جامعة الموصل / كلية الهندسة

قسم الهندسة الكهربائية

Subject Title: Electrical Networks

Subject Code: ENET 202

Level 2: Power and Machines – Electronic & Communications

Instructor : Assistant Prof. Dr. Omar Sh. AL-Yozbaky

Course Description (15 weeks) or Outlines

Week 1	Introduction; syllabus; Advantages and Disadvantages of Electrical Networks as a different circuits .
Week 2	AC Circuit Power Analysis
Week 3	Transient Response of RL Circuit / Transient Response of RC Circuit
Week 4	Transient Response of RLC Circuit / Parallel connection
Week 5	Transient Response of RLC Circuit / Series connection
Week 6	Poly-phase Circuits & Three phase circuit analysis / Balance load
Week 7	Three phase circuit analysis / Un-Balance load
Week 8	Magnetically Coupled Circuits
Week 9	Linear Transformer
Week 10	Ideal Transformers
Week 11	Complex Frequency and the Laplace Transform
Week 12	Circuit Analysis in the S-Domain
Week 13	Frequency Response
Week 14	Two-Port Networks
Week 15	Fourier Circuit Analysis

Transient Response of RL Circuit

The Natural Response of an *RL* Circuit

The natural response of an *RL* circuit can best be described in terms of the circuit shown in Fig. 1. We assume that the independent current source generates a constant current of *Is* A, and that the switch has been in a closed position for a long time. We define the phrase *a long time* more accurately later in this section.

For now, it means that all currents and voltages have reached a constant value. Thus only constant, or dc, currents can exist in the circuit just prior to the switch's being opened, and therefore the inductor appears as a short circuit *(Ldi/dt* = 0) prior to the release of the stored energy.

Because the inductor appears as a short circuit, the voltage across the inductive branch is zero, and there can be no current in either R_0 or *R*. Therefore, all the source current Is appearing in the inductive branch. Finding the natural response requires finding the voltage and current at the terminals of the resistor after the switch has been opened, that is, after the source has been disconnected and the inductor begins releasing energy. If we let *t* = 0 denote the instant when the switch is opened, the problem becomes one of finding *v(t)* and *i(t)* for t ≥ 0. For t ≥ 0, the circuit shown in Fig.1. reduces to the one shown in Fig. 2.

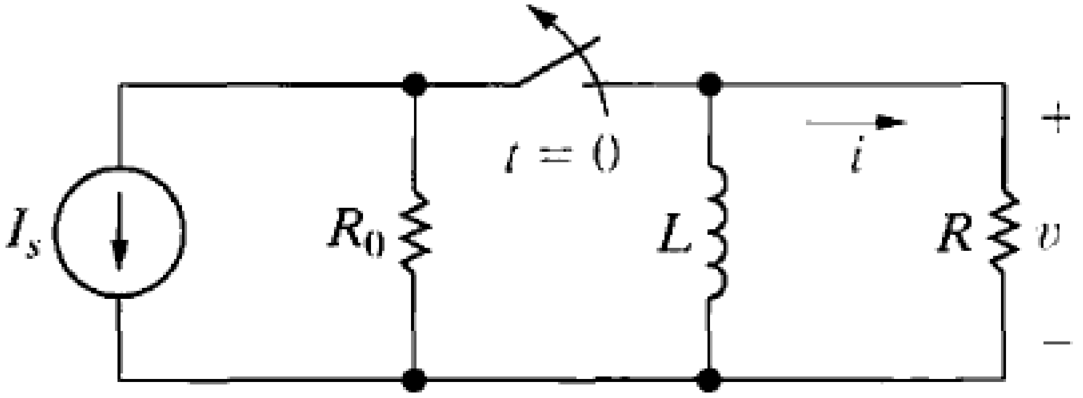


Fig.1.

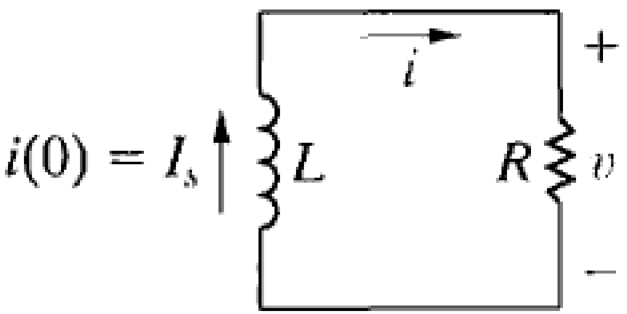


Fig.2.

