v}^2/R}{mg}$$

$$\tan\theta = \frac{\vec{v}^2}{Rg} \;\Rightarrow\; \frac{\sin\theta}{\cos\theta} = \frac{\vec{v}^2}{gR} \;\Rightarrow\; \vec{v}^2 = gR\,\frac{\sin\theta}{\cos\theta} \;\dots\dots\dots\dots(3)$$

$$\vec{v} = \sqrt{g\ R\ tan\theta}$$

$(R = L\sin\theta \quad ; \quad \vec{v} = \frac{2\pi\, L\sin\theta}{\tau})$; $\quad \vec{v} = \frac{2\pi\ R}{\tau}$ substitute in Eq. (3)

$$\cos\theta = \frac{g\,\tau^2}{4\pi^2 L}$$

$$Or \quad \tau = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

The relation refer that (θ) depends on the time of revelution (τ) and the length (L) of the cord for a given length (L), $\cos\theta$ decreases as the time is made shorter, and the angle (θ) increases, the (θ) never become (90^o), this requires that $(\tau = 0 \ or \ \vec{v} = \infty)$, the largest possible value of

$$\left[\tau = 2\pi\left(\frac{L}{g}\right)^{\frac{1}{2}} \ or \ \cos\theta = 1\right]$$

Motion in a vertical circle

Figure: represents a small body (m) attached to a cord of length (R) and whirling in a vertical circle about a fixed point O.

$$\vec{F}_{\perp} = T - \vec{F}\ cos\theta$$

$$\vec{F}_{//} = \vec{F}\ sin\theta$$

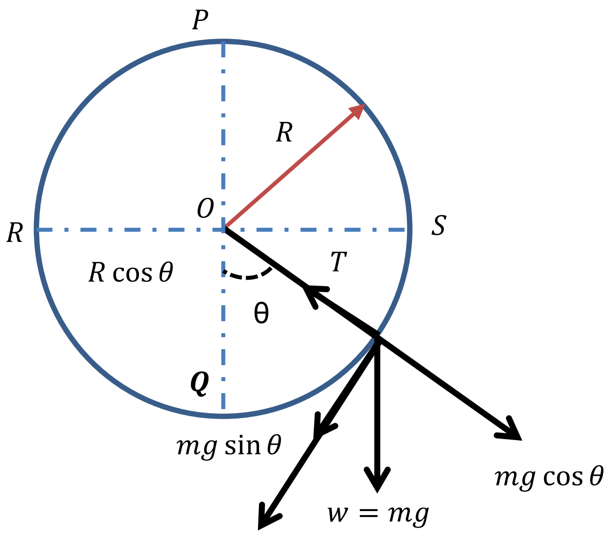
$$\vec{a}_{\perp} = \frac{\vec{F}_{\perp}}{m} = \frac{T - \vec{F}\ cos\theta}{m}$$

$$\vec{a}_c = \vec{a}_{\perp} = \frac{\vec{v}^2}{R}$$

$$\frac{T - \vec{F}\ cos\theta}{m} = \frac{\vec{v}^2}{R}$$

$$\frac{T}{m} - \frac{\vec{F}\ cos\theta}{m} = \frac{\vec{v}^2}{R}$$

$$\frac{T}{m} - \frac{mg\ cos\theta}{m} = \frac{\vec{v}^2}{R}$$



$$\frac{T}{m} = \frac{\vec{v}^2}{R} + g\,cos\theta \qquad \Rightarrow \qquad T = \frac{m\,\vec{v}^2}{R} + m\,g\,cos\theta$$

The motion, while circular is not uniform, since the Speed increases on the way down and decreases on the way up. Let the weight be resolved into a normal component of magnitude $(mg\cos\theta)$ and tangential component $(mg\sin\theta)$ and from the figure, clearly that;

$$T = \frac{m\vec{v}^2}{R} + mg\cos\theta$$

Or $\quad T = m\left(\frac{\vec{v}^2}{R} + g\cos\theta\right)$

Because of the tangent component of weight that is unequilibrim, the velocity varied in the circle path.

If $(\vec{v}_p)$ the velocity in top point (highest) and;

$$\vec{v}^2 - \vec{v}_p{}^2 = 2g(R + R\cos\theta)$$

$$\vec{v}^2 = \vec{v}_p{}^2 + 2gR(1 + \cos\theta)$$

At lowest point (Q), $\theta = 0, \quad \cos\theta = 1$, then;

$$T_Q = m\left(\frac{\vec{v}_Q{}^2}{R} + g\right)$$

$$\vec{v}_Q{}^2 = \vec{v}_p{}^2 + 4\,g\,R$$

At mid. Point (s) ; $\theta = 90^o,\ \cos 90 = 0$

$$\therefore T_s = m\frac{\vec{v}_s{}^2}{R}\ ; \qquad \Rightarrow \qquad \vec{v}_s{}^2 = \vec{v}_p{}^2 + 2\,g\,R$$

At highest point (p); $\quad \theta = 180^o \quad , \quad \cos\theta = -1$

$$T_p = m\,(\frac{\vec{v}_p{}^2}{R} - g)$$

And $\vec{v}_p = \vec{v}_Q = 0$

The minimum velocity at the point (p) should be such that the centrifugal force is just able to counterbalance the weight (mg). The body will leave the circle and will fall down along parabolic path. This minimum velocity is called critical velocity ($\vec{v}_c$). Then the tension in the string is zero, or;

$$T_p = 0 \qquad \Rightarrow \qquad \vec{v}_p = \vec{v}_c$$

Thus;
$$0 = m\left(\frac{\vec{v}_c^{\,2}}{R} - g\right)$$

$$\vec{v}_c = \sqrt{Rg}$$

The corresponding values of velocity and tension at the point (Q) is: -

$$\vec{v}_Q^{\,2} = \vec{v}_c^{\,2} + 4\,g\,R = 5gR$$

And;
$$\vec{v}_Q = \sqrt{5gR}$$

$$T_Q = m\left(\frac{5gR}{R} + g\right) = 6\,m\,g$$

Similarly; at point S is:-

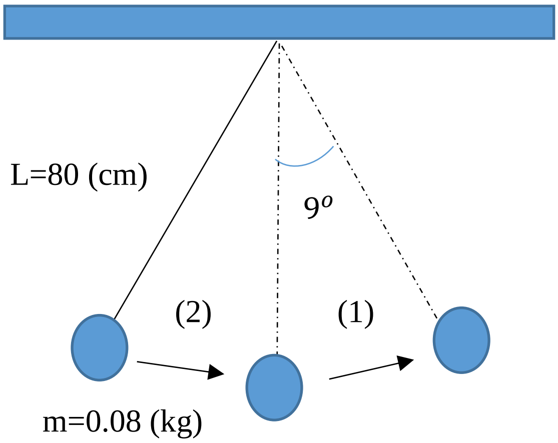
$$\vec{v}_s^{\,2} = gR + 2gR = 3gR$$

$$\vec{v}_s = \sqrt{3gR}$$

And;
$$T_s = m\frac{3gR}{R} = 3\,m\,g$$

Example (1)

In the following figure, Find the tension force of the thread and the return force of the pendulum.



Solution:

$$\vec{F}_T = mgcos\theta = 0.08 \times 9.8 \times cos9 = 0.77\ (N)$$

$$\vec{F}_r = -mgsin\theta = -0.08 \times 9.8 \times sin9 = -\ 0.122\ (N)$$

Example (2)

A string of length 0.5m carries a bob of mass 0.1kg with a period of 1.41sec. calculate the angle of the inclination of the string with vertical and tension in the string?

Solution:

$$L = 0.5\ (m)\ ;\ \ m = 0.1\ (kg)\ ;\ \ \tau = 1.41\ \ (sec)\ ;\ \ g = 9.8\ \left(\frac{m}{sec^2}\right)$$

$$T_y = ?\ \ ;\ \ \theta = ?$$

$$\tau = 2\pi\sqrt{\frac{Lcos\theta}{g}}$$

$$\tau^2 = 4\pi^2 \times \frac{Lcos\theta}{g}$$

$$cos\theta = \frac{\tau^2 g}{4\pi^2 L} = \frac{(1.41)^2 \times 9.8}{4 \times (3.14)^2 \times 0.5} = 0.9880$$

$$\theta = \cos^{-1}(0.9880) = 8.885^o$$

$$T_y cos\theta = mg$$

$$T_y = \frac{mg}{cos\theta} = \frac{0.1 \times 9.8}{0.9880} = 0.992\ (N)$$

Example (3)

A 2kg rock swings in a vertical circle of radius 8m.

(a) The speed of the rock as it passes its highest point is 10m/sec. what is tension T in rope?

(b) The speed of the rock as it passes its lowest point is 10m/sec. what is tension T in rope?

(c) What is the critical speed v_c at the top, if the 2kg mass is to continue in a circle of radius 8m?

Solution:

(a) At top:

$$mg + T = \frac{m\vec{v}^2}{R}$$

$$T = \frac{m\vec{v}^2}{R} - mg$$

$$T = \frac{2 \times (10)^2}{8} - 2 \times (9.8) = 25 - 19.6$$

$$T = 5.4\ (N)$$

(b) At bottom:

$$T - mg = \frac{m\vec{v}^2}{R}$$

$$T = \frac{mv^2}{R} + mg$$

$$T = \frac{2 \times (10)^2}{8} + 2 \times (9.8) = 25 + 19.6$$

$$T = 44.6\ (N)$$

(c) critical speed at the top:

$$mg + T = \frac{m\overrightarrow{v^2}}{R}$$

$$\vec{v}_c\ occurs\ when\ T = 0$$

$$mg = \frac{m\vec{v}^2}{R} \rightarrow \quad \vec{v}_c = \sqrt{gR}$$

$$\vec{v}_c = \sqrt{9.8 \times 8} = 8.85\ \ (m/sec)$$

Example (4)

A car is traveling at 20km/h on a level road where the coefficient of static friction between tires and road is 0.8. Find the minimum turning radius of the car?

