

Electromagnetic Theory

Magnetic Field & Magnetic Induction

Prof. Dr. Abdullah Idrees

Magnetism

Is a fundamental physical phenomenon caused by the motion of electric charges. It manifests as a force that acts on moving charges and magnetic materials. Magnetism arises from two main sources: the intrinsic magnetic moments of elementary particles and the motion of charges (such as electric currents).

Sources of Magnetism:

- *Moving Electric Charges: A current-carrying conductor produces a magnetic field around it, described by Ampère's law.*
- *Intrinsic Magnetic Moments: Particles like electrons have an intrinsic property called "spin" that contributes to magnetism.*
- *Magnetic Materials: In certain materials (e.g., iron, cobalt, nickel), atomic magnetic moments align in domains, creating strong magnetic effects.*

Types of Magnetic Materials:

- *Diamagnetic: Weakly repelled by magnetic fields (e.g., bismuth, water).*
- *Paramagnetic: Weakly attracted to magnetic fields (e.g., aluminum, oxygen).*
- *Ferromagnetic: Strongly attracted to magnetic fields and can retain magnetization (e.g., iron, cobalt, nickel).*
- *Antiferromagnetic: Magnetic moments align in opposite directions, canceling each other out (e.g., manganese oxide).*
- *Ferrimagnetic: Magnetic moments align oppositely but do not fully cancel, resulting in net magnetism (e.g., magnetite).*

Magnetostatics

Is the branch of physics that studies magnetic fields in systems where the currents are steady (time-independent)

Magnetostatics is a subset of electromagnetism, and its principles are valid when the electric fields vary slowly enough that the displacement current can be neglected ($\partial E/\partial t \approx 0$). This makes it distinct from electrodynamics, which deals with time-varying fields and electromagnetic waves.

Electromagnetism

is a branch of physics that studies the relationship between electric fields, magnetic fields, and electric charges. It describes how these fields interact and how they influence each other over time. Electromagnetism is governed by a set of fundamental laws known as Maxwell's Equations, which unify electricity and magnetism into a single framework. Magnetic Field & Magnetic Induction

Laws of Magnetism:

- **Biot-Savart Law:** *Describes the magnetic field generated by a small current element.*

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- **Ampère's Law:** Relates the circulation of the magnetic field to the current and displacement field.

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

- **Lorentz Force:** A moving charge in a magnetic field experiences a force given by: ($\mathbf{F}_{\text{magnetic}} = q(\mathbf{v} \times \mathbf{B})$) where q is the charge, \mathbf{v} is the velocity, and \mathbf{B} is the magnetic field.

Magnetic Poles:

- Magnetic fields have a north and south pole.
- Unlike electric charges, magnetic monopoles have not been observed, meaning magnetic field lines always form closed loops.

Magnetic fields

The magnetic field is a vector field that describes the influence of magnetic forces in a region of space. are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits. The magnetic field \mathbf{B} is defined in terms of force on moving charge in the Lorentz force law. The interaction of magnetic field with charge leads to many practical applications. Magnetic field sources are essentially dipolar in nature, having a north and south magnetic pole. The SI unit for magnetic field is the Tesla, which can be seen from the magnetic part of the Lorentz force law to be composed of (Newton x second)/(Coulomb x meter). A smaller magnetic field unit is the Gauss (1 Tesla = 10000 Gauss).

Laws of Magnetism:

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It will be studied in detail.

Magnetic Field Sources

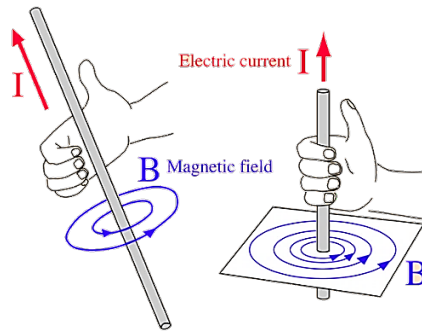
Magnetic Field due to Current in a Wire

The magnetic field lines around a long wire which carries an electric current form concentric circles around the wire. The direction of the magnetic field is perpendicular to the wire and is in the direction the fingers of your right hand would curl if you wrapped them around the wire with your thumb in the direction of the current.

