

Maxwell's Equations Symbols

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The integral form of Gauss's law

Electric charge produces an electric field, and the flux of that field passing through any closed surface is proportional to the total charge contained within that surface.

The diagram shows the integral form of Gauss's law:
$$\oint_S \vec{E} \cdot \hat{n} da = \frac{q_{\text{enc}}}{\epsilon_0}$$
 with the following annotations:

- Reminder that the electric field is a vector (points to \vec{E})
- Reminder that this integral is over a closed surface (points to the surface integral symbol \oint)
- Dot product tells you to find the part of \vec{E} parallel to \hat{n} (perpendicular to the surface) (points to the dot product \cdot)
- The unit vector normal to the surface (points to \hat{n})
- The amount of charge in coulombs (points to q_{enc})
- Reminder that only the enclosed charge contributes (points to q_{enc})
- The electric field in N/C (points to \vec{E})
- An increment of surface area in m^2 (points to da)
- The electric permittivity of the free space (points to ϵ_0)
- Tells you to sum up the contributions from each portion of the surface (points to the surface integral symbol \oint)
- Reminder that this is a surface integral (not a volume or a line integral) (points to the surface integral symbol \oint)

The differential form of Gauss's law

The electric field produced by electric charge diverges from positive charge and converges upon negative charge.

The diagram shows the differential form of Gauss's law:
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
 with the following annotations:

- Reminder that del is a vector operator (points to $\vec{\nabla}$)
- Reminder that the electric field is a vector (points to \vec{E})
- The charge density in coulombs per cubic meter (points to ρ)
- The electric permittivity of free space (points to ϵ_0)
- The differential operator called "del" or "nabla" (points to $\vec{\nabla}$)
- The electric field in N/C (points to \vec{E})
- The dot product turns the del operator into the divergence (points to the dot product \cdot)

Gauss's law for magnetic fields

The total magnetic flux passing through any closed surface is zero.

Reminder that the magnetic field is a vector

Dot product tells you to find the part of \vec{B} parallel to \hat{n} (perpendicular to the surface)

The unit vector normal to the surface

$$\oint_S \vec{B} \cdot \hat{n} \, da = 0$$

Reminder that this integral is over a closed surface

The magnetic field in Teslas

An increment of surface area in m^2

Reminder that this is a surface integral (not a volume or a line integral)

Tells you to sum up the contributions from each portion of the surface

The differential form of Gauss's law

The divergence of the magnetic field at any point is zero.

Reminder that the del operator is a vector

Reminder that the magnetic field is a vector

$$\vec{\nabla} \cdot \vec{B} = 0$$

The differential operator called "del" or "nabla"

The dot product turns the del operator into the divergence

The magnetic field in Teslas

Faraday's law

Standard form of Faraday's law:

Changing magnetic flux through a surface induces an emf in any boundary path of that surface, and a changing magnetic field induces a circulating electric field.

Diagram illustrating the standard form of Faraday's law:

$$\oint_C \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_S \vec{B} \cdot \hat{n} da$$

Annotations:

- Reminder that the electric field is a vector (points to \vec{E})
- Dot product tells you to find the part of \vec{E} parallel to $d\vec{l}$ (along path C) (points to $\vec{E} \cdot d\vec{l}$)
- An incremental segment of path C (points to $d\vec{l}$)
- The magnetic flux through any surface bounded by C (points to the right-hand side of the equation)
- The rate of change with time (points to $\frac{d}{dt}$)
- The electric field in V/m (points to \vec{E})
- Reminder that this is a line integral (not a surface or a volume integral) (points to the integral symbol \oint)
- Tells you to sum up the contributions from each portion of the closed path C in a direction given by the right-hand rule (points to the integral symbol \oint)

Faraday's law

Changing magnetic flux through a surface induces an emf in any boundary path of that surface, and a changing magnetic field induces a circulating electric field.

Alternative form of Faraday's law

This version of Faraday's law; the time derivative operates only on the magnetic field rather than on the magnetic flux

Diagram illustrating the alternative form of Faraday's law:

$$\oint_C \vec{E} \cdot d\vec{l} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} da$$

Annotations:

- Reminder that the electric field is a vector (points to \vec{E})
- Dot product tells you to find the part of \vec{E} parallel to $d\vec{l}$ (along path C) (points to $\vec{E} \cdot d\vec{l}$)
- An incremental segment of path C (points to $d\vec{l}$)
- The flux of the time rate of change of the magnetic field (points to the right-hand side of the equation)
- The rate of change of the magnetic field with time (points to $\frac{\partial \vec{B}}{\partial t}$)
- The electric field in V/m (points to \vec{E})
- Reminder that this is a line integral (not a surface or a volume integral) (points to the integral symbol \oint)
- Tells you to sum up the contributions from each portion of the closed path C (points to the integral symbol \oint)

The differential form of Faraday's law

A circulating electric field is produced by a magnetic field that changes with time

The diagram illustrates the differential form of Faraday's law, $\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$, with several annotations:

- Reminder that the del operator is a vector**: Points to the $\vec{\nabla}$ symbol.
- Reminder that the electric field is a vector**: Points to the \vec{E} symbol.
- The differential operator called "del" or "nabla"**: Points to the $\vec{\nabla}$ symbol.
- The electric field in V/m**: Points to the \vec{E} symbol.
- The cross-product turns the del operator into the curl**: Points to the \times symbol.
- The rate of change of the magnetic field with time**: Points to the $\frac{\partial \vec{B}}{\partial t}$ term.

The integral form of the Ampere–Maxwell law

An electric current or a changing electric flux through a surface produces a circulating magnetic field around any path that bounds that surface.

Reminder that the magnetic field is a vector

Dot product tells you to find the part of \vec{B} parallel to $d\vec{l}$ (along path C)

An incremental segment of path C

The electric current in amperes

The rate of change with time

The magnetic field in teslas

Tells you to sum up the contributions from each portion of closed path C in a direction given by the right-hand rule

Reminder that only the enclosed current contributes

The magnetic permeability of free space

The electric permittivity of free space

The electric flux through a surface bounded by C

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{\text{enc}} + \epsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot \hat{n} da \right)$$

The differential form of the Ampere–Maxwell law

A circulating magnetic field is produced by an electric current and by an electric field that changes with time

Reminder that the del operator is a vector

Reminder that the magnetic field is a vector

Reminder that the current density is a vector

The rate of change of the electric field with time

The differential operator called “del” or “nabla”

The magnetic field in teslas

The magnetic permeability of free space

The electric current density in amperes per square meter

The electric permittivity of free space

The cross-product turns the del operator into the curl

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$