Solution:

$$F_c = F_f$$

$$\mu = 0.8 \quad ; \quad \vec{v} = 5.55 \left(\frac{m}{sec}\right)$$

$$\frac{m\vec{v}^2}{r} = \mu_s mg$$

$$r = \frac{\vec{v}^2}{\mu_s g} = \frac{(5.55)^2}{0.8(9.8)} = 3.93\ (m)$$

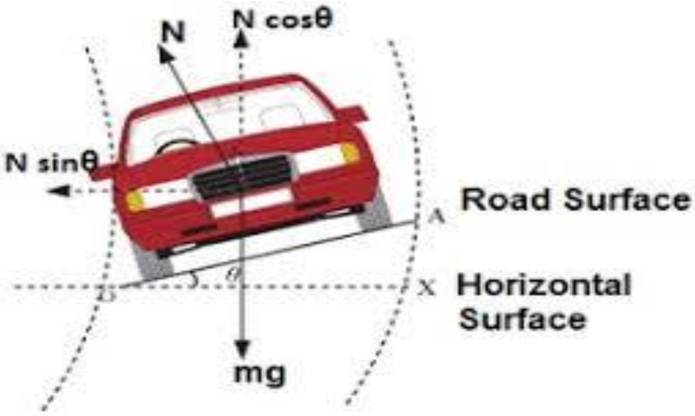
Example (5)

Consider a circular road of radius 20 meter banked at an angle of 15 degree. With what speed a car has to move on the turn so that it will have safe turn?

Solution:

$$v = \sqrt{(rg\tan\theta)} = \sqrt{20 \times 9.8 \times \tan 15} = \sqrt{20 \times 9.8 \times 0.26}$$
$$= 7.1\ \langle\frac{m}{sec}\rangle$$

Banking of tracks



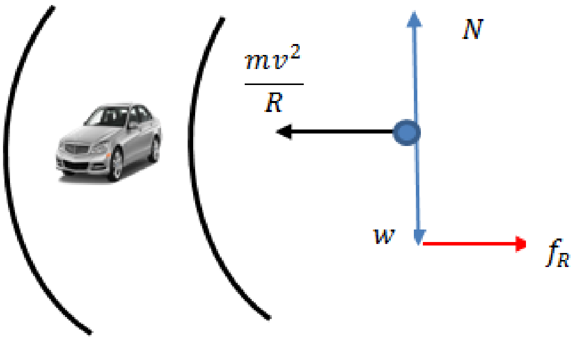
When a vehicle such as motor car a bend on a level track, the forces acting upon it are:-

1- Centrifugal force = $(\frac{m\vec{v}^2}{R})$ at the center of gravity away from the center of the curved track.
2- Friction force on the wheels action along the axis of the wheel toward the center of the curved path.
3- The weight (mg) acting vertically downwards.
4- Reaction of the grounds acting on the wheel vertically upwards.

The maximum speed that the vehicle attains without skidding off is given by:

$$\frac{m\vec{v}^2}{R} = \vec{F}_R = \mu mg$$

$$\vec{v} = \sqrt{\mu g R}\ ,$$



When a vehicle moves round a bend on a banked track. The component $(N\cos\theta)$ should be able to counterbalance the centrifugal force or;

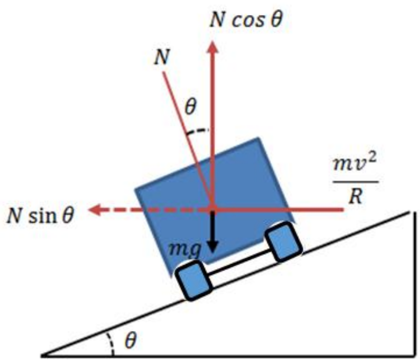
$$\frac{m\vec{v}^2}{R} = N\sin\theta$$

And; $mg = N\cos\theta$

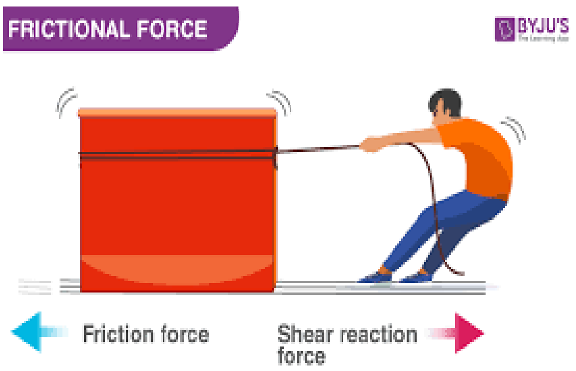
$$\frac{\vec{v}^2}{g\,R} = \tan\theta$$

The maximum safe speed of the vehicle is:-

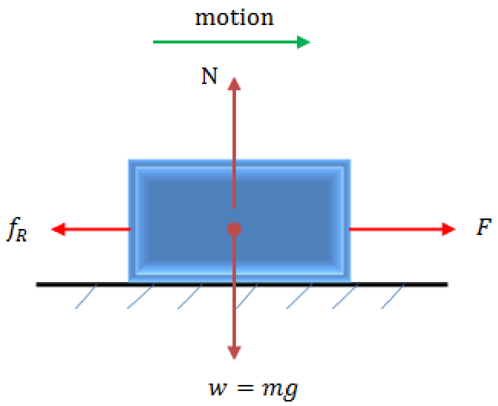
$$\left[\vec{v}_{max} = \sqrt{g\,R\tan\theta}\right]$$



Frictional forces



When two bodies are in relative motion to each other a force opposing the motion we call this force friction force or simply friction denoted $(\vec{F}_R)$.



This force is due to the interaction between the molecules of the two bodies; when the two bodies of the same material cohesion interaction and when a different materials adhesion.

❖ The force of friction always opposes the motion and depends on the normal force this means that:

$$F_R \propto N \qquad ; \quad N : normal\ force$$

And; $$F_R = \mu N \qquad ; \quad \mu : cofefficient\ of\ friction$$

❖ There are two kinds of friction:

1- **static friction(F_s):** the minimum force needed to initiate motion.

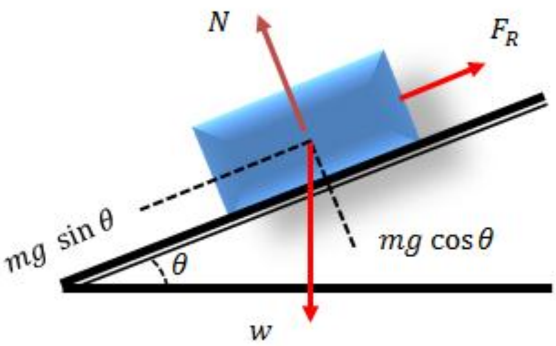
2- **kinetic friction(F_k):** force needed to maintain motion for most material.

And; $$F_s > F_k$$

$$F_R < 1$$

❖ **Angle of friction:**

Consider a body of weight (w) resting on an inclined plane, as shown in Figure.



The body is in equilibrium under the action of the following force:-

1- Weight (w) of the body, acting vertically downwards.

2- Frictional force (F_R) acting upwards along the plane.

3- Normal reaction (N) acting at right angles to the plane.

Resolving the weight into its components:

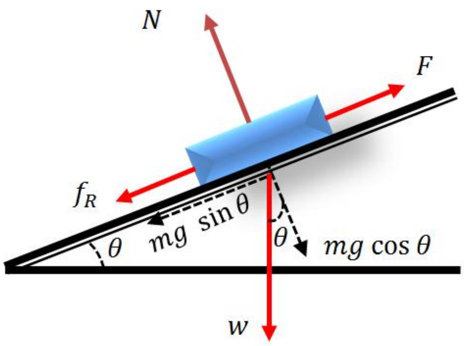
$$\sum \vec{F}_y = N - mg\cos\theta = 0$$

$$\sum \vec{F}_x = f_s - mg\sin\theta = 0 \qquad (F_R = f_s)$$

$$\because \quad \mu_s = \frac{f_s}{N} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

$$\therefore \ \mu_s = \tan\theta_s$$

If the body in motion:-



a- motion upward (Frictional force downward)

$$\sum \vec{F}_y = N - mg\cos\theta = 0$$

Or; $\quad N = mg\cos\theta \qquad \ldots\ldots\ldots\ldots\ldots(1)$

$$\sum \vec{F}_x = \vec{F} - mg\sin\theta - f_R = m\vec{a}$$

Or $\quad \vec{F} = m\vec{a} + mg\sin\theta + F_R \qquad \ldots\ldots\ldots\ldots\ldots(2)$

$$\because \ F_R = \mu N$$

$$\therefore \ \vec{F} = m\vec{a} + mg\sin\theta + \mu\, mg\cos\theta$$

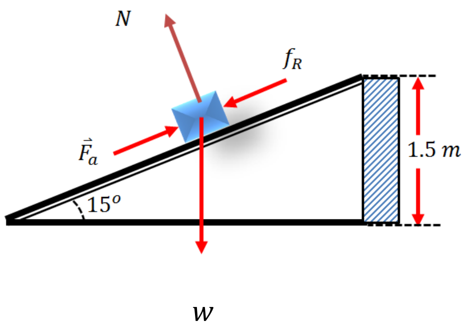
$$\vec{F} = m\,[\vec{a} + g\sin\theta + \mu\, g\cos\theta]$$

b- motion downward: in the same way we find that:-

$$\vec{F} = m[\vec{a} - g\sin\theta + \mu g\cos\theta]$$

Example (1):- (Forces acting on the body as it is moved up the incline)

A box of mass (510 kg) pulled up an incline, the coefficient of static friction along the incline is ($\mu_s = 0.42$) and the coefficient of kinetic friction along the incline is ($\mu_k = 0.33$), the ramp forms an angle ($\theta = 15^o$) above the horizontal.