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A straight, infinitely long wire carrying current I generates a magnetic field that circles the wire. The magnitude and direction of the magnetic field can be described as:

Magnitude:

$$B = \frac{\mu_0 I}{2\pi r}$$

where:

- B is the magnetic field in teslas (T),
- $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$ is the permeability of free space,
- I is the current through the wire (in amperes),
- r is the distance from the wire (in meters).

Example

A long, straight wire carries a current of $I=10 \text{ A}$. Find the magnetic field at a point (5 cm) away from the wire.

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \frac{(4\pi \times 10^{-7}) \cdot 10}{2\pi \cdot 0.05}$$

$$B = \frac{4 \cdot 10^{-7} \cdot 10}{0.1}$$

$$B = \frac{4 \cdot 10^{-6}}{0.1} = 4 \times 10^{-5} \text{ T}$$

$$B = 40 \mu\text{T}$$

*Once the magnetic field has been calculated, the **magnetic force** expression can be used to calculate the force. The direction is obtained from the right-*

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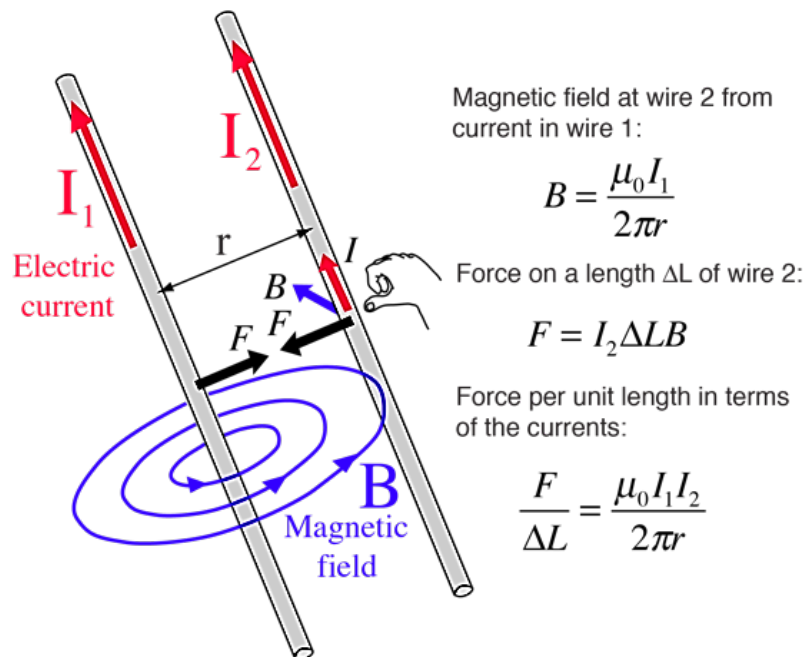
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hand rule. Note that two wires carrying current in the same direction attract each other, and they repel if the currents are opposite in direction. The calculation below applies only to long straight wires, but is at least useful for estimating forces in the ordinary circumstances of short wires. Once you have calculated the force on wire 2, of course the force on wire 1 must be exactly the same magnitude and in the opposite direction according to Newton's third law.

Magnetic Force Between Wires

The force per unit length (F/L) between two long, straight, parallel wires separated by a distance r , each carrying a current I_1 and I_2 is given by:

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

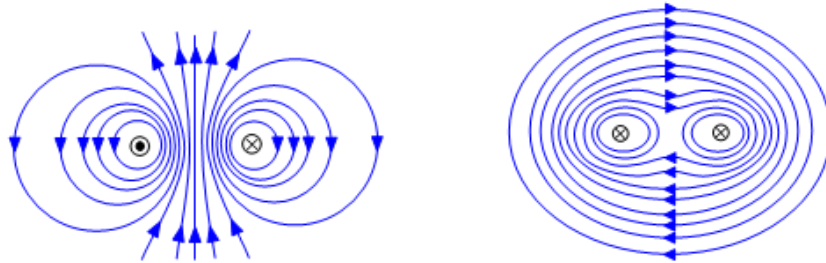


- If the currents I_1 and I_2 flow in the **same direction**, the wires **attract** each other.

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- If the currents flow in **opposite directions**, the wires **repel** each other.



