جامعة الموصل / كلية الهندسة

قسم الهندسة الكهربائية

Subject Title: **Numerical Analysis**

Subject Code: ENGE 320

Class 1: General

Instructor :Dr. Riyadh Zaki

Course Description (15 weeks) or Outlines

- **Concepts and role for the numerical method in engineering, approximations, and errors, the definition of Round-off error and truncation error**
- **Numerical Solution of Nonlinear Algebraic Equations (Roots of Equations):Bracketing Methods (Bisection, and False-Position method)**
- **Numerical Solution of linear algebraic equations (system)**
- **The gauss-Seidel iterative method, Gauss-Seidel iterative with the relaxation factor method. Tri-diagonal systems and its solution.**
- **Curve Fitting: Classification of Curve Fitting (Regression and Interpolation), the concepts of regression, and Least Square Criterion, Linear Regression.**
- **Introduction another to another methods (finite difference, finite volume, finite element method**

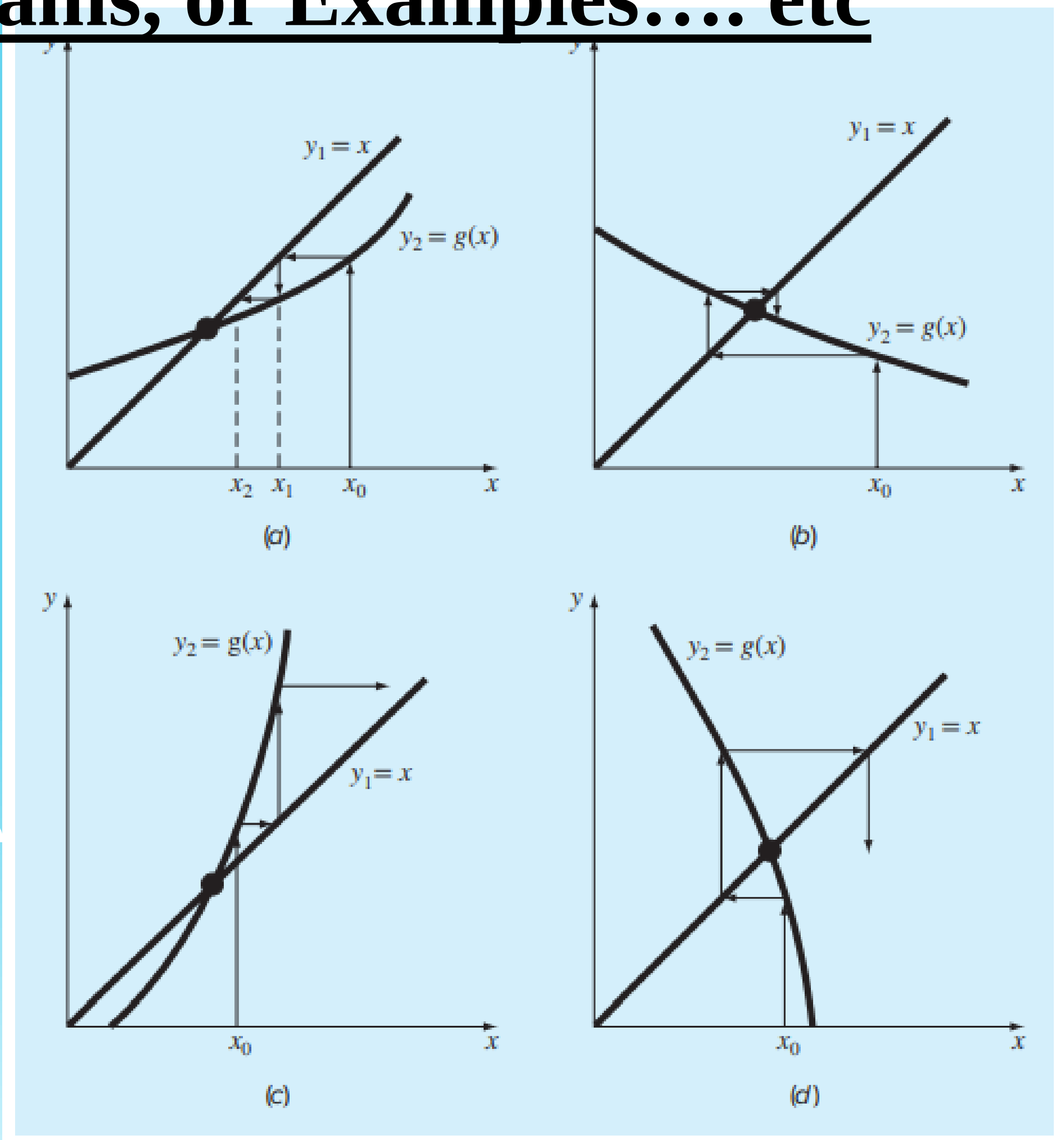
Figures, Diagrams, or Examples…. etc

- 1-Bracketing methods
  - 1-Bisection method
  - 2-False position method
- 2-Open end methods
  - 1-Newton-Raphson method
  - 2-Secant method
  - 3-Mullers method
  - 4-Fixed-point method
  - 5-Bairstows method

Figures, Diagrams, or Examples…. etc

divergence

convergence



6.1 Employ fixed-point iteration to locate the root of

$$f(x) = \sin(\sqrt{x}) - x$$

Use an initial guess of $x_0 = 0.5$ and iterate until $\varepsilon_a \le 0.01\%$. Verify that the process is linearly convergent as described at the end of Sec. 6.1.

