
Magnetism II

Magnetic Field Strength H

The magnetization of a material is expressed in terms of density of net magnetic dipole moments m in the material. We define a vector quantity called the magnetization M

Then the total magnetic field B in the material is given by:

$$\mathbf{B} = \mathbf{B_0} + \mu_0 \mathbf{M}$$

where μ_0 is the magnetic permeability of space and B_0 is the externally applied magnetic field.

When magnetic fields inside of materials are calculated using Ampere's law or the Biot-Savart law, then the μ_0 in those equations is typically replaced by just μ with the definition:

$$\mu = K_m \mu_0$$

where K_m is called the relative permeability. If the material does not respond to the external magnetic field by producing any magnetization, then $K_m = 1$. Another commonly used magnetic quantity is the magnetic susceptibility which specifies how much the relative permeability differs from one.

Magnetic Susceptibility (χ)

***Magnetic susceptibility** is a dimensionless quantity that indicates how a material responds to an applied magnetic field. It shows whether a material becomes magnetized and the degree of its magnetization in response to an external magnetic field.*

$$\chi = \frac{M}{H}$$

Where:

- *χ: Magnetic susceptibility (dimensionless),*
- *M: Magnetization (magnetic moment per unit volume, A/m),*
- *H: Applied magnetic field strength (A/m).*

Types of Magnetic Materials Based on χ:

1. ***Diamagnetic Materials ($\chi < 0$):***
   - *Weakly repelled by a magnetic field.*
   - *Magnetic susceptibility is small and negative.*
   - *Examples: Copper, gold, bismuth.*
2. ***Paramagnetic Materials ($\chi > 0$):***
   - *Weakly attracted to a magnetic field.*

- *Magnetic susceptibility is small and positive.*
- *Examples: Aluminum, oxygen, platinum.*

3. ***Ferromagnetic Materials ($\chi \gg 1$):***

- *Strongly attracted to a magnetic field.*
- *Magnetic susceptibility is very large and positive.*
- *Examples: Iron, cobalt, nickel.*

*Magnetic susceptibility is related to the **relative permeability** (μ_r) of a material as:*

$$\mu_r = 1 + \chi$$

Example

A paramagnetic material is placed in an external magnetic field of $H=500$ A/m. If the material has a magnetic susceptibility $\chi=0.002$, the magnetization M of the material.

$$M = \chi \cdot H$$

$$M = 0.002 \cdot 500 = 1 \text{ A/m}.$$

Total Magnetic Field (B)

The magnetic fields generated by currents and calculated from Ampere's Law or the Biot-Savart Law are characterized by the magnetic field B measured in Tesla. But when the generated fields pass through magnetic materials which themselves contribute internal magnetic fields, ambiguities can arise about what part of the field comes from the external currents and what comes from the material itself. It has been common practice to define another magnetic field quantity, usually called the "magnetic field strength" designated by H. It can be defined by the relationship

$$\boldsymbol{H = B/\mu_m}$$

and has the value of unambiguously designating the driving magnetic influence from external currents in a material, independent of the material's magnetic response. The relationship for B can be written in the equivalent form

$$\boldsymbol{B = \mu_0(H + M)}$$

H and M will have the same units, amperes/meter. To further distinguish B from H, B is sometimes called the magnetic flux density or the magnetic *induction. The quantity M in these relationships is called the magnetization of the material.*

*The unit for the magnetic field strength **H** can be derived from its relationship to the magnetic field **B**, $\mathbf{B} = \mu\mathbf{H}$. Since the unit of magnetic permeability μ is N/A^2, then the unit for the magnetic field strength is:*

$$T/(N/A^2) = (N/Am)/(N/A^2) = A/m$$

An older unit for magnetic field strength is the oersted: 1 A/m = 0.01257 Oersted

*For **linear magnetic materials**, the magnetization is proportional to the applied field:*

$$M=\chi_m H$$

*χ_m is the **magnetic susceptibility** of the material (dimensionless), which indicates how much the material becomes magnetized in response to the magnetic field.*
Using this, we can rewrite the total field B as:

$$B = \mu_0 (1+\chi_m)H = \mu H$$

Where:

$$\mu = \mu_0 (1+ \chi_m),$$

*is the **magnetic permeability** of the material, which describes how easily the material can be magnetized.*