Example:

Two parallel wires are separated by a distance of $r=0.1$ m. Each wire carries a current of $I_1=I_2=5$ A. Find the magnetic force per unit length between the wires. Is the force attractive or repulsive?

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$
$$\frac{F}{L} = \frac{(4\pi \times 10^{-7}) \cdot 5 \cdot 5}{2\pi \cdot 0.1}$$
$$\frac{F}{L} = 50 \mu\text{N/m}$$

Since the currents flow in the **same direction**, the force is **attractive**.

Magnetic Field of Current Loop

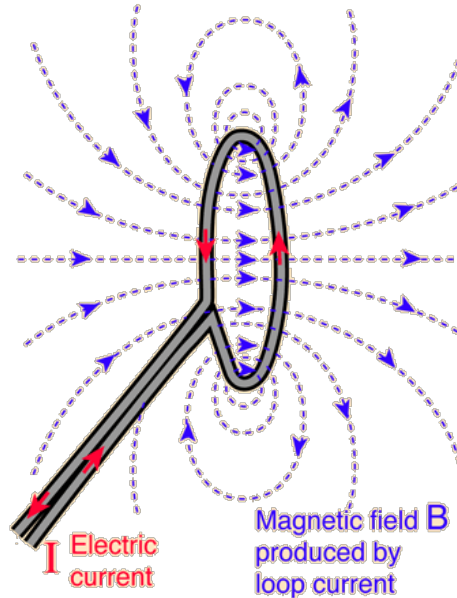
Examining the direction of the magnetic field produced by a current-carrying segment of wire shows that all parts of the loop contribute magnetic field in the same direction inside the loop.

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For a circular loop of radius R carrying a current I , the magnetic field at the center of the loop is:

$$B = \frac{\mu_0 I}{2R}$$

B is the magnetic field at the center (in Tesla's, T),

R is the radius of the loop (in meters).

At a distance x along the axis of the loop, the magnetic field is given by:

$$B_x = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

Example:

A circular loop of radius $R=0.1$ m carries a current $I=5$ A.

Find:

1. The magnetic field at the center of the loop.
2. The magnetic field along the axis at a distance $x=0.2$ m from the center.

Part 1: Magnetic Field at the Center

$$B = \frac{\mu_0 I}{2R}$$

$$B = \frac{(4\pi \times 10^{-7}) \cdot 5}{2 \cdot 0.1}$$

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$$B = 10^{-5} \pi \text{ T}$$

Part 2: Magnetic Field Along the Axis

$$B_x = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

$$B_x = \frac{(4\pi \times 10^{-7}) \cdot 5 \cdot (0.1)^2}{2 \cdot [(0.1)^2 + (0.2)^2]^{3/2}}$$

$$B_x = \frac{\pi \times 10^{-8}}{0.01118} \approx 2.81 \times 10^{-7} \text{ T or } 0.281 \mu\text{T}.$$

Electric current in a circular loop creates a magnetic field which is more concentrated in the center of the loop than outside the loop. Stacking multiple loops concentrates the field even more into what is called a solenoid.

Solenoid

A long straight coil of wire can be used to generate a nearly uniform magnetic field similar to that of a bar magnet. Such coils, called solenoids, have an enormous number of practical applications. The field can be greatly strengthened by the addition of an iron core. Such cores are typical in electromagnets.

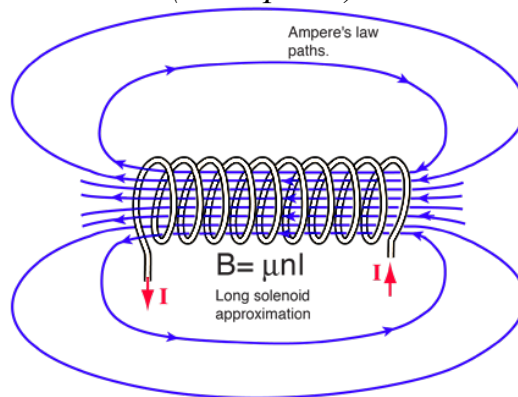
Expression for the Magnetic Field

The magnetic field inside a solenoid is given by:

$$B = \mu_0 n I$$

Where:

- *B: Magnetic field inside the solenoid (in teslas, T),*
- *n: Number of turns per unit length ($n = N/L$ where N is the total number of turns and L is the length of the solenoid),*
- *I: Current through the solenoid (in amperes).*



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Properties of the Magnetic Field

1. Inside the Solenoid:

- The field is strong and nearly uniform.
- Direction is along the solenoid's axis (use the right-hand rule: curl your fingers in the direction of current flow, and your thumb points in the direction of the field).