(a) Find the minimum force needed to move the box.

(b) To slide the box up at a constant speed with what magnitude force must the moves push?

Solution:

(a) $\sum \vec{F}_y = N - mg\cos\theta = 0 \quad \Longrightarrow \quad N = mg\cos\theta$

$$\sum \vec{F}_x = \vec{F}_a + f_s + w = \vec{F}_a - \mu mg\cos\theta - mg\sin\theta = 0$$

$$\vec{F}_a = mg(\mu\cos\theta + \sin\theta) = 0$$

$$\vec{F}_a = (510 \times 9.8)(0.42\cos 15 + \sin 15) = 3300\ (N)$$

An applied force that exceeds 3300N starts to move.

(b) $\sum \vec{F}_y = -\mu_k N = -\mu_k mg\cos\theta$

$$\sum \vec{F}_x = \vec{F}_a - \mu_k mg\cos\theta - mg\sin\theta = 0$$

$$\vec{F}_a = mg(\mu_k\cos\theta + \sin\theta)$$

$$\vec{F}_a = (510 \times 9.8)(0.33\cos 15 - \sin 15) = 2900\ (N)$$

Example (2):-

The coefficient of sliding friction between the tires of a car and the road surface is $(\mu = 0.5)$ the driver bracks sharply and locks the wheels. If the velocity of the car before braking was $\left(\vec{v}_0 = 60\,\frac{km}{h}\right)$,

a) How much time will the car take to stop?

b) What is the stopping distance?

Solution:

$$\sum \vec{F}_x = \mathrm{m}\vec{\mathrm{a}} = -f_R$$

$$f_R = \mu N \rightarrow -\mu N = m\vec{a} \qquad \ldots\ldots\ldots\ldots..(1)$$

$$\sum \vec{F}_y = \mathrm{N} - \mathrm{mg} = 0$$

$$N = mg \qquad \ldots\ldots\ldots\ldots(2)$$

$$\vec{a} = -\mu g \qquad \ldots\ldots\ldots\ldots(3)$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{v}_0 = 60\ \ (kmh^{-1})\,; \quad \vec{v} = 0\,; \quad \vec{a} = -\mu g$$

$$0 = \vec{v}_0 - \mu g t$$

$$t = \frac{\vec{v}_0}{\mu g} \quad ; \ \vec{v}_0 = 60\left(\frac{km}{h}\right) = 16.67\left(\frac{m}{sec}\right)$$

$$t = \frac{16.67}{0.5 \times 9.8} = 3.4\ (sec)$$

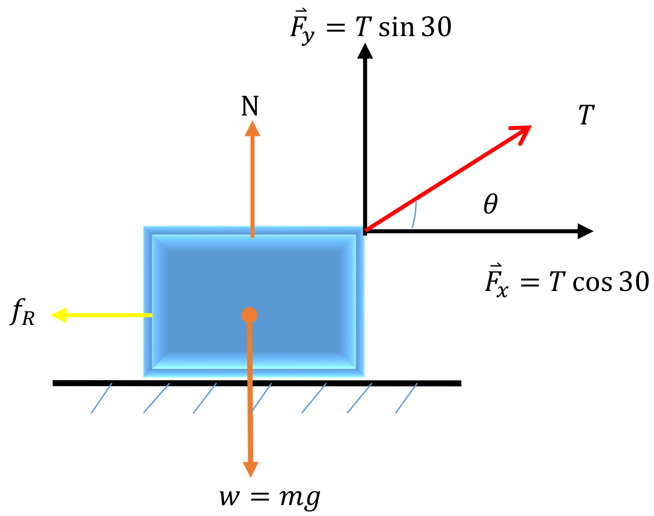
The stopping distance is :

$$\vec{x} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2\ = (16.67)(3.4) - \frac{1}{2}(0.5)(9.8)(3.4)^2 = 28.4\ (m)$$

Example (3)

What force (T), at an angle of (30^o) above the horizontal is required to drag a block weighing (20 N) to the right at constant speed. If the coefficient of kinetic friction between block and the surface is (o.2)?

Solution:



$$\sum \vec{F}_x = T\cos 30 - f_R = 0$$

$$T\cos 30 = f_R = \mu N$$

$$T\cos 30 = 0.2N \quad \text{..........} \ (1)$$

$$\sum \vec{F}_y = T\sin 30 + N - w = 0$$

$$T\sin 30 = w - N$$

$$T\sin 30 = 20 - N \ \text{..........}(2)$$

Dividing eq. (2) to eq. (1), find:

$$\frac{T\sin 30}{T\cos 30} = \frac{20 - N}{0.2N}$$

$$0.2(\sin 30)\,N = (20 - N)\cos 30$$

$$N = 17.9 \ \ (Newtons)$$

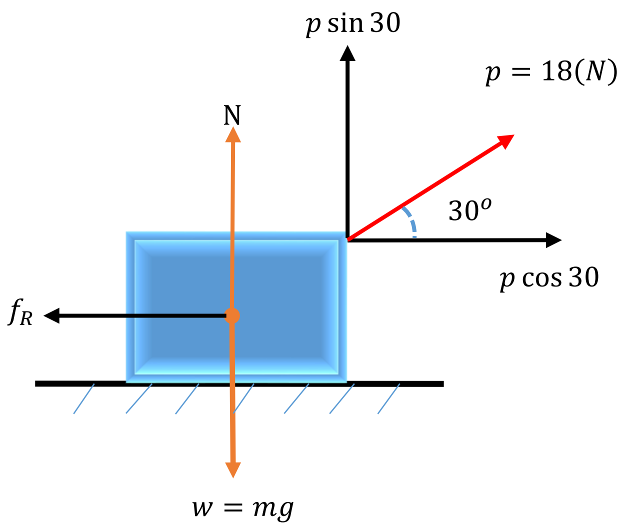
Substituting value of N in eq. (1), we find

$$T = \frac{0.2(17.9)}{\cos 30} = 4.14 \ (Newtons)$$

Example (4)

A body, resting on a rough horizontal plane required a pull of (18 N) inclined at (30^o) to the plane just to remove it. It was found that a push of (22 N) inclined at (30^o) to the plane just removed the body. Determine the weight of the body and the coefficient of friction.

Solution:-



$$\sum \vec{F}_x = p\cos 30 - f_R = 0$$

$$f_R = 18\cos 30 = 15.6\ (N)$$

$$\sum \vec{F}_y = N + p\sin 30 - w = 0$$

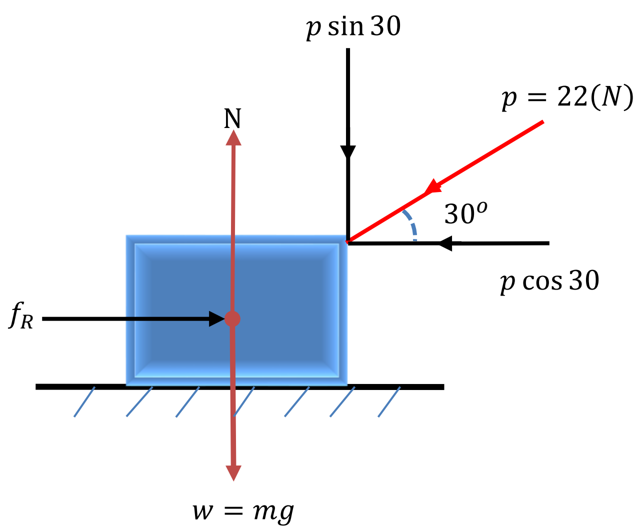
$$N = w - 18\sin 30 = w - 9$$

$$f_R = \mu N$$

$$15.6 = \mu(w-9)\ \ldots\ldots\ldots\ldots\ldots(1)$$

$$\sum \vec{F}_x = -pco\,s\,\theta + f_R = 0$$

$$f_R = 22\cos 30 = 19.05$$



$$\sum \vec{F}_y = N - pco\,s\,30 - w = 0$$

$$N = w + 22\sin 30$$

$$N = w + 11$$

$$f_R = \mu N$$

$$19.05 = \mu(w+11)\ \ \ldots\ldots\ldots\ldots(2)$$

Dividing eq. (1) by eq.(2)

$$\frac{15.6}{19.05} = \frac{\mu(w-9)}{\mu(w+11)} \quad \Longrightarrow \quad w = 99.4\ (Newtons)$$

Now substituting this value of w in eq. (1)

$$15.6 = \mu(99.4 - 9) = 90.4\,\mu \qquad \Longrightarrow \qquad \mu = \frac{15.6}{90.4} = 0.1726$$

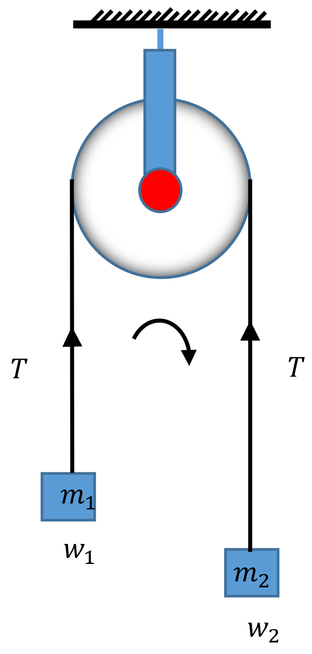
Applications second law of newton

(Atwood machine and similar systems)

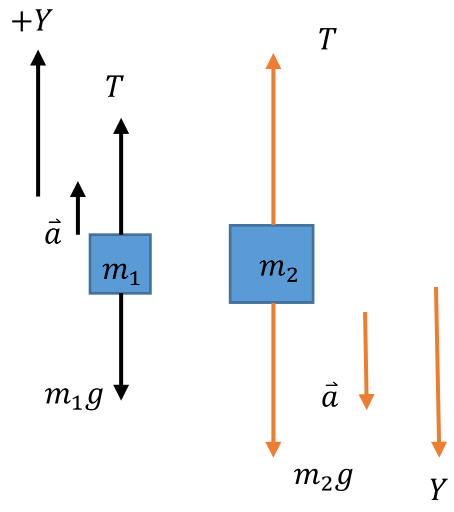
1-Two bodies hanging on a pulley:

Consider light inextensible string passing over a smooth pulley, so that the tension (T) in both the strings may be the same. Let (m_2) be greater than (m_1), the downward force of gravity is stronger on the right side then on the left. We expect (m_2) acceleration to be downward and (m_1) to be upward:-

We treat each body as a separate system, draw free body diagrams for each and then apply newton's second law to each.



