Coulomb's law

The force between two charged bodies was studied by Coulomb in 1785.

Coulomb's law states that the force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The direction of forces is along the line joining the two point charges.

Let q1 and q2 be two point charges placed in air or vacuum at a distance r apart . Then, according to Coulomb's law,

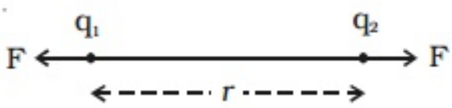


Fig 1.3a Coulomb forces

$$F \propto \frac{q_1 q_2}{r^2} \quad \text{or} \quad F = k\frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} \qquad \ldots(1)$$

and $\dfrac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

In the above equation, if $q_1 = q_2 = 1\text{C}$ and $r = 1\text{m}$ then,

$$F = (9 \times 10^9)\frac{1 \times 1}{1^2} = 9 \times 10^9 \text{ N}$$

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \qquad \ldots(2)$$

Dividing equation (1) by (2)

$$\frac{F}{F_m} = \frac{\varepsilon}{\varepsilon_o} = \varepsilon_r$$

The ratio $\dfrac{\varepsilon}{\varepsilon_o} = \varepsilon_r$, is called the relative permittivity or dielectric constant of the medium. The value of ε_r for air or vacuum is 1.

$$\therefore \quad \varepsilon = \varepsilon_o \varepsilon_r$$

Since $F_m = \dfrac{F}{\varepsilon_r}$, the force between two point charges depends on the nature of the medium in which the two charges are situated.

If $\vec{F}_{21}$ is the force exerted on charge q_2 by charge q_1 (Fig.1.3b),

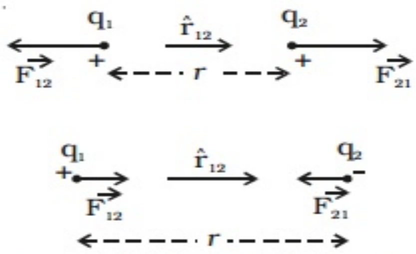


Fig 1.3b Coulomb's law in vector form

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12}$$

where $\hat{r}_{12}$ is the unit vector from q_1 to q_2.

If $\vec{F}_{12}$ is the force exerted on q_1 due to q_2,

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{21}^2}\,\hat{r}_{21}$$

where $\hat{r}_{21}$ is the unit vector from q_2 to q_1.

[Both $\hat{r}_{21}$ and $\hat{r}_{12}$ have the same magnitude, and are oppositely directed]

$$\therefore \quad \vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\,(-\hat{r}_{12})$$

$$\text{or} \quad \vec{F}_{12} = -k\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12}$$

$$\text{or} \quad \vec{F}_{12} = -\vec{F}_{21}$$

So, the forces exerted by charges on each other are equal in magnitude and opposite in direction.

Principle of Superposition

The principle of superposition is to calculate the electric force experienced by a charge q_1 due to other charges q_2, q_3 ..... q_n.

The total force on a given charge is the vector sum of the forces exerted on it due to all other charges.

The force on q_1 due to q_2

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{21}^2}\,\hat{r}_{21}$$

Similarly, force on q_1 due to q_3

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{r_{31}^2}\,\hat{r}_{31}$$

The total force F_1 on the charge q_1 by all other charges is,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} \,\ldots\ldots\ldots + \vec{F}_{1n}$$

Therefore,

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + \ldots\ldots \frac{q_1 q_n}{r_{n1}^2}\hat{r}_{n1}\right]$$

Electric Field

Electric field due to a charge is the space around the test charge in which it experiences a force. The presence of an electric field around a charge cannot be detected unless another charge is brought towards it.

When a test charge q_o is placed near a charge q, which is the source of electric field, an electrostatic force F will act on the test charge.

Electric Field Intensity (E)

Electric field at a point is measured in terms of electric field intensity. Electric field intensity at a point, in an electric field is defined as the force experienced by a unit positive charge kept at that point.

It is a vector quantity.

$$|\vec{E}| = \frac{|\vec{F}|}{q_o}.$$

The unit of electric field intensity is N/C.

The electric field intensity is also referred as electric field strength or simply electric field. So, the force exerted by an electric field on a charge is F = q_oE.