Example:
Let's say we have a ferromagnetic material with a magnetic susceptibility χ_m=500 placed in an external magnetic field H=1000 A/m. The total magnetic field B inside the material would be:

$$\mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H}$$

$$\mathbf{B} = (4\pi \times 10^{-7}) \cdot (1 + 500) \cdot 1000$$

$$\mathbf{B} = (4\pi \times 10^{-7}) \cdot 501 \cdot 1000$$

$$\mathbf{B} \approx 6.28 \times 10^{-1}\,\mathrm{T} = 0.628\,\mathrm{T}.$$

Lorentz Force Law

*The **Lorentz Force Law** describes the force experienced by a charged particle in the presence of electric and magnetic fields. It is fundamental to understanding the behavior of charged particles in electromagnetic fields.*

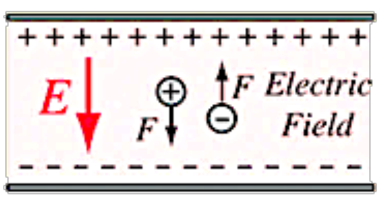
Both the electric field and magnetic field can be defined from the Lorentz force law:

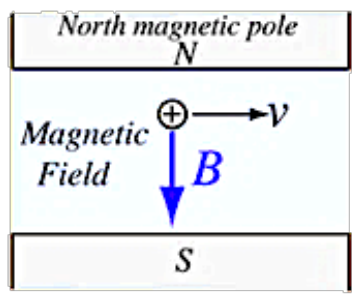
$$\boldsymbol{F} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$$

where:

***F**: Force acting on the particle (vector).*
q: Electric charge of the particle.
***E**: Electric field (vector).*
***B**: Magnetic field (vector).*
***v**: Velocity of the particle (vector).*

The electric force is straightforward, being in the direction of the electric field if the charge q is positive, but the direction of the magnetic part of the force is given by the right-hand rule.





The right-hand rule is a useful mnemonic for visualizing the direction of a magnetic force as given by the Lorentz force law. The diagrams above are two of the forms used to visualize the force on a moving positive charge. The force is in the opposite direction for a negative charge moving in the direction shown. One fact to keep in mind is that the magnetic force is perpendicular to both the magnetic field and the charge velocity, but that leaves two possibilities. The right-hand rule just helps you pin down which of the two directions applies.
The magnetic force exerted on a moving charge takes the form of a vector product.

Electric Force (qE):
A charged particle in an electric field experiences a force in the direction of the field for a positive charge, and opposite to the field for a negative charge.

Example:

A particle with charge q=2 μC is placed in an electric field $\boldsymbol{E}=5\,\boldsymbol{a}_x$ V/m. Find the force on the particle.

$$\mathbf{F} = q\mathbf{E}$$

$$\mathbf{F} = (2\times10^{-6})\cdot(5\,\mathbf{a}_x) = 10\times10^{-6}\,\mathbf{a}_x = 10\,\mu N\,\mathbf{a}_x.$$

Magnetic Force (qv×B):

The magnetic field B is defined from the Lorentz Force Law, and specifically from the magnetic force on a moving charge:

The implications of this expression include:

I. *The direction of the force is given by the right-hand rule. The force relationship above is in the form of a vector product.*
II. *The magnetic force depends on the particle's velocity and is perpendicular to both $\vec{v}$ and $\vec{B}$ (determined by the right-hand rule).*
III. *Magnetic force does no work, as it only changes the direction of motion, not the speed.*
IV. *The magnitude of the force is $F = qvB\sin\theta$ where θ is the angle < 180 degrees between the velocity and the magnetic field. This implies that the magnetic force on a stationary charge or a charge moving parallel to the magnetic field is zero.*

Example:

An electron ($q=-1.6\times10^{-19}$) moves with a velocity $v=2\,\boldsymbol{a}_x+3\,\boldsymbol{a}_y$ m/s in a magnetic field $\boldsymbol{B}=0.1\,\boldsymbol{a}_z$ T. The magnetic force is:

$$\boldsymbol{F} = q(\boldsymbol{v}\times\boldsymbol{B})$$

$$\boldsymbol{v}\times\boldsymbol{B} = \begin{vmatrix} \mathbf{a_x} & \mathbf{a_y} & \mathbf{a_z} \\ 2 & 3 & 0 \\ 0 & 0 & 0.1 \end{vmatrix} = 0.3\,\mathbf{a_x} - 0.2\,\mathbf{a_y}$$

$$\boldsymbol{F} = (-1.6\times10^{-19})(0.3\,\mathbf{a_x} - 0.2\,\mathbf{a_y}) = -4.8\times10^{-20}\,\mathbf{a_x} + 3.2\times10^{-20}\,\mathbf{a_y}\ \mathrm{N}$$

Example:

A particle with charge $q=2\times10^{-19}$ C and mass $m=3\times10^{-25}$ kgm is moving with velocity v=4 ax+3 ay m/s through a region with:

- *Electric field $E=100\,a_x+200\,a_y$ N/C*
- *Magnetic field $B=0.5\,a_z$ T*

find the total force on the particle, as well as the acceleration of the particle.

$$\boldsymbol{F_E} = q\boldsymbol{E}.$$

$$\boldsymbol{F_E} = (2 \times 10^{-19})(100\,\mathbf{a_x} + 200\,\mathbf{a_y})$$

$$\boldsymbol{F_E} = 2 \times 10^{-17}\,\mathbf{a_x} + 4 \times 10^{-17}\,\mathbf{a_y}\,\mathrm{N}$$

$$\boldsymbol{F_B} = q(\boldsymbol{v} \times \boldsymbol{B}).$$

$$\boldsymbol{v} \times \boldsymbol{B} = \begin{vmatrix} \mathbf{a_x} & \mathbf{a_y} & \mathbf{a_z} \\ 4 & 3 & 0 \\ 0 & 0 & 0.5 \end{vmatrix} = (1.5\,\mathbf{a_x} - 2\,\mathbf{a_y})\,\mathrm{m/s \cdot T}$$

$$\boldsymbol{F_B} = (2 \times 10^{-19})(1.5\,\mathbf{a_x} - 2\,\mathbf{a_y})$$

$$\boldsymbol{F_B} = 3 \times 10^{-19}\,\mathbf{a_x} - 4 \times 10^{-19}\,\mathbf{a_y}\,\mathrm{N}$$

The total force is the vector sum of the electric and magnetic forces:

$$\boldsymbol{F} = \boldsymbol{F_E} + \boldsymbol{F_B}$$

$$\boldsymbol{F} = (2 \times 10^{-17}\,\mathbf{a_x} + 4 \times 10^{-17}\,\mathbf{a_y}) + (3 \times 10^{-19}\,\mathbf{a_x} - 4 \times 10^{-19}\,\mathbf{a_y})$$

$$\boldsymbol{F} = (2.03 \times 10^{-17}\,\mathbf{a_x} + 3.96 \times 10^{-17}\,\mathbf{a_y})\,\mathrm{N}$$

*The acceleration **a** is given by: **a**=**F**/m*

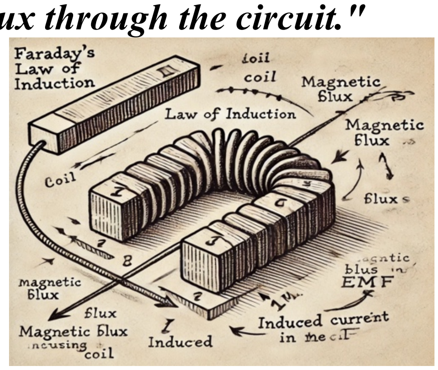
$$\boldsymbol{a} = \frac{2.03 \times 10^{-17}\,\mathbf{a_x} + 3.96 \times 10^{-17}\,\mathbf{a_y}}{3 \times 10^{-25}}$$

$$\boldsymbol{a} = 6.77 \times 10^{7}\,\mathbf{a_x} + 1.32 \times 10^{8}\,\mathbf{a_y}\,\mathrm{m/s^2}$$

Faraday's Law

The original text of Faraday's Law of Induction, as written by Michael Faraday in 1831, can be summarized as follows:

"The induction of an electromotive force in a circuit is proportional to the rate of change of the magnetic flux through the circuit."