The newton's second law for (m_1);



$$\sum \vec{F}_1 = T - m_1 g = m_1 \vec{a}$$

$$or \quad T = m_1 \vec{a} + m_1 g \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$also\ for\ (m_2)$$

$$\sum \vec{F}_2 = m_2 g - T = m_2 \vec{a}$$

$$then \ \ T = m_2 g - m_2 \vec{a} \quad \ldots\ldots\ldots\ldots.\ (2)$$

From eq.(1) and eq.(2) find;

$$m_2 g - m_1 g = m_2 \vec{a} + m_1 \vec{a}$$

Solving for $(\vec{a})$ we find;

$$\vec{a} = \frac{(m_2 - m_1)g}{m_2 + m_1}$$

To find T we can substitute the expression for $(\vec{a})$ into either of the two original equations.

Using first equation;

$$T - m_1 g = m_1 \frac{(m_2 - m_1)\ g}{m_2 + m_1}$$

Solving for T yields;

$$T = \frac{2\ m_1 m_2\ g}{m_2 + m_1}$$

Example (1)

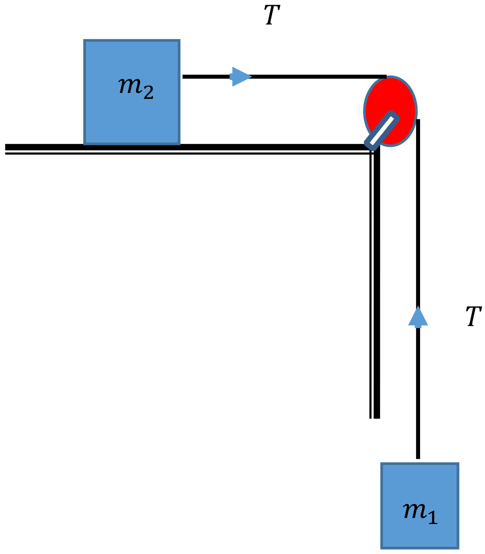
If the $(m_1 = 26\ kg)$ and $(m_2 = 42\ kg)$ in a above Figure, what are the accelerations of each body and the tension in the string.

Solution:-

$$\vec{a} = \left(\frac{42 - 26}{42 + 26}\right)(9.8) = 2.31\left(\frac{m}{sec^2}\right)$$

$$T = \frac{2 \times 26 \times 42}{68} \times 9.8 = 315\ (N)$$

2-motion of two bodies connected by a string, one of which hanging free and the other lying on a smooth horizontal plane.



Force acting on m_2 is;

$$\sum \vec{F}_2 = T = m_2\vec{a} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

And also acting force for $m_1$is

$$\sum \vec{F}_1 = m_1 g - T = m_1\vec{a}$$

$$or \quad T = m_1 g - m_1\vec{a} \ \ldots\ldots\ldots\ldots(2)$$

From eq. (1), find;

$$m_2\vec{a} = m_1 g - m_1\vec{a}$$

$$\vec{a}(m_2 + m_1) = m_1 g$$

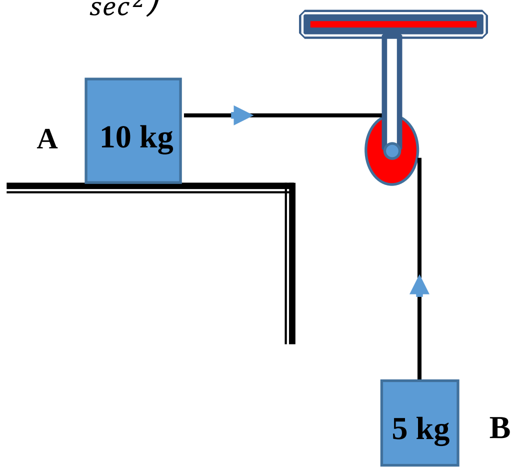
$$\vec{a} = \frac{m_1 g}{m_2 + m_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Then we find T from equation (1) that:

$$T = \left(\frac{m_2 m_1}{m_1 + m_2}\right) g$$

Example (2)

Find the acceleration of a solid body A of a mass (10 kg), when it is being pulled by another body B of mass (5 kg) along a smooth horizontal plane, as shown in the following figure. Also find the tension in the string. Assuming the string to be inextensible. If $\left(g = 9.8\ \frac{m}{sec^2}\right)$.



Solution:

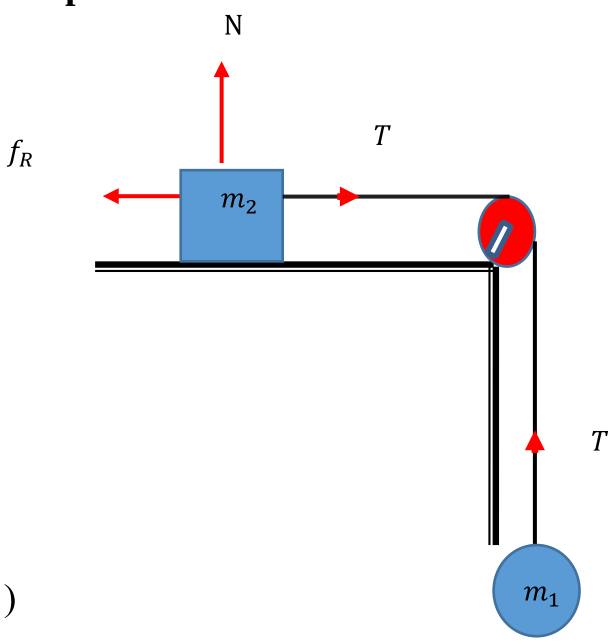
$$\vec{a} = \frac{m_1 g}{m_2 + m_1} = \frac{5 \times 9.8}{5 + 10} = 3.27\left(\frac{m}{sec^2}\right)$$

Tension in the string:

$$T = \frac{g \times m_1 \times m_2}{m_1 + m_2} = \frac{9.8 \times 10 \times 5}{10 + 5}$$

$$= 32.7(\ N)$$

3-Motion of two bodies connected by a string, one of which is hanging free and other lying on a rough horizontal plane.



Where;

$N = normal\ reaction$

$N = m_2 g$

$F_R = \mu N = \mu m_2 g$

From newton's second law;

$\sum \vec{F}_2 = T - \mu m_2 g = m_2 \vec{a}$

$then\ \ T = m_2 \vec{a} + \mu m_2 g$ ....(1)

Apply newton law for (m),

$$\sum \vec{F}_1 = m_1 g - T = m_1 \vec{a}$$

$$and\quad T = m_1 g - m_1 \vec{a} \;\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving eq. (1) and (2) find;

$$\vec{a} = \frac{g(m_1 - \mu m_2)}{m_1 + m_2}$$

Substituting this value of $(\vec{a})$ in equation (2);

$$T = \frac{m_1 m_2 (1 + \mu)}{m_1 + m_2} g$$

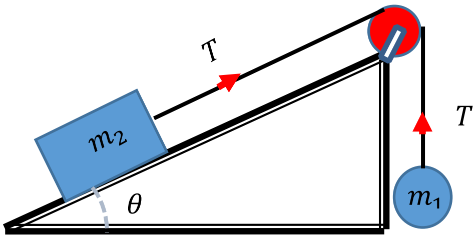
4-A pulley, an incline, and two blocks a.(an inclined smooth)

$$\sum \vec{F}_2 = T - m_2 g \sin\theta = m_2 \vec{a}$$

$$T = m_2 \vec{a} + m_2 g \sin\theta \;\ldots\ldots\ldots\ldots(1)$$

$$\sum \vec{F}_1 = m_1 g - T = m_1 \vec{a}$$

$$or\quad T = m_1 g - m_1 \vec{a} \;\ldots\ldots\ldots\ldots(2)$$



Adding eq. (1) and (2), obtain;

$$\vec{a} = \frac{g(m_1 - m_2 \sin\theta)}{m_1 + m_2}$$

Substituting this value of $(\vec{a})$ in equation (2);

$$T = \frac{m_1 m_2 (1 + \sin\theta) g}{m_1 + m_2}$$

Example (3)

A body of mass (50 kg), lying on a smooth plane inclined at (15^o) to the horizontal, is being pulled by a body of mass (20 kg). Find the acceleration; and find the tension in string.