Deriving the Expression for the Current

To find *i(i),* we use Kirchhoff s voltage law to obtain an expression involving *i*, *R*, and L. Summing the voltages around the closed loop gives:

$$L\frac{di}{dt} + Ri = 0, \qquad 7.1$$

where we use the passive sign convention. Equation 7.1 is known as a first order ordinary differential equation, because it contains terms involving the ordinary derivative of the unknown, that is, di/dt. The highest order derivative appearing in the equation is 1; hence the term first-order. We can go one step further in describing this equation. The coefficients in the equation, R and L, are constants; that is, they are not functions of either the dependent variable i or the independent variable t. Thus the equation can also be described as an ordinary differential equation with constant coefficients. To solve Eq. 1, we divide by L, transpose the term involving i to the right-hand side, and then multiply both sides by a differential time dt. The result is :

$$\frac{di}{dt}dt = -\frac{R}{L}i\,dt.$$

Next, we recognize the left-hand side of Eq.2 as a differential change in the current i, that is, di. We now divide through by i, getting:

$$\frac{di}{i} = -\frac{R}{L}dt.$$

We obtain an explicit expression for i as a function of f by integrating both sides of Eq.3. Using x and y as variables of integration yields:

$$\int_{i(t_0)}^{i(t)} \frac{dx}{x} = -\frac{R}{L}\int_{t_0}^{t} dy,$$

in which $i(t_0)$ is the current corresponding to time t_0 and $i(t)$ is the current corresponding to time t. Here, $t_0 = 0$. Therefore, carrying out the indicated integration gives:

$$\ln\frac{i(t)}{i(0)} = -\frac{R}{L}t.$$

Based on the definition of the natural logarithm,

$$i(t) = i(0)e^{-(R/L)t}.$$

Therefore, in the first instant after the switch has been opened, the current in the inductor remains unchanged. If we use 0− to denote the time just prior to switching, and 0+ for the time immediately following switching, then

$$i(0^-) = i(0^+) = I_0,$$

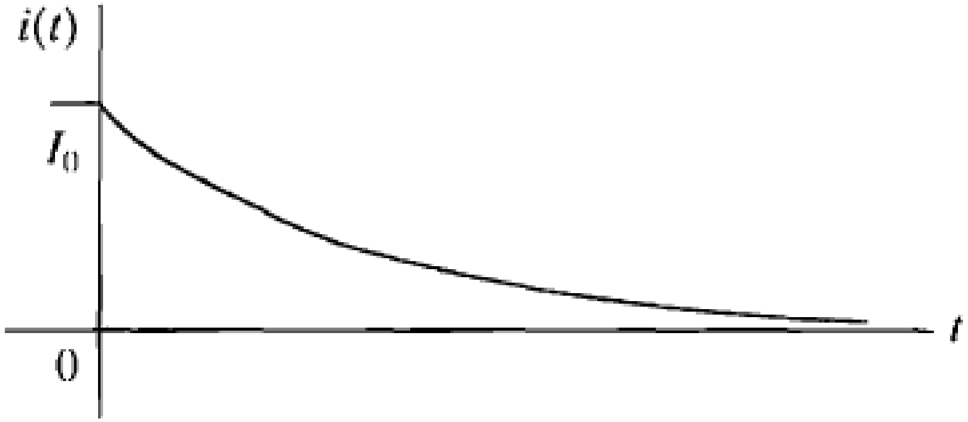
◀ **Initial inductor current**

$$i(t) = I_0e^{-(R/L)t}, \quad t \geq 0,$$

◀ **Natural response of an *RL* circuit**

$$v(0^-) = 0,$$

$$v(0^+) = I_0 R,$$



The current response for the circuit shown in Fig 3

We derive the power dissipated in the resistor from any of the following expressions:

$$p = vi, \quad p = i^2R, \quad \text{or} \quad p = \frac{v^2}{R}.$$

$$p = I_0^2 R e^{-2(R/L)t}, \quad t \geq 0^+.$$

The energy delivered to the resistor during any interval of time after the switch has been opened is:

$$w = \int_0^t p\,dx = \int_0^t I_0^2 R e^{-2(R/L)x}\,dx$$

$$= \frac{1}{2(R/L)} I_0^2 R(1 - e^{-2(R/L)t})$$

$$= \frac{1}{2} L I_0^2 (1 - e^{-2(R/L)t}), \quad t \geq 0.$$

The Significance of the Time Constant

The coefficient of t—namely, R/L —determines the rate at which the current or voltage approaches zero. The reciprocal of this ratio is the time constant of the circuit, denoted:

$$\tau = \text{time constant} = \frac{L}{R}.$$

Using the time-constant concept, we write the expressions for current, voltage, power, and energy as:

$$i(t) = I_0 e^{-t/\tau}, \quad t \geq 0,$$

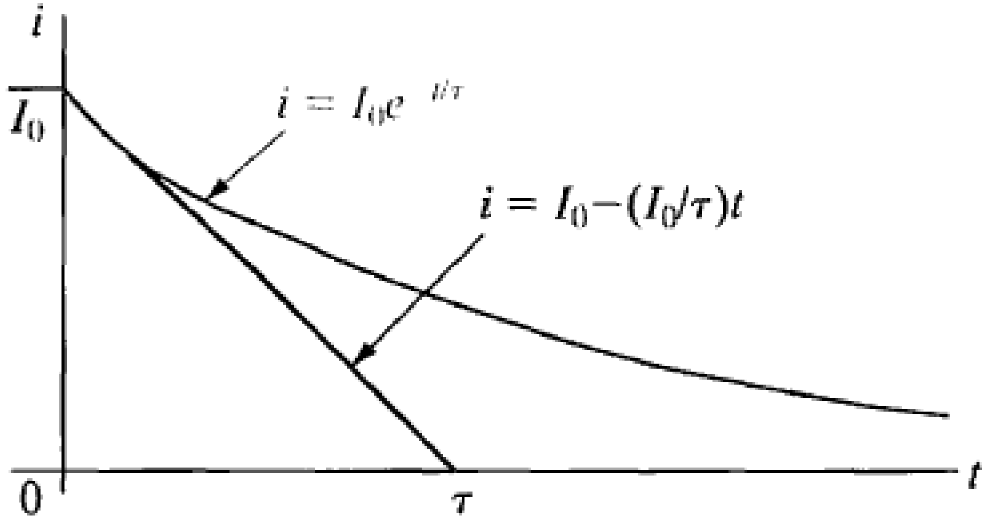
$$v(t) = I_0 R e^{-t/\tau}, \quad t \geq 0^+,$$

$$p = I_0^2 R e^{-2t/\tau}, \quad t \geq 0^+,$$

$$w = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau}), \quad t \geq 0.$$

The time constant is an important parameter for first-order circuits, so mentioning several of its characteristics is worthwhile. First, it is convenient to think of the time elapsed after switching in terms of integral multiples of τ. Thus one time constant after the inductor has begun to release its stored energy to the resistor, the current has been reduced to e−1, or approximately 0.37 of its initial value.

t	$e^{-t/\tau}$	t	$e^{-t/\tau}$
τ	3.6788×10^{-1}	6τ	2.4788×10^{-3}
2τ	1.3534×10^{-1}	7τ	9.1188×10^{-4}
3τ	4.9787×10^{-2}	8τ	3.3546×10^{-4}
4τ	1.8316×10^{-2}	9τ	1.2341×10^{-4}
5τ	6.7379×10^{-3}	10τ	4.5400×10^{-5}



Transient Response of RC Circuit

Following the same reasoning as with the RL circuit, will be satisfied when $s = -1/RC$ and therefore,