1. Electric field due to a point charge

Fig 1.4 Electric field due to a point charge

Let q be the point charge placed at O in air (Fig.1.4). A test charge qo is placed at P at a distance r from q. According to Coulomb's law, the force acting on qo due to q is

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q\,q_o}{r^2}$$

The electric field at a point P is, by definition, the force per unit test charge.

$$E = \frac{F}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

The direction of E is along the line joining O and P, pointing away from q, if q is positive and towards q, if q is negative.

In vector notation $\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$, where $\hat{r}$ is a unit vector pointing away from q.

2. Electric field due to system of charges

If there are a number of stationary charges, the net electric field (intensity) at a point is the vector sum of the individual electric fields due to each charge.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \ldots\ldots \vec{E}_n$$

$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + \ldots\ldots\ldots\right]$$

7

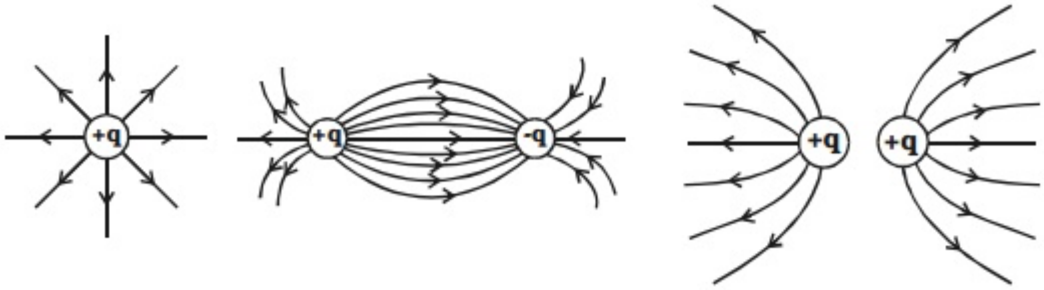
3. Definition of Electric lines of force

Definition of Electric lines of force and Properties of lines of forces:

The concept of field lines was introduced by Michael Faraday as an aid in visualizing electric and magnetic fields.

Electric line of force is an imaginary straight or curved path along which a unit positive charge tends to move in an electric field.

The electric field due to simple arrangements of point charges are shown in diagram.



(a) Isolated charge (b) Unlike charges (c) Like charges

Fig1.5 Lines of Forces

Properties of lines of forces:

- Lines of force start from positive charge and terminate at negative charge.
- Lines of force never intersect.
- The tangent to a line of force at any point gives the direction of the electric field (E) at that point.
- The number of lines per unit area, through a plane at right angles to the lines, is proportional to the magnitude of E. This means that, where the lines of force are close together, E is large and where they are far apart, E is small.
- Each unit positive charge gives rise to $1/\varepsilon_o$ lines of force in free space. Hence number of lines of force originating from a point q charge q is $N = q/\varepsilon_o$ in free space

4. Electric dipole and electric dipole moment

Two equal and opposite charges separated by a very small distance constitute an electric dipole.

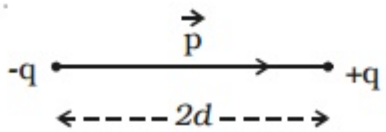


Fig 1.6 Electric dipole

Water, ammonia, carbon?dioxide and chloroform molecules are some examples of permanent electric dipoles. These molecules behave like electric dipole, because the centres of positive and negative charge do not coincide and are separated by a small distance.

Two point charges +q and -q are kept at a distance 2d apart (Fig.1.6). The magnitude of the dipole moment is given by the product of the magnitude of the one of the charges and the distance between them.

? Electric dipole moment, p = q2d or 2qd.

It is a vector quantity and acts from -q to +q. The unit of dipole moment is C m.

5. Electric field due to an electric dipole at a point on its axial line.

AB is an electric dipole of two point charges -q and +q separated by a small distance 2d (Fig 1.7). P is a point along the axial line of the dipole at a distance r from the midpoint O of the electric dipole.

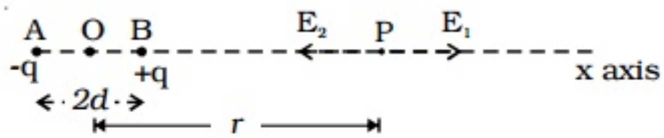


Fig 1.7 Electric field at a point on the axial line

The electric field at the point P due to +q placed at B is,

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} \quad \text{(along BP)}$$

The electric field at the point P due to xq placed at A is,

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+d)^2} \quad \text{(along PA)}$$

E_1 and E_2 act in opposite directions.