Solution:-

$$\vec{a} = \frac{9.8(20 - 50\sin 15)}{20 + 50} = 0.988\left(\frac{m}{sec^2}\right)$$

$$T = \frac{(20)(50)(1 + \sin 15)(9.8)}{20 + 50}$$

$$T = \frac{1000(1 + 0.2588)}{70} = 176.2\ (N)$$

Example (4)

Two blocks have weights (A=8 kg) and (B=4 kg) and the coefficient of friction between block (A) and the horizontal plane, $(\mu = 0.25)$. If the system is released from rest and the block (B) falls through a vertical distance of (1 m), what is the velocity a required by it? **Solution:-**

$$\vec{a} = \frac{g(m_1 - m_2\mu)}{m_1 + m_2} = \frac{9.8(4 - 0.25 \times 8)}{4 + 8} = 1.63\left(\frac{m}{sec^2}\right)$$

$$\vec{v}^2 = \vec{v}_o{}^2 + 2\vec{a}\vec{x}$$

$$\vec{v}^2 = 0 + 2(1.63)(1) = 3.26$$

$$\vec{v} = 1.81\left(\frac{m}{sec}\right)$$

Example (5)

A body A of mass (30 kg) is lying on a horizontal table (1.2 m) from its edge it is attached to a string, whose other end is carrying a body (B) of mass (3 kg). If the coefficient of friction between the body (A) and the table is $\left(\frac{1}{16}\right)$. Find the acceleration of the system, and time required to fall over the edge? (g=9.81)

Solution:

$$\vec{a} = \frac{g(m_1 - \mu m_2)}{m_1 + m_2} = \frac{9.81(3 - 30 \times \frac{1}{16})}{30 + 3} = 0.334 \left(\frac{m}{sec^2}\right)$$

$$\vec{x} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2 \qquad \Longrightarrow \qquad 1.2 = 0 + \frac{1}{2}(0.334)(t)^2 = 0.167t^2$$

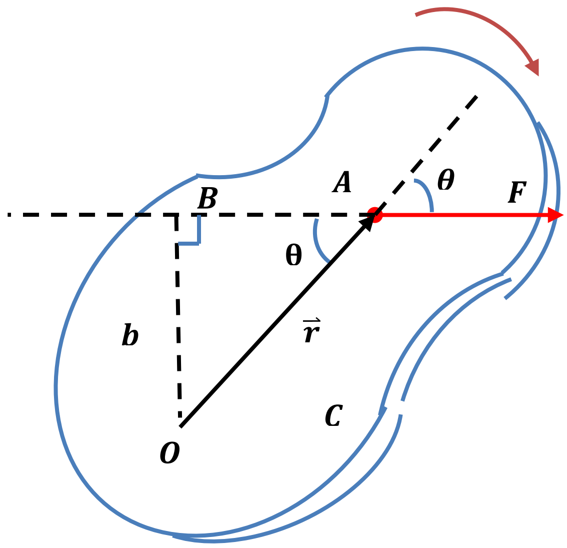
$$t = \sqrt{\frac{1.2}{0.167}} = 2.68\ (sec)$$

Chapter 5

Torque

Torque

Consider a force $(\vec{F})$ applied at point (A), acting on a body (C) that can rotate about point (O) (show Fig.),



The net effect of the force will be to rotate the body around (O); that the rotating effectiveness of (F) increases with the perpendicular distance (called lever arm) (b=OB) from (O) to the line of action of the force.

This is so called torque, according;

$$\vec{\tau} = \vec{F} b$$

Or $(torque = force \times lever)$; it is expressed as the product of a unit force and distance (Newton-meter) or (N.m).

From the Fig.:- $\qquad b = \vec{r} \sin\theta$

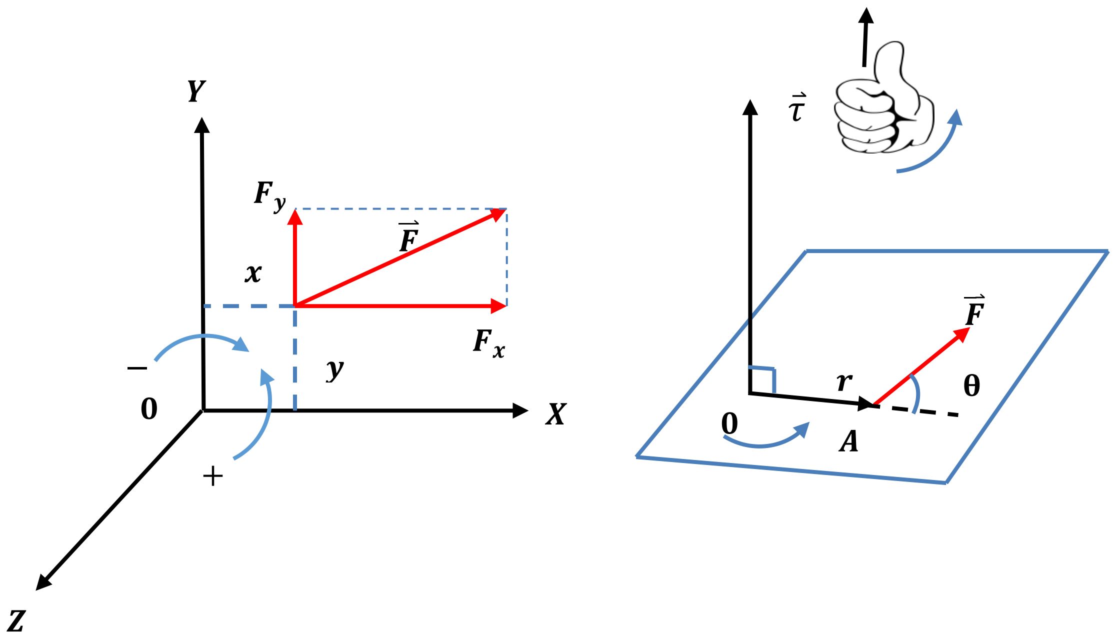
$$\vec{\tau} = \vec{F}\vec{r} \sin\theta$$

We conclude then that the torque may be considered as a vector quantity given by a vector product:-

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Where $(\vec{r})$ vector position, the torque is a vector perpendicular to both $(\vec{r})$ and $(\vec{F})$, the torque has the same direction of thumb of the right hand, when the fingers are curled toward the rotation product by $(\vec{F})$around (O).

Note:- The torque of a force is always defined relative to a certain point, if the point of reference is changed, usually the torque of the force changed.



Vector relation between torque, force, and position vector

If $(\vec{r})$ *and* $(\vec{F})$ are in the (X Y) plane, the direction of $(\vec{\tau} = \vec{r} \times \vec{F})$ is parallel to(Z) axis then the torque also given as:

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ x & y & 0 \\ F_x & F_y & 0 \end{vmatrix} = k(x\,F_y - y\,F_x)$$

The component force, of torque given as:

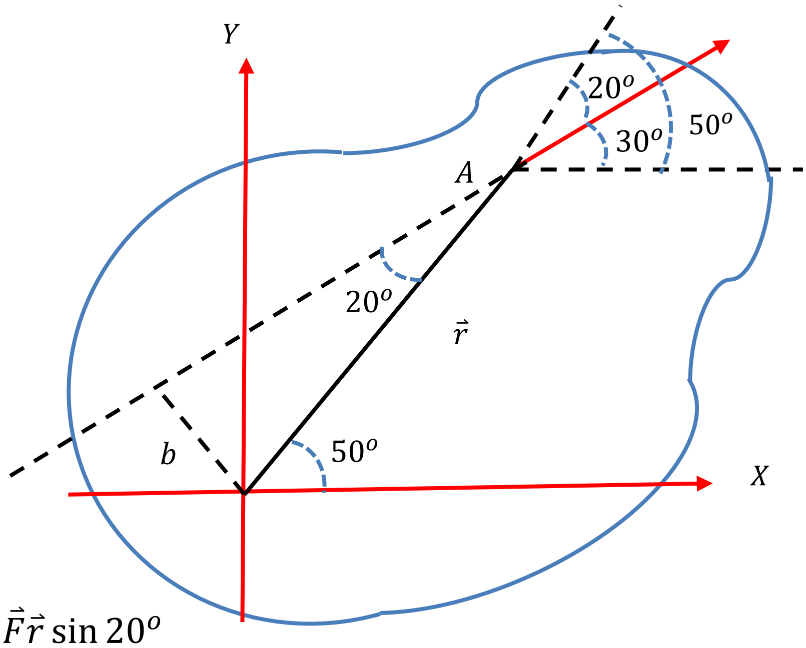
$$\vec{\tau}\ \ of\ \ \tau_x = -y\,F_x\,, \qquad \vec{\tau}\ \ of\ \ \tau_y = x\,F_y$$

Then the torque of F is:

$$\vec{\tau} = xF_y - yF_x \implies magnitude\ form$$

Example (1)

Determine the torque applied to the body as shown in figure, where $(\vec{F} = 6\ N)$ and makes an angle of (30^o) with the x-axis and $(\vec{r} = 0.45\ m)$ long and makes an angle of (50^o) with +x-axis.



Solution:-

$\vec{\tau} = \vec{F}b = \vec{F}\vec{r}\sin\theta = \vec{F}\vec{r}\sin 20^o$

$\vec{\tau} = (6\ N)(0.45)(0.342)$

$\vec{\tau} = 0.924\ (N.m)$

We must write (- 0.924N.m) the rotation around O is clockwise.