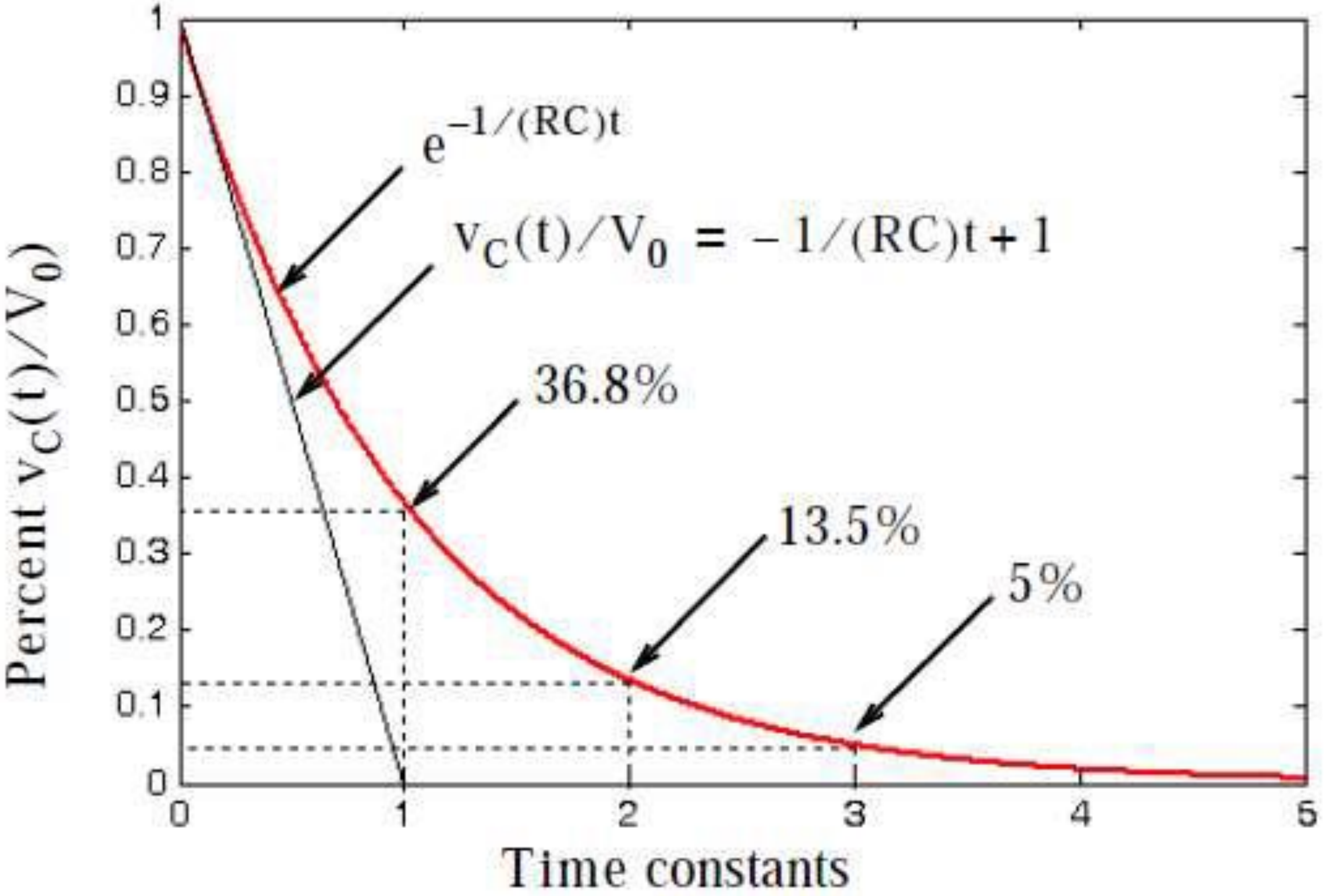
$$v_C(t) = Ae^{-(1/RC)t}$$

The constant A is evaluated from the initial condition, i.e., $v_C(0) = V_0 = Ae^0$ or $A = V_0$. Therefore, the natural response of the RC circuit is

$$\boxed{v_C(t) = V_0 e^{-(1/RC)t}}$$

$$\frac{v_C(t)}{V_0} = e^{-(1/RC)t}$$

and we sketch it as shown in Figure



From Figure we observe that at $t = 0$, $v_C/V_0 = 1$, and $i \to 0$ as $t \to \infty$

The initial rate (slope) of decay is found from the derivative of $v_C(t)/V_0$ evaluated at $t = 0$, that is,

$$\frac{d}{dt}\left(\frac{v_C}{V_0}\right)\Bigg|_{t=0} = -\frac{1}{RC}e^{-(1/RC)t}\Bigg|_{t=0} = -\frac{1}{RC}$$

and thus the slope of the initial rate of decay is $-1/(RC)$

Next, we define the *time constant* τ as the time required for $v_C(t)/V_0$ to drop from unity to zero assuming that the initial rate of decay remains constant. This constant rate of decay is represented by the straight line equation

$$\frac{v_C(t)}{V_0} = -\frac{1}{RC}t + 1$$

and at $t = \tau$, $v_C(t)/V_0 = 0$. Then,

$$0 = -\frac{1}{RC}\tau + 1$$

or

$$\boxed{\begin{array}{c}\tau = RC \\ \text{Time Constant for RC Circuit}\end{array}}$$

at $t = \tau = RC$, we obtain

$$\frac{v_C(\tau)}{V_0} = e^{-\tau/RC} = e^{-RC/RC} = e^{-1} = 0.368$$

or

$$\boxed{v_C(\tau) = 0.368V_0}$$

Therefore, *in one time constant, the response has dropped to approximately 36.8% of its initial value.*

If we express the rate of decay in time constant intervals , we find that $v_C(t)/V_0 \approx 0$ after $t = 5\tau$, that is, it reaches its final value after five time constants.

In the examples that follow, we will make use of the fact that

$$\boxed{v_C(0^-) = v_C(0) = v_C(0^+)}$$

Textbook or References

* Engineering Circuit Analysis Eight Edition (William H. Hayt) 2012

* Electric Circuits Tenth Edition (James W. Nilsson) 2015

* Fundamentals of Electric Circuits (Charles K. Alexander) 2009