6.1 The function can be set up for fixed-poin

$$x_{i+1} = \sin\left(\sqrt{x_i}\right)$$

Using an initial guess of $x_0 = 0.5$, the fi

$$x_1 = \sin\left(\sqrt{0.5}\right) = 0.649637$$

$$|\varepsilon_a| = \left|\frac{0.649637 - 0.5}{0.649637}\right| \times 100\% = 23\%$$

Second iteration:

$$x_2 = \sin\left(\sqrt{0.649637}\right) = 0.721524$$

$$|\varepsilon_a| = \left|\frac{0.721524 - 0.649637}{0.721524}\right| \times 100\% = 9.96\%$$

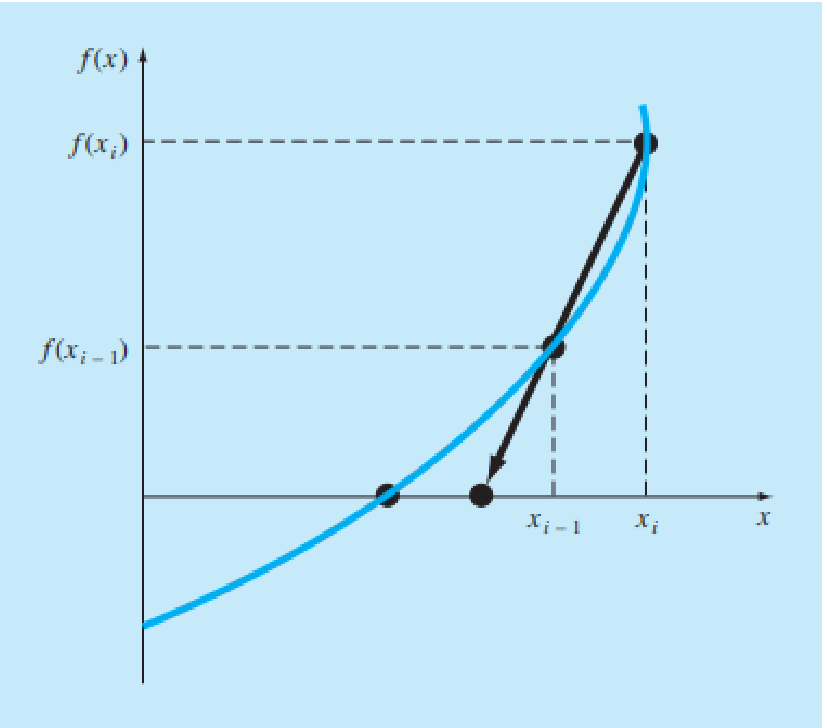
The process can be continued as tabulated below:

iteration	x_i	$\lvert\varepsilon_a\rvert$
0	0.500000	
1	0.649637	23.0339%
2	0.721524	9.9632%
3	0.750901	3.9123%
4	0.762097	1.4691%
5	0.766248	0.5418%
6	0.767772	0.1984%
7	0.768329	0.0725%
8	0.768532	0.0265%
9	0.768606	0.0097%

Thus, after nine iterations, the root is estimated to be 0.768606 with an approximate error of 0.0097%.

The Secant method:

One of the drawbacks of Newton-Raphson method is that you have to evaluate the derivative of the function. Although this is not inconvenient for polynomials and many other functions, there are certain functions whose derivatives may be extremely difficult or inconvenient to evaluate. To overcome this drawback, the derivative of the function $f(x)$ is approximated as



$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \qquad \dots (1)$$

Sub. The above equation into Newton-Raphson formula,

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})} \qquad \dots\dots\dots (2)}$$

This equation is called the secant formula.

This method now requires two initial guesses, but unlike the bisection method, the two initial guesses do not need to bracket the root of the equation. The secant method is an open method and may or may not converge.