A second method, to find τ

$x = \vec{r}\cos 50^o = 0.289\ (m)$

$y = \vec{r}\sin 50^o = 0.345m$

$F_x = \vec{F}\cos 30^o = 5.196\ (N)$

$F_y = \vec{F}\sin 30^o = 3.0\ (N)$

$\vec{\tau} = x\ F_y - y\ F_x$

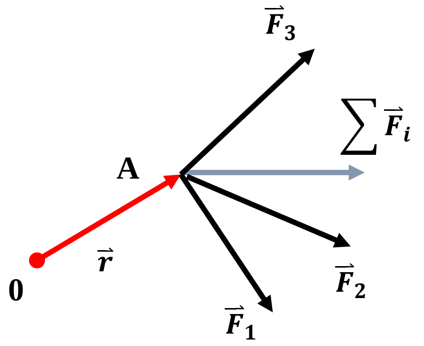
$\vec{\tau} = (0.289)(3) - (0.345)(5.196)$

$\vec{\tau} = 0.867 - 1.792 = -0.925\ (N.m)$

In agreement with our previous result.

Torque of several forces

Let $\vec{F}_1, \vec{F}_2, \vec{F}_3 \dots\dots\dots$ forces acting on a point (A) (Shown in figure). The torque of each $\left(\vec{\tau}_i = \vec{r} \times \vec{F}_i\right)$. The torque of resultant $\left(\sum \vec{F}\right)$ given;



$$\vec{\tau} = \vec{r} \times \sum_{i}^{n} \vec{F}_i \quad , \qquad where \sum_{i}^{n} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots\dots\dots$$

Or $\qquad \vec{\tau} = \vec{r} \times (\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \dots\dots.)$

$$\vec{\tau} = \vec{r} \times \overrightarrow{F_1} + \vec{r} \times \overrightarrow{F_2} + \vec{r} \times \overrightarrow{F_3}$$

Therefore; $\quad \vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \dots\dots. = \sum \vec{\tau}_i$

That is, the torque of the resultant force is equal to the vector sum of the torques of the component forces.

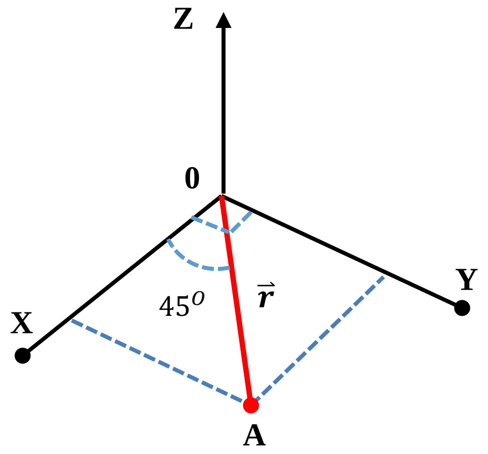
Example (2)

Consider three forces applied at a point A, (shown in figure), with ($\vec{r} = 1.5\ m$), and;

$\overrightarrow{F_1} = i(6) + j(3)\ \ Net.$

$\vec{F}_2 = i(-2) + j(7)$

$\vec{F}_3 = i(5) + j(-8)$



Solution:

The resultant torque due to these forces;

$$\vec{\tau} = \vec{r} \times \sum \overrightarrow{F_i}$$

$$\sum \vec{F}_i = i(6 - 2 + 5) + j(3 + 7 - 8)$$

$$\sum \vec{F}_i = i(9) + j(2) \quad \text{Net.}$$

$$\because\ \vec{r} = ix + jy$$

$$x = \vec{r}\cos\theta = \vec{r}\cos 45 = 1.5\cos 45^o = 1.06\ (m)$$

$$y = \vec{r}\sin\theta = \vec{r}\sin 45^o = 1.5\ \sin 45^o = 1.06\ (m)$$

$$\therefore\ \vec{\tau} = \vec{r} \times \sum \overrightarrow{F_i} = k(x\,F_{iy} - y\,F_{ix})$$

$$\vec{\tau} = k[1.06\ (2) - 1.06(9)]$$

$$\vec{\tau} = k(2.12 - 9.54) = k(-7.42) \qquad (N.m)$$

The same result can obtain by using;

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3\ ; \qquad we\ have:$$

$$\vec{\tau}_1 = \vec{r} \times \overrightarrow{F_1} = k(xF_1y - yF_1x)$$

$$\vec{\tau}_1 = k(1.06 \times 3 - 1.06 \times 6) = k(-3.18) \quad (N.m)$$

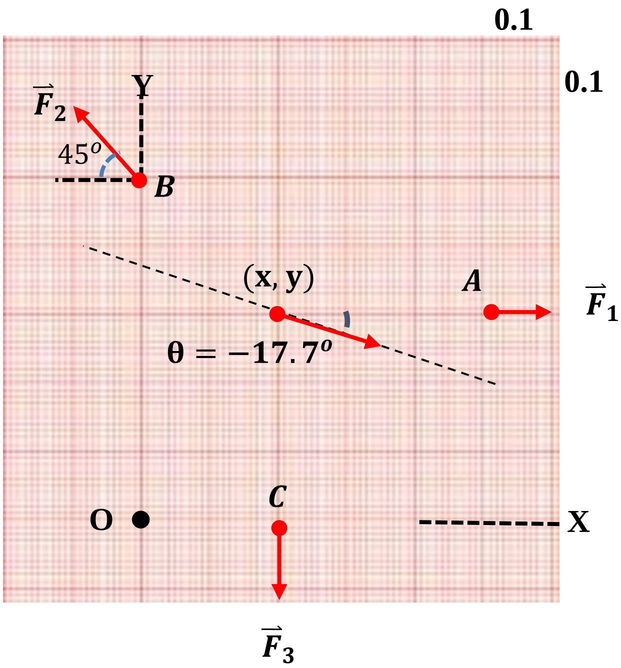
$\vec{\tau}_2 = k(1.06 \times 7 - 1.06 \times -2) = k(9.54) \quad (N.m)$

$\vec{\tau}_3 = k(1.06 \times -8 - 1.06 \times 5) = k(-13.78)\ (N.m)$

$\vec{\tau} = k(-3.18 + 9.54 - 13.78) = k(-7.42) \quad (N.m)$

Example (3)

Determine of resultant of the system forces shown in figure; acting in one plane. The magnitude of the forces: $(\vec{F}_1 = 10\ N)$; $(\vec{F}_2 = 8\ N)$; $(\vec{F}_3 = 7N)$. The side of each square is (0.1 m).



Solution:

The vector form of (F) is

$\vec{F}_1 = i(10)\ N$

$\vec{F}_2 = i(8\cos 135^o) + j(8\sin 135^o) = i(-5.66) + j(5.66)\ \ N$

$\vec{F}_3 = -j(7)N.$

The resultant force:

$$\sum \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\sum \vec{F}_i = i(4.34) + j(-1.34)$$

$$\vec{F} = \sqrt{(4.34)^2 + (-1.34)^2}$$

$\vec{F} = 4.54\ \ (N)(magnitude\ of\ Net\ force)$: It make angle θ

$\theta = \tan^{-1}\dfrac{-1.34}{4.34} = -17.1^{o}$

The coordinate of the points

A =(0.5,0.3) ; B=(0,0.5) ; C=(0.2,0)

To find (τ) we compute:-

$\vec{\tau} = (x\,F_y - y\,F_x)$

$\vec{\tau}_1 = -(0.3)(10) = -3\ (N.m)$

$\tau_2 = -(0.5)(-5.66) = 2.83\ (N.m)$

$\vec{\tau}_3 = (0.2)(-7) = -1.4\ (N.m)$

$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 = (-3) + (2.83) + (-1.4) = -1.57\ (N.m)$

To find the point of application of the (τ) resultant having coordinate:

$\vec{\tau} = \left(xF_{iy} - yF_{ix}\right) = (0.2)(-1.34) - (0.3)(4.34) = -0.268 - 1.302$

$\vec{\tau} = -1.57\ (N.m)$

Since there are numerous values of (x, y) that satisfy this relation.

Composition of parallel forces

Consider a system of parallel forces $\left(\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots\ldots.\right)$ all act in direction perpendicular to the x-axis at points $(x_1, x_2, x_3, \ldots\ldots)$ from the origin O.

The magnitude of resultant forces is:

$$\sum \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

The total torque of all forces is:-

$$\vec{\tau} = \sum x_i \vec{F}_i = x_1\vec{F}_1 + x_2\vec{F}_2 + x_3\vec{F}_3 + \cdots$$

The resultant force $\left(\sum \vec{F}_i\right)$ must be applied at a point (C) a distance (x_c) from (O) or;

$$\vec{\tau} = x_c \sum \vec{F_i}$$

Or;
$$x_c = \frac{\sum x_i \vec{F}_i}{\sum \vec{F}i} = \frac{x_1\vec{F}_1 + x_2\vec{F}_2 + x_3\vec{F}_3 + \cdots}{\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots}$$

In general, the coordinate of (C) are given by:

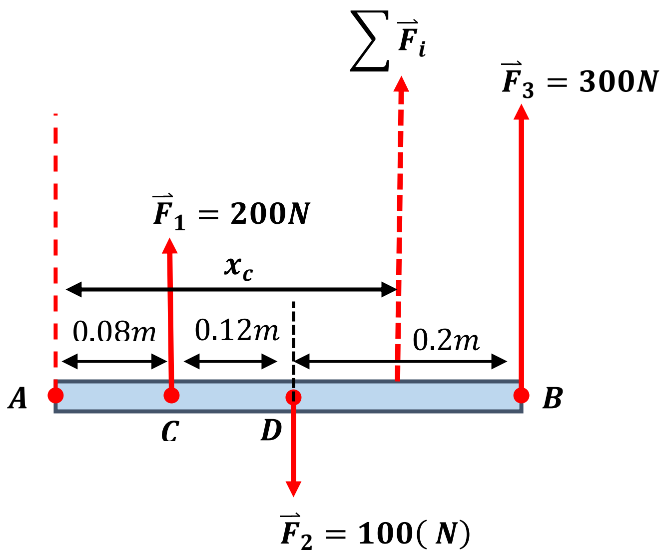
$$x_c = \frac{\sum x_i \vec{F_i}}{\sum \vec{F_i}} \quad ; \quad y_c = \frac{\sum y_i \vec{F_i}}{\sum \vec{F_i}} \quad ; \quad z_c = \frac{\sum z_i \vec{F_i}}{\sum \vec{F_i}}$$