Therefore, the magnitude of resultant electric field (E) acts in the direction of the vector with a greater magnitude. The resultant electric field at P is,

$$E = E_1 + (-E_2)$$

$$E = \left[\frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} - \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+d)^2}\right] \text{along BP.}$$

$$E = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right] \text{ along BP}$$

$$E = \frac{q}{4\pi\varepsilon_o}\left[\frac{4rd}{(r^2-d^2)^2}\right] \text{along BP.}$$

If the point P is far away from the dipole, then d <<r

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_o}\frac{4rd}{r^4} = \frac{q}{4\pi\varepsilon_o}\frac{4d}{r^3}$$

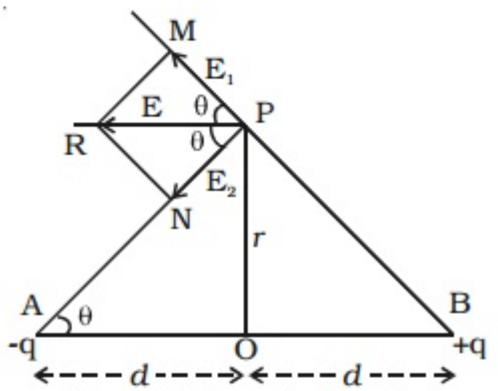
$$E = \frac{1}{4\pi\varepsilon_o}\frac{2p}{r^3} \text{ along BP.}$$

[? Electric dipole moment p = q x 2d]

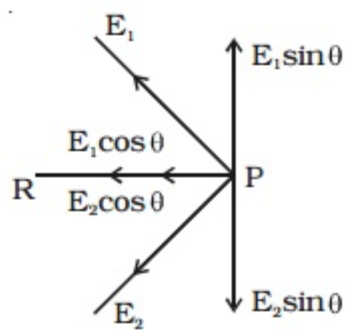
E acts in the direction of dipole moment.

6. Electric field due to an electric dipole at a point on the equatorial line.

Consider an electric dipole AB. Let 2d be the dipole distance and p be the dipole moment. P is a point on the equatorial line at a distance r from the midpoint O of the dipole (Fig 1.8a).



(a) Electric field at a point on equatorial line



(b) The components of the electric field

Fig 1.8

Electric field at a point P due to the charge +q of the dipole,

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{BP^2} \text{ along BP.}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+d^2)} \text{ along BP } (\because BP^2 = OP^2 + OB^2)$$

Electric field (E_2) at a point P due to the charge $-q$ of the dipole

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{AP^2} \text{ along PA}$$

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+d^2)} \text{ along PA}$$

The magnitudes of E1 and E2 are equal. Resolving E1 and E2 into their horizontal and vertical components (Fig 1.8b), the vertical components E1 sin θ and E2 sin θ are equal and opposite, therefore they cancel each other.

The horizontal components E1 cos θ and E2 cos θ will get added along PR.

Resultant electric field at the point P due to the dipole is

E = E1 cos θ + E2 cos θ (along PR)

= 2 E1cos θ (?E1 = E2)

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+d^2)} \times 2\cos\theta$$

But $\cos\theta = \dfrac{d}{\sqrt{r^2+d^2}}$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+d^2)} \times \frac{2d}{(r^2+d^2)^{1/2}} = \frac{1}{4\pi\varepsilon_o}\frac{q2d}{(r^2+d^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{p}{(r^2+d^2)^{3/2}} \qquad (\because p = q2d)$$

For a dipole, d is very small when compared to r

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^3}$$

The direction of E is along PR, parallel to the axis of the dipole and directed opposite to the direction of dipole moment.

7. Electric dipole in a uniform electric field

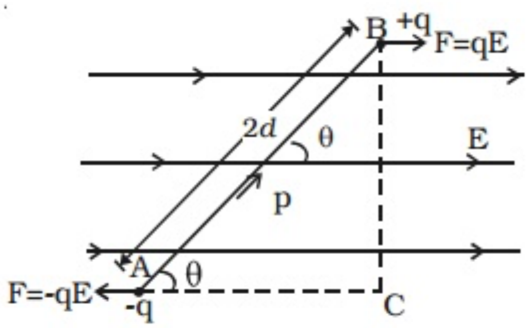


Fig 1.9 Dipole in a uniform field

Consider a dipole AB of dipole moment p placed at an angle ? in an uniform electric field E (Fig.1.9). The charge +q experiences a force qE in the direction of the field. The charge -q experiences an equal force in the opposite direction. Thus the net force on the dipole is zero.

