جامعة الموصل / كلية الهندسة

قسم الهندسة الكهربائية

Subject Title: Digital Communications 1

Subject Code: DICOM301

Class 3: Electronic and Communication

Instructor : Dr. Saad Wasmi Osman Luhaib

Course layout (15 Weeks)

- Introduction of Random signals and Probability Theory
- Deterministic system and Random system
- The axioms of probability
- Conditional probabilities, joint probabilities and Bayes’s rule
- Cumulative distributions and probability density functions
- Probability Density Function (PDF)
- Statistical average, variance
- Covariance and correlation
- Continuous distributions and densities
- Q-function:
- Random processes spectral characteristics

Books and References

- Introduction to Analog and Digital Communications
  2nd edition, by Simon Haykin and Michael Moher

- Introduction to communication systems, 3rd edition, by Ferrel Stremler.

- John J. Proakis, “Communication Systems Engineering”, 2nd edition
  2001.

- Bruce Carlson, “Communication Systems”, 4th edition, , 2002

- A. Kattoush, “digital communication” 2005

- **Example :** In throwing a fair dice, the probability of

A = (the outcome is greater than 3)

Is P(A) = P(4) +P(5) + P(6) = ½

- The probability of B = (The outcome is even) is

P(B) = P(2)+ P(4) +P(6) =1/2

- In this case
- $P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(4)+P(6)}{1/2} = \frac{2}{3}$

Probability Density Function (PDF)

- The pdf is defined as the derivative of the CDF
- $f_X = \frac{dF_X(x)}{dx}$
- It is following that:
- $P(x_1 < X \leq x_2) = P(X \leq x_2) - P(X \leq x_1)$

$$= F_{\mathbf{x}}(x_2) - F_{\mathbf{x}}(x_1) = \int_{x_1}^{x_2} f_{\mathbf{x}}(x) \mathrm{d}x.$$

- Basic properties of pdf:
  1. $f_{\mathbf{x}}(x) \geq 0.$
  2. $\int_{-\infty}^{\infty} f_{\mathbf{x}}(x) \mathrm{d}x = 1.$
  3. In general, $P(\mathbf{x} \in \mathcal{A}) = \int_{\mathcal{A}} f_{\mathbf{x}}(x) \mathrm{d}x.$
- For discrete random variables, it is more common to define the *probability mass function* (pmf): $p_i = P(\mathbf{x} = x_i)$.
- Note that, for all i, one has $p_i \geq 0$ and $\sum_i p_i = 1$.

Ex/ Let X~N(−5,4).

a) Find P(X<0).

b) Find P(−7<X<−3).

c) Find P(X>−3|X>−5)

Solution

N(μ_x,σ^2) , μ_x=-5 , σ=2

- $P\{X < 0\} = \int_{-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}}\, e^{\left(-\frac{(x-m_x)^2}{2\sigma^2}\right)}\, dx$
- Let $z = \frac{x-\mu}{\sigma} \quad \therefore\ \sigma dz = dx$
- $P\{X < 0\} = \int_{-\infty}^{5/2} \frac{1}{\sqrt{2\pi}}\, e^{\left(-\frac{(z)^2}{2}\right)}\, dz = 1 - Q\left(\frac{5}{2}\right) = 1 - \frac{6.209}{1000} = 0.99$
- $z1 = \frac{-7+5}{2} = -1,\ z2 = \frac{-3+5}{2} = 1$
- =1-2Q(1)=1-2*0.158=0.68

Power Density Spectrum & Properties

- The power density spectrum (power spectral density) PSD.