The point (C) is called center of parallel forces in vector form is written:

$$\vec{r} = \frac{\sum \vec{r_i} \vec{F_i}}{\sum \vec{F_i}}$$

Example (4)

Find the resultant of the forces acting on the bar as shown in figure:



Solution:

$$\sum \vec{F_i} = \vec{F_1} - \vec{F_2} + \vec{F_3} = 200 - 100 + 300 = 400\ (N)$$

Take point A as the origin, we obtain:

$$x_c = \frac{\sum x_i \vec{F_i}}{\sum \vec{F_i}}$$

$$x_c = \frac{\sum x_i \vec{F}_i}{\sum \vec{F}_i}$$

$$x_c = \frac{\mp\vec{F}_1 x_1 \mp \vec{F}_2 x_2 \mp \vec{F}_3 x_3}{\mp\vec{F}_1 \mp \vec{F}_2 \mp \vec{F}_3 \mp} = \frac{(200)(0.08)) + (-100)(0.2) + (300)(0.4)}{400}$$

$$x_c = 0.29\ (m)$$

Take point (D) as the origin:

$$x_c = \frac{(200)(-0.12) + (-100)(0) + (300)(0.2)}{400} = 0.09\ (m)$$

This point (x_c) is the same as before.

Center of mass

We consider a system consisting of (N) particles of masses $(m_1, m_2, m_3 \dots, m_N)$. The total masses is:

$$M = m_1 + m_2 + m_3 + \cdots, m_N = \sum m_N$$

Each particle in the system can represented by its mass, and its location $(\vec{r})$ (whose components X, Y, and Z). The center mass defined as:

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots + m_N \vec{r}_N}{m_1 + m_2 + \cdots + m_N}$$

$$\vec{r}_{cm} = \frac{1}{M} \sum m_N \vec{r}_N$$

In term component:

$$X_{cm} = \frac{1}{M}(m_1 x_1 + m_2 x_2 + \cdots + m_N x_N)$$

$$Y_{cm} = \frac{1}{M}(m_1 y_1 + m_2 y_2 + \cdots + m_N y_N)$$

$$Z_{cm} = \frac{1}{M}(m_1 z_1 + m_2 z_2 + \cdots + m_N z_N)$$

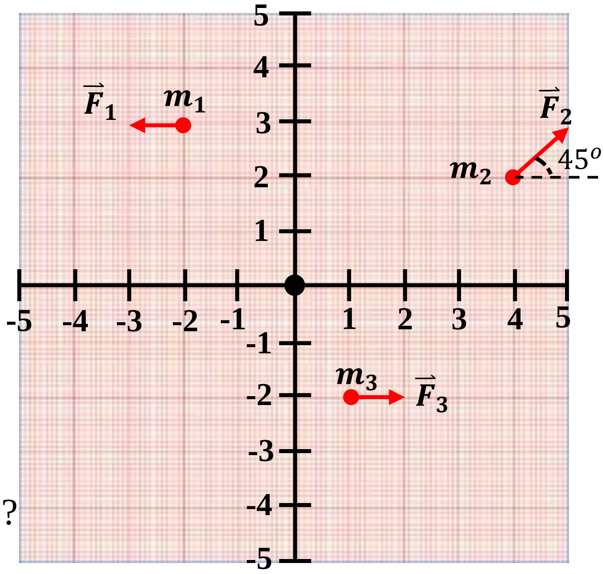
Example (5)

In figure:

System of three particles initially at rest.

$(m_1 = 4.1\ kg,\ m_2 = 8.2\ kg,\ m_3 = 4.1 kg)$

and $(\vec{F}_1 = 6\ N,\ \vec{F}_2 = 12\ N,\ \vec{F}_3 = 14\ N)$



1- What is the center mass of this system?

2- What is the acceleration of the center mass?

Solution:

$$X_{cm} = \frac{1}{M}(m_1 x_1 + m_2 x_2 + m_3 x_3)$$

$$X_{cm} = \frac{1}{16.4}[(4.1)(-2) + (8.2)(4) + (4.1)(1)] = 1.8\ (cm)$$

$$Y_{cm} = \frac{1}{M}(m_1 y_1 + m_2 y_2 + m_3 y_3)$$

$$Y_{cm} = \frac{1}{16.4}[(4.1)(3) + (8.2)(2) + (4.1)(-2)] = 1.3\ (cm)$$

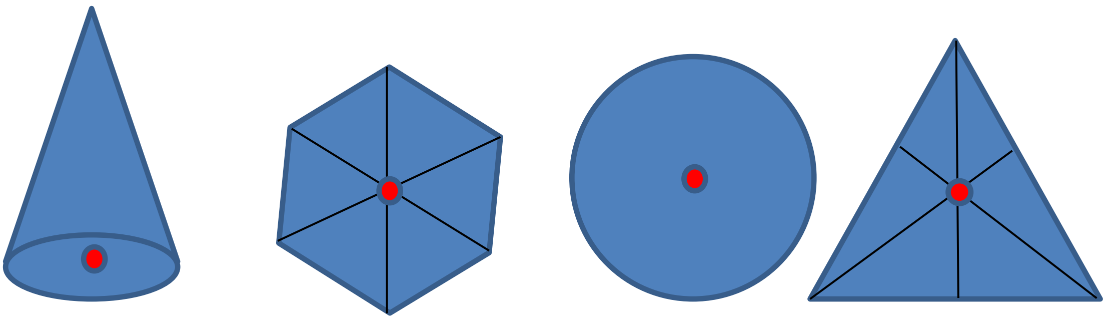
$$\sum F_x = -6\ + 12\cos 45^o + 14 = 16.5\ (N)$$

$$\sum F_y = 0 + 12\sin 45 + 0 = 8.5\ (N)$$

$$F = \sqrt{(16.5)^2 + (8.5)^2} = 18.6\ (N)$$

$$\theta = \tan^{-1}\frac{8.5}{16.5} = 27^o$$

$$|\vec{a}| = \frac{\vec{F}}{M} = \frac{18.6}{16.4} = 1.1\left(\frac{m}{sec^2}\right)$$



Example (6)

Find the center of mass of the particles located as shown in figure, the values of masses are:-

The values of masses are:

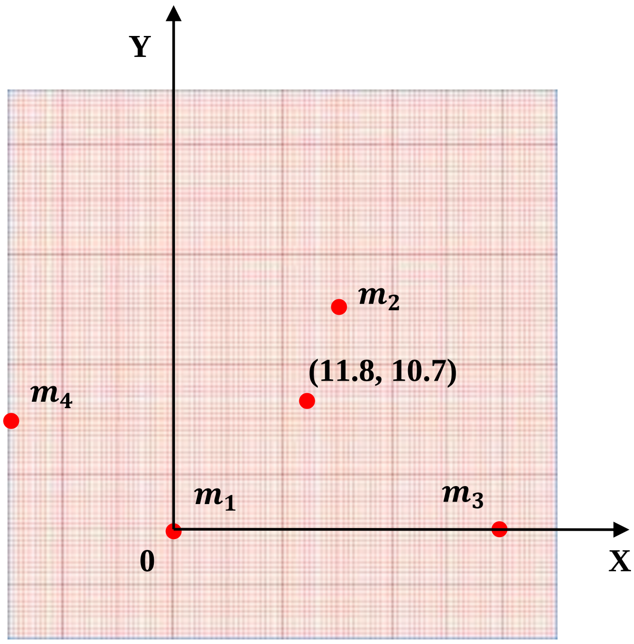
$m_1 = 5\ (kg)$

$m_2 = 30\ (kg)$

$m_3 = 20\ (kg)$

$m_4 = 15(\ kg)$

The side of each square is (5cm)



Solution:

Total mass $M = m_1 + m_2 + m_3 + m_4 = 5 + 30 + 20 + 15 = 70\ (kg)$

$$X_{cm} = \frac{(5)(0) + (30)(15) + (20)(30) + (15)(-15)}{70} = 11.8\ (cm)$$

$$Y_{cm} = \frac{(5)(0) + (30)(20) + (20)(0) + (15)(10)}{70} = 10.7\ (cm)$$

$$\therefore\ \ \vec{r}_{cm} = (11.8, 10.7)$$

Equilibrium of a particle

A particle is in equilibrium when the sum of all the forces acting on it is zero:

$$\sum_i \vec{F}_i = 0 \quad ; \quad is\ equivalent\ to$$

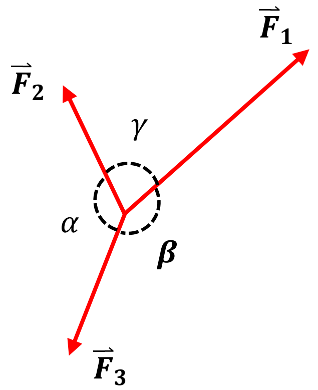
$$\sum_i \vec{F}_{ix} = 0\ ;\ \sum_i \vec{F}_{iy} = 0\ ;\ \sum_i F_{iz} = 0$$

Example (7)

Equilibrium of three forces acting on a particle:

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

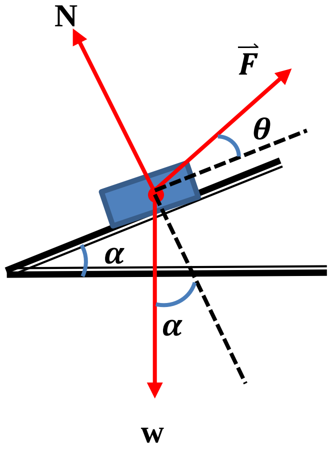
As shown in Fig.;



Applying the law of sine's

$$\frac{\vec{F}_1}{\sin\alpha} = \frac{\vec{F}_2}{\sin\beta} = \frac{\vec{F}_3}{\sin\gamma}$$

If the particle on a smooth inclined plane.