The two equal and unlike parallel forces are not passing through the same point, resulting in a torque on the dipole, which tends to set the dipole in the direction of

the electric field.

The magnitude of torque is,

τ = One of the forces x perpendicular distance between the forces

$= F \times 2d \sin\theta$

$= qE \times 2d \sin\theta = pE \sin\theta$

$(q \times 2d = P)$

In vector notation, $\vec{\tau} = \vec{p} \times \vec{E}$

Note : If the dipole is placed in a non?uniform electric field at an angle θ,, in addition to a torque, it also experiences a force.

8. Electric potential energy of an electric dipole in an electric field.

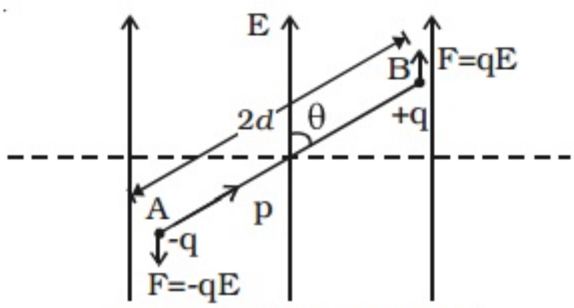


Fig 1.10 Electric potential energy of dipole

Electric potential energy of an electric dipole in an electrostatic field is the work done in rotating the dipole to the desired position in the field.

When an electric dipole of dipole moment p is at an angle ? with the electric field E, the torque on the dipole is

$\tau = pE \sin\theta$

Work done in rotating the dipole through $d\theta$,

$dw = \tau . d\theta$

$= pE \sin\theta . d\theta$

The total work done in rotating the dipole through an angle θ is

$W = \int dw$

W = pE ∫sinθ.dθ = �'pE cos θ

This work done is the potential energy (U) of the dipole.

∴ U = �' pE cos θ

When the dipole is aligned parallel to the field, θ = 0o

∴U = �'pE

This shows that the dipole has a minimum potential energy when it is aligned with the field. A dipole in the electric field experiences a torque (�'τ = �'p x�'E) which tends to align the dipole in the field direction, dissipating its potential energy in the form of heat to the surroundings.

Microwave oven

It is used to cook the food in a short time. When the oven is operated, the microwaves are generated, which in turn produce a non?uniform oscillating electric field. The water molecules in the food which are the electric dipoles are excited by an oscillating torque. Hence few bonds in the water molecules are broken, and heat energy is produced. This is used to cook food.

Electric potential

Let a charge +q be placed at a E points, in the electric field. When a unit Electric potential positive charge is moved from A to B against the electric force, work is done. This work is the potential difference between these two points. i.e., dV = $W_{A \to B}$.

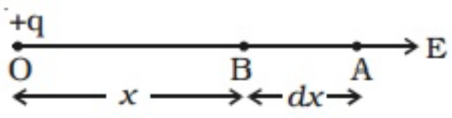


Fig1.11 Electric potential

The potential difference between two points in an electric field is defined as the amount of work done in moving a unit positive charge from one point to the other against the electric force.

The unit of potential difference is volt. The potential difference between two points is 1 volt if 1 joule of work is done in moving 1 Coulomb of charge from one point to another against the electric force.

The electric potential in an electric field at a point is defined as the amount of work done in moving a unit positive charge from infinity to that point against the electric forces.

Relation between electric field and potential

Let the small distance between A and B be dx. Work done in moving a unit positive charge from A to B is dV = E.dx.

The work has to be done against the force of repulsion in moving a unit positive charge towards the charge +q. Hence,

dV = -E.dx

E = -dV/dx

The change of potential with distance is known as potential gradient, hence the electric field is equal to the negative gradient of potential.

The negative sign indicates that the potential decreases in the direction of electric field. The unit of electric intensity can also be expressed as Vm^{-1}.