- Parsaval's Theorem:

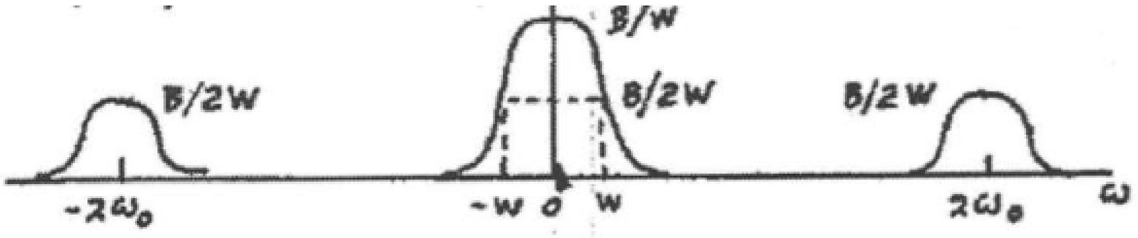
$$P_{XX} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} E[X^2(t)]dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \lim_{T\to\infty} \frac{E[|X_T(\omega)|^2]}{2T} d\omega$$

$$S_{XX}(\omega) = \lim_{T\to\infty} \frac{E[|X_T(\omega)|^2]}{2T}$$

$$P_{XX} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{XX}(\omega) d\omega$$

- The $\frac{1}{2\pi}$ factor is there because we are using ω.
- Power is related to the time average of the second moment $A\{E[X^2(t)]\}$, for w.s.s. $= \overline{X^2}$

Solution



$R_{XX}(\tau) = B\cos^2(\omega_0\tau)\exp(-W|\tau|)$

(a) expand $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) = \frac{1}{2} + \frac{1}{4}e^{j2x} + \frac{1}{4}e^{-j2x}$

- Using the given pairs and relations
- $S_{XX}(\omega) = \frac{B \cdot W}{W^2+\omega^2} + \frac{\left(\frac{B \cdot W}{2}\right)}{W^2+(\omega-2\omega_0)^2} + \frac{\left(\frac{B \cdot W}{2}\right)}{W^2+(\omega+2\omega_0)^2}$
- $S_{XX}(\omega) =$ lowpass part + bandpass part

(b) with $\omega_0 \gg W$, we have

- $P_{lowpass} = \frac{BW}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{W^2+\omega^2} = \frac{B}{2}$
- $P_{bandpass} = 2 \times \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\left(\frac{B \cdot W}{2}\right)d\omega}{W^2+(\omega-2\omega_0)^2} = \frac{B}{2}$
- Note that the bandpass and the lowpass components each preserve half of the total power

Recall:

$$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega-\omega_0)$$

$$e^{-\alpha|t|} \leftrightarrow \frac{2\alpha}{\alpha^2+\omega^2}$$

$$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$$

Example

- Autocorrelation of a Random Cosine Process.
  • Let X(t) be a random process defined by X(t) = A cos(2πft+θ) where the amplitude A and frequency f are known, but θ is uniformly distributed on the interval between 0 and 2π.
- This is a special type of random process where a single parameter θ defines the sample function for all time.
- The requirement is to find the autocorrelation of X(t)?

The autocorrelation is given by

$$R_X(t, t-\tau) = \mathbf{E}[X(t)X(t-\tau)]$$
$$= A^2\mathbf{E}[\cos(2\pi ft + \theta)\cos(2\pi f(t-\tau) + \theta)]$$

Applying the trigonometric identity $2\cos A\cos B = \cos(A-B) + \cos(A+B)$ to this relation, we obtain

$$R_X(t, t-\tau) = \frac{A^2}{2}\cos(2\pi f\tau) + \frac{A^2}{2}\mathbf{E}[\cos(4\pi ft - 2\pi f\tau + 2\theta)]$$

Since θ is uniformly distributed between 0 and 2π, we have

$$\mathrm{E}[\cos(4\pi ft - 2\pi f\tau + 2\theta)] = \frac{1}{2\pi}\int_0^{2\pi} \cos(4\pi ft - 2\pi f\tau + 2\theta)\, d\theta$$

$$= \frac{1}{4\pi}\sin(4\pi ft - 2\pi f\tau + 2\theta)\Big|_0^{2\pi}$$

$$= 0$$

Consequently, the expression for the autocorrelation reduces to

$$R_X(t, t - \tau) = \frac{A^2}{2}\cos(2\pi f\tau)$$

The autocorrelation clearly only depends upon the time difference τ in this example, and the process can be shown to be wide-sense stationary.