The following forces:

W =weight,

$\vec{F} = pull$;

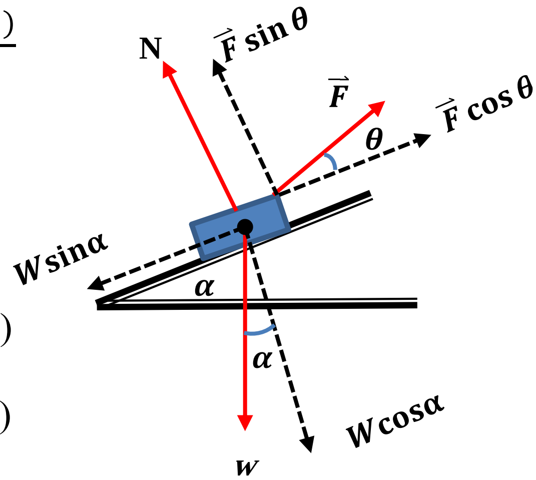
N = normal reaction

We have;

$$\frac{\vec{F}}{\sin(180-\alpha)} = \frac{N}{\sin(90+\alpha+\theta)} = \frac{W}{\sin(90-\theta)}$$

$$\frac{\vec{F}}{\sin\alpha} = \frac{N}{\cos(\alpha+\theta)} = \frac{W}{\cos\theta}$$

Then :- $\vec{F} = \frac{w\sin\alpha}{\cos\theta}$; $N = \frac{w\cos(\alpha+\theta)}{\cos\theta}$



As an alternative procedure: -

By using component of forces: -

$$\sum \vec{F}_{ix} = \vec{F}\cos\theta - w\sin\alpha = 0 \ \ldots\ldots\ldots\ldots(1)$$

$$\sum F_{iy} = F\sin\theta - w\cos\alpha + N = 0 \ \ldots\ldots\ldots(2)$$

From Equ. (1) we obtain:

$$\vec{F}\cos\theta = w\sin\alpha \quad or \quad F = \frac{w\sin\alpha}{\cos\theta}$$

From equation (2) found:

$$N = w\cos\alpha - \vec{F}\sin\theta = w\cos\alpha - \frac{w\sin\alpha\sin\theta}{\cos\theta}$$

$$N = \frac{w(\cos\alpha\cos\theta - \sin\alpha\sin\theta)}{\cos\theta}$$

$$N = \frac{w\cos(\alpha+\theta)}{\cos\theta}$$

It is agreement with our previous results.

Equilibrium of a solid body (Bar)

Therefore, the two following conditions are required:-

1- The sum of all the forces must be zero (transitional equilibrium)

$$\sum \vec{F}_i = 0$$

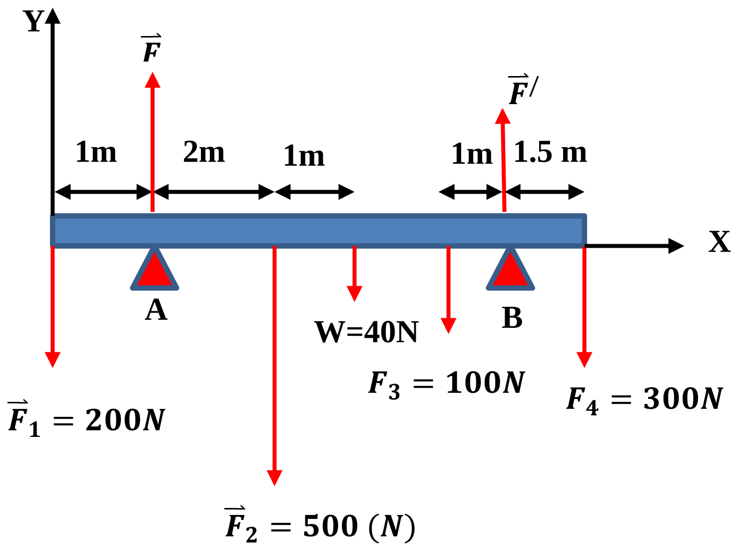
Or; $\sum \vec{F}_{ix} = 0\ ;\ \ \sum \vec{F}_{iy} = 0$

2- The sum of all the torques must be zero (rotational equilibrium)

$$\sum \vec{\tau}_i = 0$$

Example (8)

The bar (as shown in Fig.) is in equilibrium on points (A and B). Under action forces indicated. Find the forces exerted on the bar at points (A and B) The bar weight (40 N), and its length is (8 m).



Solution:-

$$\sum \vec{F}_i = \vec{F} + \vec{F}' - 200 - 500 - 40 - 100 - 300 = 0$$

Or; $\vec{F} + \vec{F}' = 1140\ (N)$

We apply rotation equilibrium; $\sum \vec{\tau}_i = 0$

Choice point A, as rotation point,

$$\sum \vec{\tau_i} = (-200)(-1) + \vec{F}(0) + (-500)(2) + (-40)(3)$$

$$+ (-100)(4.5) + \vec{F}'(5.5) + (-300)(7) = 0$$

Solving find that;

$$\vec{F}' = 630.9 \quad (N)$$

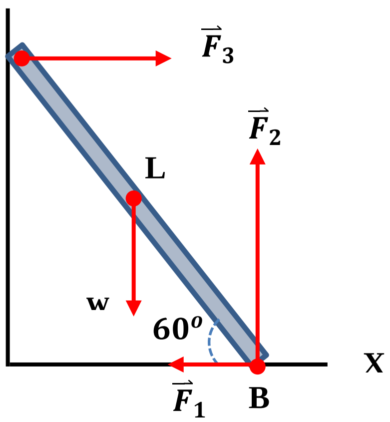
Combining this result with Equ.(1) obtain;

$$\vec{F} = 509.1\ (N)$$

Example (9)

A ladder (A B) weighing (160 N) rest against a vertical wall, making an angle of (60^o) with the floor. Find the forces on the ladder at (A and B) .

Using conditions of equilibrium:



$$\sum \vec{F}_{ix} = -\vec{F}_1 + \vec{F}_3 = 0 \quad \Longrightarrow \quad \vec{F}_1 = \vec{F}_3$$

$$\sum \vec{F}_{iy} = -w + \vec{F}_2 = 0 \quad \Longrightarrow \quad w = \vec{F}_2$$

Take torques around point B.

$$\sum \vec{\tau}_i = w\left(\frac{1}{2}L\cos 60^o\right) - \vec{F}_3 L \sin 60^o = 0$$

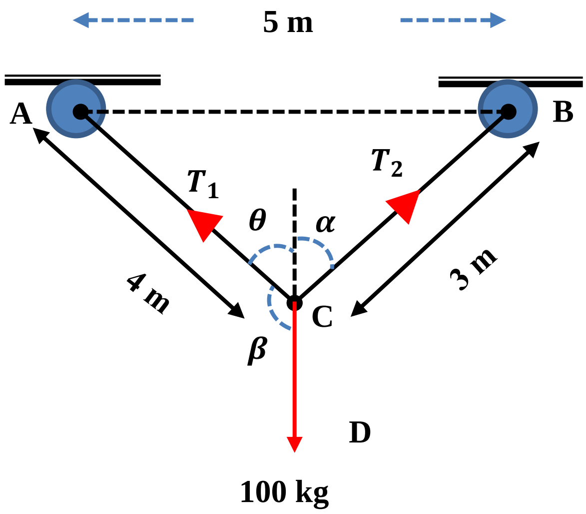
$$\vec{F}_3 = \frac{w\cos 60^o}{2\sin 60^o} = 46.1\ (N)$$

$$\because\ \vec{F}_1 = \vec{F}_3 = 138.6\ (N)\ ; \quad and \quad \vec{F}_2 = w = 160\ (N)$$

Example (10)

A body of mass (100 kg) is suspended by tow strings of (4 m and 3 m) lengths attached at the same horizontal level (5 m) apart. Find the tension in strings?

Solution:-



Given load a C=100 (kg)

Length of AC= 4 (m)

Length of BC= 3 (m)

Length of AB= 5 (m)

Since in triangle ABC $(5^2 = 4^2 + 3^2)$

$\therefore \ \sphericalangle ABC = 90^o$

$\sin A = \frac{3}{5} = 0.6$

$\therefore \ \sphericalangle A = 36^O.52'$

And $\sphericalangle B = 90^O - (36^O.52') = 53^O.8'$

$T_1 = tension\ AB$

And ; $T_2 = tension\ BC$

From the geometry, we find that

$\sphericalangle\theta = \sphericalangle B; \ \sphericalangle\alpha = \sphericalangle A;$

$\therefore \ \sphericalangle BCD = 180^o - \alpha = 180 - (36^o.52')$

$\sphericalangle ACD = 180^o - \theta = 180 - (53^o.8')$

And $\sphericalangle ACB = 90^O$

Applying lama's equation at point (C)

$$\frac{T_1}{\sin(180 - 36^O.52')} = \frac{T_2}{\sin(180 - 53^O.8')} = \frac{100}{\sin 90^o}$$

$$\frac{T_1}{\sin 36^O.52'} = \frac{T_2}{\sin 53^O.8'} = \frac{100}{1}$$

$$\frac{T_1}{0.6} = \frac{T_2}{0.8} = 100$$

$$T_1 = 60kg$$

$$T_2 = 80kg$$