1. Electric potential at a point due to a point charge

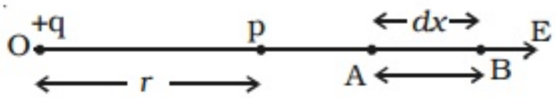


Fig 1.12 Electric potential due to a point charge

Let +q be an isolated point charge situated in air at O. P is a point at a distance r from +q. Consider two points A and B at distances x and x + dx from the point O (Fig.1.12).

The potential difference between A and B is,

dV = -E dx

The force experienced by a unit positive charge placed at A is

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{x^2}$$

$$\therefore \qquad dV = -\frac{1}{4\pi\varepsilon_o}\frac{q}{x^2} \cdot dx$$

The negative sign indicates that the work is done against the electric force.

The electric potential at the point P due to the charge +q is the total work done in moving a unit positive charge from infinity to that point.

$$V = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_o x^2} \cdot dx = \frac{q}{4\pi\varepsilon_o r}$$

2. Electric potential at a point due to an electric dipole

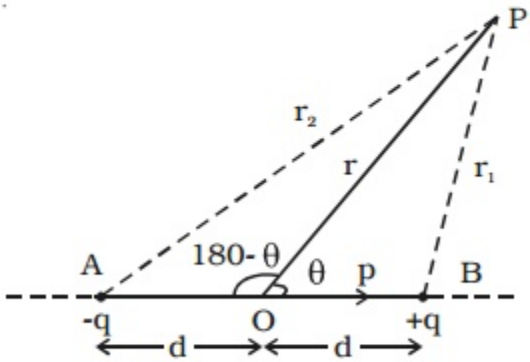


Fig 1.13 Potential due to a dipole

Two charges xq at A and +q at B separated by a small distance 2d constitute an electric dipole and its dipole moment is p (Fig 1.13).

Let P be the point at a distance r from the midpoint of the dipole O and ? be the angle between PO and the axis of the dipole OB. Let r_1 and r_2 be the distances of the point P from +q and xq charges respectively.

Potential at P due to charge (+q) = $\frac{1}{4\pi\varepsilon_o}\frac{q}{r_1}$

Potential at P due to charge (–q) = $\frac{1}{4\pi\varepsilon_o}\left(-\frac{q}{r_2}\right)$

Total potential at P due to dipole is, V = $\frac{1}{4\pi\varepsilon_o}\frac{q}{r_1} - \frac{1}{4\pi\varepsilon_o}\frac{q}{r_2}$

$$V = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad ...(1)$$

Applying cosine law,

$$r_1^2 = r^2 + d^2 - 2rd\cos\theta$$

$$r_1^2 = r^2\left(1 - 2d\frac{\cos\theta}{r} + \frac{d^2}{r^2}\right)$$

Since d is very much smaller than r, $\frac{d^2}{r^2}$ can be neglected.

$$\therefore \qquad r_1 = r\left(1 - \frac{2d}{r}\cos\theta\right)^{\frac{1}{2}}$$

or $$\frac{1}{r_1}=\frac{1}{r}\left(1-\frac{2d}{r}\cos\theta\right)^{-1/2}$$

Using the Binomial theorem and neglecting higher powers,

$$\therefore \quad \frac{1}{r_1}=\frac{1}{r}\left(1+\frac{d}{r}\cos\theta\right) \qquad \ldots(2)$$

Similarly,

$$r_2^2 = r^2 + d^2 - 2rd\cos(180-\theta)$$

or $$r_2^2 = r^2 + d^2 + 2rd\cos\theta.$$

$$r_2 = r\left(1+\frac{2d}{r}\cos\theta\right)^{1/2} \qquad \left(\because \frac{d^2}{r^2} \text{ is negligible}\right)$$

or $$\frac{1}{r_2}=\frac{1}{r}\left(1+\frac{2d}{r}\cos\theta\right)^{-1/2}$$

Using the Binomial theorem and neglecting higher powers,

$$\frac{1}{r_2}=\frac{1}{r}\left(1-\frac{d}{r}\cos\theta\right) \qquad \ldots(3)$$

Substituting equation (2) and (3) in equation (1) and simplifying

$$V = \frac{q}{4\pi\varepsilon_o}\frac{1}{r}\left(1+\frac{d}{r}\cos\theta-1+\frac{d}{r}\cos\theta\right)$$

$$\therefore \quad V = \frac{q\,2d\cos\theta}{4\pi\varepsilon_o\,.\,r^2}=\frac{1}{4\pi\varepsilon_o}\frac{p\,.\,\cos\theta}{r^2} \qquad \ldots(4)$$