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Chapter Two

Indirect or iterative methods

Gauss-Seidel iterative method

2022-2023

Dr. Riyadh Zaki

Wednesday, December 7, 2022

GAUSS-SEIDEL ITERATIVE METHOD:

$$x_1^{k+1} = \frac{1}{a_{11}}\left[b_1 - a_{12}x_2^k - a_{13}x_3^k - a_{14}x_4^k - \cdots\cdots\cdots - a_{1n}x_n^k\right]$$

$$x_2^{k+1} = \frac{1}{a_{22}}\left[b_2 - a_{21}x_1^{k+1} - a_{23}x_3^k - a_{24}x_4^k - \cdots\cdots\cdots - a_{2n}x_n^k\right]$$

$$x_3^{k+1} = \frac{1}{a_{33}}\left[b_3 - a_{31}x_1^{k+1} - a_{32}x_2^{k+1} - a_{34}x_4^k - \cdots\cdots\cdots - a_{3n}x_n^k\right]$$

$$x_4^{k+1} = \frac{1}{a_{44}}\left[b_4 - a_{41}x_1^{k+1} - a_{42}x_2^{k+1} - a_{43}x_3^{k+1} - \cdots\cdots\cdots - a_{4n}x_n^k\right]$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$x_n^{k+1} = \frac{1}{a_{nn}}\left[b_n - a_{n1}x_1^{k+1} - a_{n2}x_2^{k+1} - a_{n3}x_3^{k+1} - a_{n4}x_4^{k+1} - \cdots\cdots\cdots - a_{nn-1}x_{n-1}^{k+1}\right]$$

The general form of Gauss-Seidel iterative method,

for $i = 1$ to n,

$$x_i^{k+1} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{j=i-1} a_{ij}x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij}x_j^k\right] \qquad \ldots\ldots(2)$$

Curve Fitting Classification

Regression:

- Linear Regression.
- Nonlinear Regression.

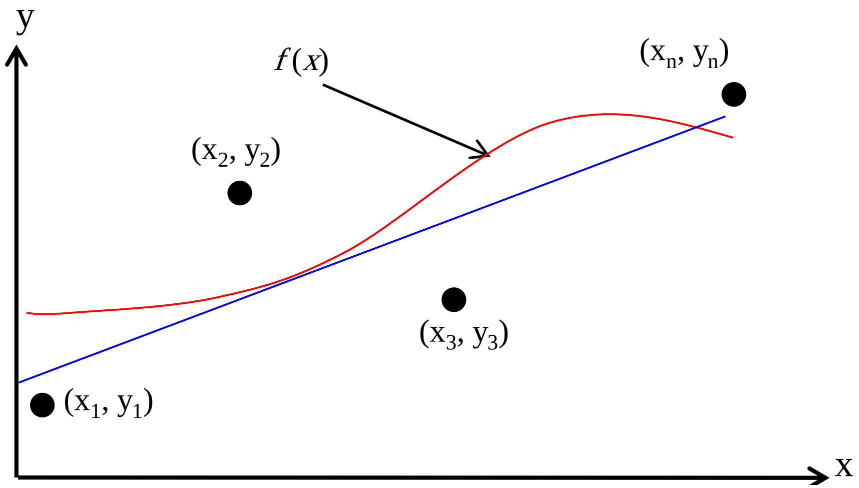

Figure. Regression

Interpolation:

- Linear Interpolation.
- Nonlinear Interpolation.

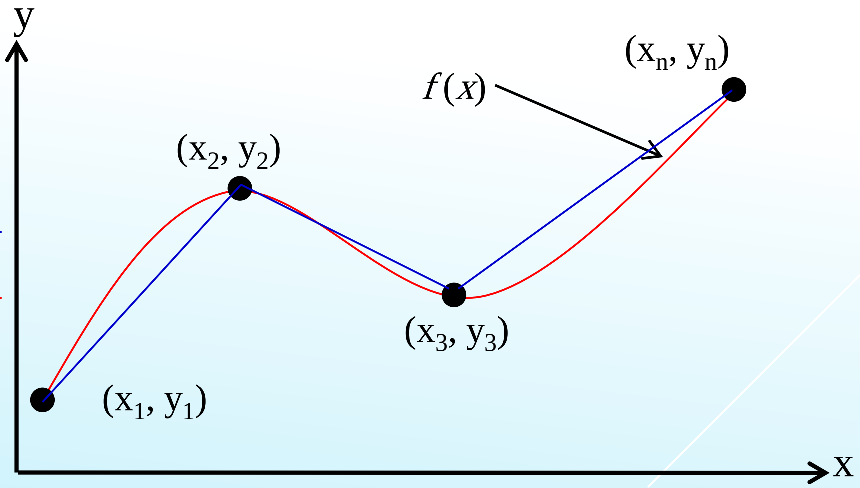

Figure. Interpolation

TEXTBOOK OR REFERENCES

- "Numerical Methods for Engineers with software and Programming Applications", 4 th Edition, Steven C. Chapra and Raymond P. Canale, 2002
- "Applied Numerical Methods with MATLAB for Engineers and Scientists" Steven C. Chapra ,2018