2. Outside the Solenoid:

- The field is very weak and approximates zero for long solenoids (practically negligible).

Example:

A solenoid has $N=500$ turns, a length of $L=0.25$ m, and carries a current of $I=2$ A. Find the magnetic field inside the solenoid.

$$B = \mu_0 n I$$
$$n = \frac{N}{L} = \frac{500}{0.25} = 2000 \text{ turns/m.}$$
$$B = (4\pi \times 10^{-7}) \cdot 2000 \cdot 2$$
$$B \approx 2.51 \times 10^{-3} \text{ T.}$$

Magnetic Dipole Moment

The magnetic dipole moment (m) quantifies the strength and orientation of a magnetic source. For a current-carrying loop, it is given by:

$$m = I \cdot A$$

Where:

- I is the current through the loop (in amperes),
- A is the area of the loop (in square meters).

Thus, the unit of magnetic dipole moment is:
Ampere-meter² ($A \cdot m^2$).

Example:

A circular loop of radius $r=0.1$ m carrying a current $I=2$ A has a magnetic dipole moment:

$$m = I \cdot A = 2 \cdot \pi (0.1)^2 = 0.0628 \text{ A} \cdot \text{m}^2.$$

Common Units of Magnetic Moment:

1. the SI unit naturally becomes: $A \cdot m^2$

2. **In the CGS System:**

The unit is **erg per gauss (erg/G)**

Conversion to SI: $1 \text{ erg/G} = 10^{-3} \text{ A} \cdot \text{m}^2$

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3. For Atomic and Molecular Systems:

he unit often used is the **Bohr magneton (μ_B)**, which is:

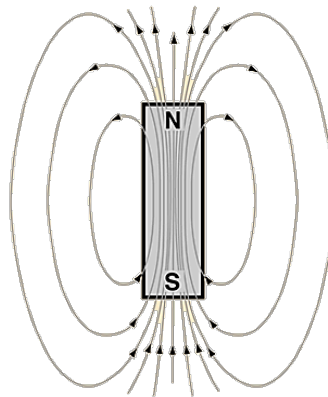
$$\mu_B = \frac{e\hbar}{2m_e}$$

Where e is the charge of an electron, \hbar is the reduced Planck constant, and m_e is the mass of an electron.

$$1 \mu_B \approx 9.274 \times 10^{-24} \text{ A} \cdot \text{m}^2$$

Bar Magnet

The magnetic field lines of a bar magnet can be traced out with the use of a compass. The needle of a compass is itself a permanent magnet and the north indicator of the compass is a magnetic north pole. The north pole of a magnet will tend to line up with the magnetic field, so a suspended compass needle will rotate until it lines up with the magnetic field. Unlike magnetic poles attract, so the north indicator of the compass will point toward the south pole of a magnet. In response to the Earth's magnetic field, the compass will point toward the geographic North Pole of the Earth because it is in fact a magnetic south pole. The magnetic field lines of the Earth enter the Earth near the geographic North Pole.



The lines of magnetic field from a bar magnet form closed lines. By convention, the field direction is taken to be outward from the North pole and in to the South pole of the magnet. Permanent magnets can be made from ferromagnetic materials.

As can be visualized with the magnetic field lines, the magnetic field is strongest inside the magnetic material. The strongest external magnetic fields are near the poles. A magnetic north pole will attract the south pole of another magnet, and repel a north pole.

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Magnetic Field of a Bar Magnet

Magnetic Dipole Moment (m):

A bar magnet can be thought of as a magnetic dipole with a magnetic moment m , which points from its south pole to its north pole. The strength of the bar magnet is defined by its magnetic moment, given by:

$$\mathbf{m} = m_p \cdot \mathbf{l}$$

where:

- m_p : Magnetic pole strength,
- l : Distance between the poles (length of the magnet).