Special cases :

1. When the point P lies on the axial line of the dipole on the side of $+q$, then $\theta = 0$

$$\therefore V = \frac{p}{4\pi\varepsilon_o r^2}$$

2. When the point P lies on the axial line of the dipole on the side of $-q$, then $\theta = 180$

$$\therefore V = -\frac{p}{4\pi\varepsilon_o r^2}$$

3. When the point P lies on the equatorial line of the dipole, then, $\theta = 90^o$,

$$\therefore V = 0$$

3. Electric potential Energy

The electric potential energy of two point charges is equal to the work done to assemble the charges or workdone in bringing each charge or work done in bringing a charge from infinite distance.

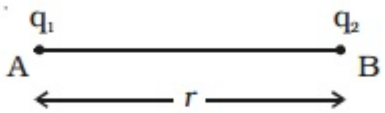


Fig 1.14a Electric potential energy

Let us consider a point charge q_1, placed at A (Fig 1.14a].

The potential at a point B at a distance r from the charge q1 is

$$V = \frac{q_1}{4\pi\varepsilon_o r}$$

Another point charge q_2 is brought from infinity to the point B.

Now the work done on the charge q_2 is stored as electrostatic potential energy (U) in the system of charges q_1 and q_2.

$$\therefore \quad \text{work done, } w = Vq_2$$

$$\text{Potential energy (U)} = \frac{q_1 q_2}{4\pi\varepsilon_o r}$$

Keeping q_2 at B, if the charge q_1 is imagined to be brought from infinity to the point A, the same amount of work is done.

Also, if both the charges q_1 and q_2 are brought from infinity, to points A and B respectively, separated by a distance r, then potential energy of the system is the same as the previous cases.

For a system containing more than two charges (Fig 1.14b), the potential energy (U) is given by

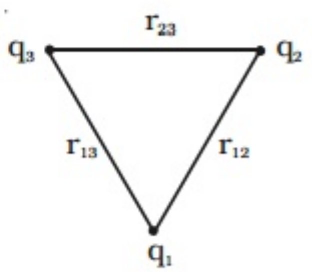


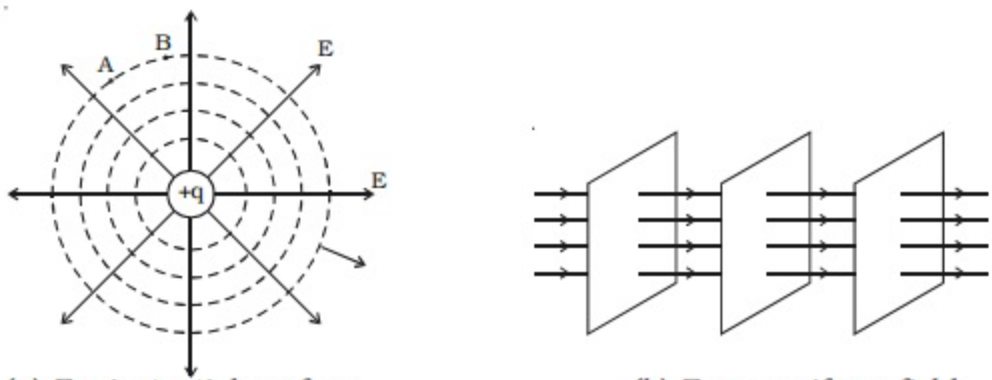
Fig 1.14b Potential energy of system of charges

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

4. Equipotential Surface

If all the points of a surface are at the same electric potential, then the surface is called an equipotential surface.

(i) In case of an isolated point charge, all points equidistant from the charge are at same potential. Thus, equipotential surfaces in this



(a) Equipotential surface (spherical) Fig 1.15 (b) For a uniform field (plane)

case will be a series of concentric spheres with the point charge as their centre (Fig 1.15a). The potential, will however be different for different spheres.

If the charge is to be moved between any two points on an equipotential surface through any path, the work done is zero. This is because the potential difference between two points A and B is defined as $V_B - V_A = W_{AB}/q$. If $V_A = V_B$ then $W_{AB} = 0$. Hence the electric field lines must be normal to an equipotential surface.

(ii) In case of uniform field, equipotential surfaces are the parallel planes with their surfaces perpendicular to the lines of force as shown in Fig 1.15b.