Faraday's law was originally described in terms of the behavior of magnetic fields and the generation of electric currents. The mathematical form of the law, as stated in modern terms, is:

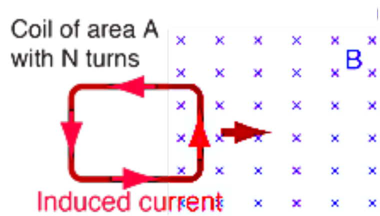
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

where:

- *ε is the induced electromotive force (emf),*
- *Φ_B is the magnetic flux,*
- *t is time.*

The negative sign represents Lenz's Law, indicating that the direction of the induced emf opposes the change in magnetic flux.

i.e. Any change in the magnetic environment of a coil of wire will cause a voltage (emf) to be "induced" in the coil. No matter how the change is produced, the voltage will be generated. The change could be produced by changing the magnetic field strength, moving a magnet toward or away from the coil, moving the coil into or out of the magnetic field, rotating the coil relative to the magnet, etc.



Example:

*Consider a **conductive coil** of wire with **N turns**, moving at a constant velocity through a **uniform magnetic field**. The coil is positioned so that its plane is perpendicular to the magnetic field lines. As the coil moves through the field, the amount of magnetic flux passing through it changes, which induces an electromotive force (emf).*

***Magnetic Flux Change**: As the coil moves through the magnetic field, the area through which the magnetic field lines pass changes. This causes the magnetic flux Φ_B to change.*

The magnetic flux is given by:

$$\Phi_B = B \cdot A$$

***Induced EMF**: According to Faraday's Law, the induced emf ε is proportional to the rate of change of magnetic flux $d\Phi_B/dt$.*

This induced emf drives a current through the coil if the coil is connected in a circuit.

The emf induced in the coil is given by:

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

where N is the number of turns in the coil.
The negative sign indicates the direction of the induced emf (as per Lenz's Law), opposing the change in flux.

Example:

Let the magnetic field strength B=0.5 T. The coil has N=100 turns. The area of the coil A=0.01 m2. The coil moves at a velocity such that the magnetic flux changes in 0.1 seconds. calculate the change in flux

$$\Delta\Phi_B = B \cdot A = 0.5 \cdot 0.01 = 0.005\,\text{Wb (Weber)}$$

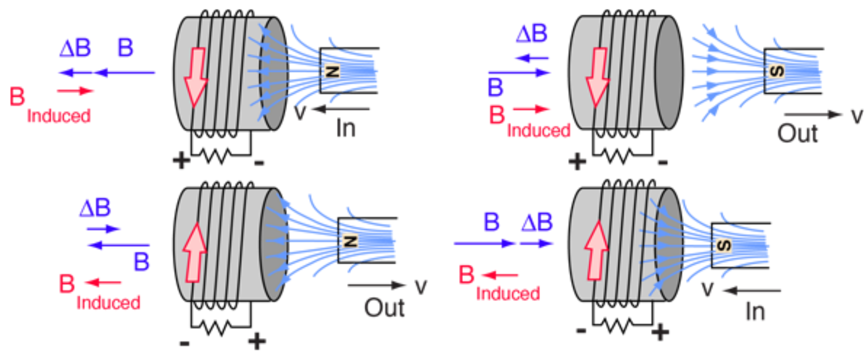
$$\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t} = -100 \times \frac{0.005}{0.1} = -5\,\text{V}$$

Lenz's Law

Lenz's Law *is a fundamental principle that describes the direction of the induced current (or electromotive force, emf) in response to a change in magnetic flux. It states:*

"The direction of the induced emf (and thus the current in a closed loop) is such that it opposes the change in magnetic flux that produced it."

When an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change which produces it. The induced magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant. In the examples below, if the B field is increasing, the induced field acts in opposition to it. If it is decreasing, the induced field acts in the direction of the applied field to try to keep it constant.