Magnetic Field Around a Bar Magnet:

1. *The magnetic field outside the bar magnet resembles the field lines of a magnetic dipole.*
2. *The field lines emerge from the north pole and curve back to enter the south pole, forming closed loops.*
3. *At a point along the axis (axial line) of the magnet, the magnetic field is stronger than at a point on the perpendicular bisector (equatorial line).*

Magnetic Field Formulas

1. **On the Axial Line** (line extending through the poles):

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \frac{2m}{r^3}$$

m : Magnetic moment

($m = M \cdot V$), where M is magnetization and V is the volume of the magnet),

r : Distance from the center of the magnet.

- On the Equatorial Line** (perpendicular to the axis, passing through the center):

$$B_{\text{equatorial}} = \frac{\mu_0}{4\pi} \frac{m}{r^3}$$

Properties of a Bar Magnet

1. **Poles:**

- *The north pole is the source of field lines.*
- *The south pole is where the field lines terminate.*

2. **Magnetic Field Lines:**

- *Field lines are denser near the poles, indicating a stronger field.*
- *The lines never cross or intersect.*

3. **Torque in a Magnetic Field:** *When placed in an external magnetic field B , a bar magnet experiences a torque:*

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$$

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This torque tends to align the magnet's dipole moment m with the external field.

Example:

A bar magnet has a magnetic moment of $m=1.5 \text{ A}\cdot\text{m}^2$. Calculate the magnetic field strength at a distance $r=0.1 \text{ m}$ from the center of the magnet along the axial line.

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \frac{2m}{r^3}$$

$$B_{\text{axial}} = \frac{(4\pi \times 10^{-7})}{4\pi} \cdot \frac{2 \cdot 1.5}{(0.1)^3}$$

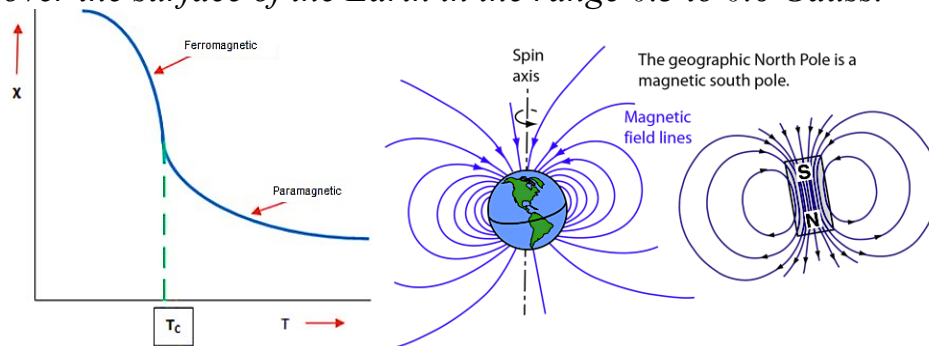
$$B_{\text{axial}} = 3 \times 10^{-4} \text{ T}$$

Magnetic Field of the Earth

The Earth's magnetic field is similar to that of a bar magnet tilted 11 degrees from the spin axis of the Earth. The problem with that picture is that the Curie temperature of iron is about 770° C .

The Earth's core is hotter than that and therefore not magnetic. So how did the Earth get its magnetic field?

Magnetic fields surround electric currents, so we surmise that circulating electric currents in the Earth's molten metallic core are the origin of the magnetic field. A current loop gives a field similar to that of the earth. The magnetic field magnitude measured at the surface of the Earth is about half a Gauss and dips toward the Earth in the northern hemisphere. The magnitude varies over the surface of the Earth in the range 0.3 to 0.6 Gauss.



Earth's Magnetic Dipole Moment: *Earth's magnetic dipole moment is approximately:*

$$m_{\text{Earth}} \approx 8 \times 10^{22} \text{ A} \cdot \text{m}^2$$

As a comparison, a loop of current $I=3 \text{ A}$ with an area $A=0.02 \text{ m}^2$ has:

$$m = I \cdot A = 3 \cdot 0.02 = 0.06 \text{ A} \cdot \text{m}^2.$$