Examples:

1. ***Magnet Moving Towards a Coil:***
   - *If a magnet is moved towards a coil, the magnetic flux through the coil increases.*
   - *According to Lenz's Law, the induced current in the coil will flow in such a way that it produces a magnetic field that opposes the approaching magnet (i.e., it creates a magnetic field that repels the magnet).*
2. ***Magnet Moving Away from a Coil:***
   - *If the magnet moves away from the coil, the magnetic flux through the coil decreases.*
   - *The induced current will flow in the opposite direction, creating a magnetic field that tries to "pull" the magnet back (i.e., it attracts the magnet).*
3. ***Dropping a Magnet Through a Conductive Tube***
   - *If you drop a magnet through a long conductive tube (e.g., copper or aluminum), the magnetic flux through the tube changes as the magnet moves. According to Lenz's Law:*
   - *As the magnet enters the tube, the magnetic field inside the tube increases.*
   - *The tube will induce a current that creates a magnetic field opposing the motion of the magnet (it repels the magnet).*
   - *This resistance slows down the magnet's fall due to the opposing force exerted by the induced current. This is a practical demonstration of Lenz's Law at work, where the induced current generates a magnetic force that opposes the magnet's motion.*

Biot-Savart Law

The Biot-Savart Law is a fundamental law in electromagnetism that describes the magnetic field generated by an electric current. It gives the magnetic field at a point in space due to a small segment of current-carrying wire.

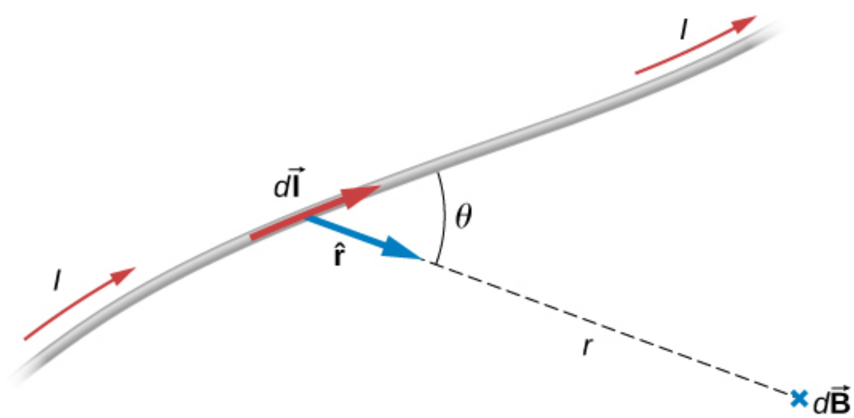
Biot-Savart Law relates magnetic fields to the currents which are their sources. In a similar manner, Coulomb's law relates electric fields to the point charges which are their sources. Finding the magnetic field resulting from a current distribution involves the vector product, and is inherently a calculus problem when the distance from the current to the field point is continuously changing.

The law is particularly useful for calculating the magnetic field due to arbitrary current distributions.

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I\, d\mathbf{l} \times \hat{\mathbf{r}}}{|\mathbf{r}|^2}$$

Where:

- ***B(r)** is the magnetic field at a point r,*
- *I is the current in the wire,*
- *d**l** is a differential length element of the current-carrying wire,*
- *$\hat{r}$ is the unit vector pointing from the current element dl to the observation point r*
- ***|r|** is the distance from the current element to the observation point*



Key Points:

1. ***Current Element**: The magnetic field produced by a current depends on small segments of current. The dl term represents a small segment of the wire through which the current I is flowing.*
2. ***Cross Product**: The magnetic field is proportional to the cross product of dl and $\hat{r}$^, meaning the magnetic field is perpendicular to both the current direction and the line connecting the current element to the observation point.*
3. ***Distance Dependence**: The magnetic field strength decreases with the square of the distance from the current element ($1/|r|^2$).*
4. ***Integral Form**: Since the Biot-Savart law involves an integral, the total magnetic field is the sum of the contributions from all infinitesimal current elements in the entire current distribution.*
5. *The law is especially useful when the current is not uniformly distributed (i.e., in cases where the geometry of the current path is complex), allowing for the calculation of the magnetic field produced by various current distributions like loops, coils, or solenoids.*

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6. *For example, in a **current loop**, the Biot-Savart law can be used to calculate the magnetic field at any point along the axis of the loop, which is important for understanding electromagnets.*

Example:
Magnetic field due to a finite current-carring wire of length L at a point P, at a distance a away from the wire along the its bisector.

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$$

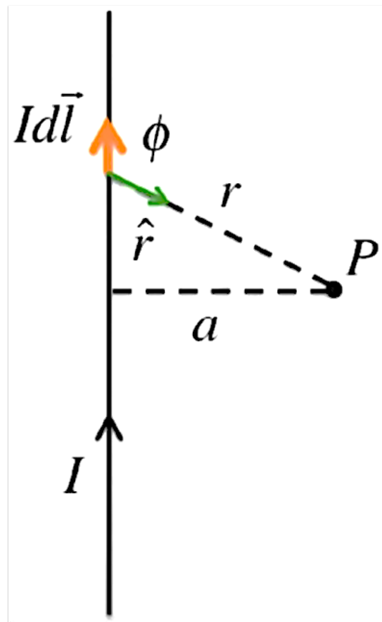
$$dl = dy$$

$$r^2 = a^2 + y^2$$

$$\sin\phi = \frac{a}{r} = \frac{a}{\sqrt{a^2+y^2}}$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idy}{a^2+y^2}\frac{a}{\sqrt{a^2+y^2}}$$

$$= \frac{\mu_0}{4\pi}\frac{Iady}{\left(a^2+y^2\right)^{3/2}}$$



$$B = \int_{-L/2}^{L/2} dB = \int_{-L/2}^{L/2}\frac{\mu_0}{4\pi}\frac{Iady}{\left(a^2+y^2\right)^{3/2}} = \frac{\mu_0 Ia}{4\pi}\int_{-L/2}^{L/2}\frac{dy}{\left(a^2+y^2\right)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi a}\frac{L}{\sqrt{L^2/4+a^2}}$$

By taking the limit as ***L*** approach to infinity and ***a*** approach to ***r***

For magnetic field a distance r away from an infinitely long wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

Example:

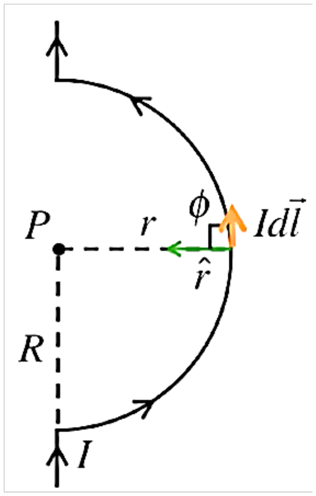
Magnetic field due to current-carring arc of wire with radius R at a point P, at the center of the arc

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$$

$$dl = Rd\theta$$

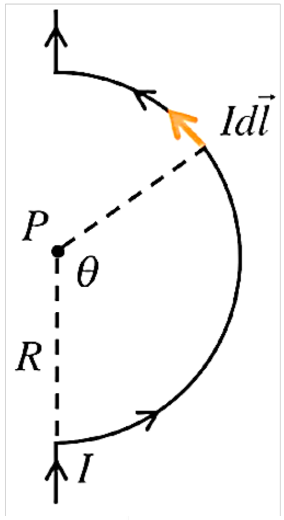
$$r = R$$

$$\phi = 90^\circ \Rightarrow \sin\phi = 1$$



$$dB = \frac{\mu_0}{4\pi}\frac{IRd\theta}{R^2} = \frac{\mu_0}{4\pi}\frac{Id\theta}{R}$$

$$B = \int_{\theta_i}^{\theta_f} dB = \int_{\theta_i}^{\theta_f} \frac{\mu_0}{4\pi}\frac{Id\theta}{R} = \frac{\mu_0}{4\pi}\frac{I}{R}\int_{\theta_i}^{\theta_f} d\theta = \frac{\mu_0}{4\pi}\frac{I}{R}\Delta\theta$$



For magnetic field at center of circular loop of radius R:

$$\Delta\theta = 2\pi$$

$$B = \frac{\mu_0 I}{2R}$$