Applications on Maxwell's 1st Equation

Find the electric flux through the surface of a sphere containing 15 protons and 10 electrons.

$$\Phi_E = \oint_S \vec{E} \circ \hat{n}\ da = q_{enc}/\varepsilon_0$$

$$q_{enc} = \sum_i q_i = 15(1.6\times10^{-19}C) + 10(-1.6\times10^{-19}C)$$

$$= (2.4\times10^{-18} - 1.6\times10^{-18})C$$

$$= 8\times10^{-19}C$$

$$\Phi_E = q_{enc}/\varepsilon_0 = \frac{8\times10^{-19}\,C}{8.85\times10^{-12}\,C/Vm}$$

$$= 9.04\times10^{-8}\,Vm$$

A cube of side L contains a flat plate with variable surface charge density of σ = -3xy. If the plate extends from x = 0 to x = L and from y = 0 to y = L, what is the total electric flux through the walls of the cube?

$$\Phi_E = \oint_S \vec{E} \circ \hat{n}\ da = q_{enc}/\varepsilon_0 \qquad q_{enc} = \int_S \sigma\ da \qquad \sigma = -3xy,$$

$$q_{enc} = \int_{y=0}^{L}\int_{x=0}^{L}(-3xy)\ dx\,dy$$

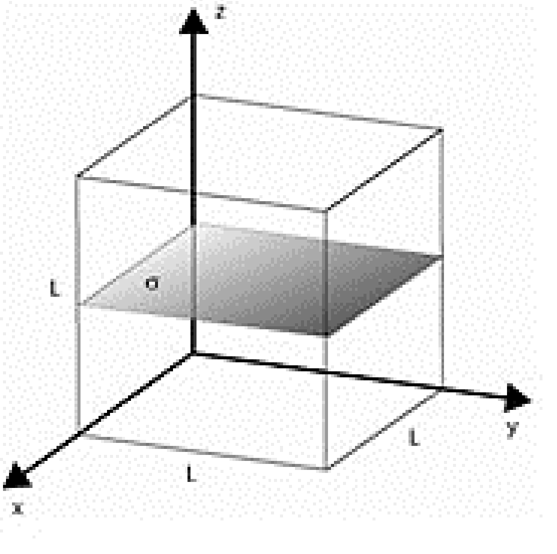
$$= -3\int_{y=0}^{L} y\left(\frac{1}{2}x^2\ \Big]_0^L\right)dy$$

$$= -\frac{3}{2}\int_{y=0}^{L} L^2 y\ dy$$

$$= -\frac{3L^2}{2}\left(\frac{1}{2}y^2\ \Big]_0^L\right)$$

$$= -\frac{3}{4}L^4\ \text{C}$$

$$\boxed{\Phi_E = q_{enc}/\varepsilon_0 = -3L^4/4\varepsilon_0}$$



Find the total electric flux through a closed cylinder containing a line charge along its axis with linear charge density $\lambda = \lambda_0(1\text{-}x/h)$ C/m if the cylinder and the line charge extend from x = 0 to x = h.

$$\Phi_E = \oint_S \bar{E} \circ \hat{n}\ da = q_{enc}/\varepsilon_0 \qquad q_{enc} = \int_L \lambda\ dl$$

$$q_{enc} = \int_{x=0}^{h} \lambda_0 (1 - \frac{x}{h})\ dx$$

$$= \int_{x=0}^{h} \lambda_0\ dx - \int_{x=0}^{h} \lambda_0 \frac{x}{h}\ dx$$

$$= \lambda_0 x\Big|_0^h - \frac{\lambda_0}{h}(\frac{1}{2}x^2\Big|_0^h)$$

$$= \lambda_0 h - \frac{\lambda_0}{h}(\frac{1}{2}h^2) = \lambda_0 h - \frac{\lambda_0}{2}h = \frac{\lambda_0}{2}h$$

$$\Phi_E = q_{enc}/\varepsilon_0 = \frac{\lambda_0 h}{2\varepsilon_0}$$

What is the flux through any closed surface surrounding a charged sphere of radius a_0 with volume charge density of $\rho = \rho_0(r/a_0)$, where r is the distance from the center of the sphere?

$$\Phi_E = \oint_S \bar{E} \circ \hat{n}\ da = q_{enc}/\varepsilon_0 \qquad q_{enc} = \int_V \rho\ dV$$

$$q_{enc} = \int_{r=0}^{a_0} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho_0 (\frac{r}{a_0}) r^2\ sin\theta\ dr\, d\theta\, d\phi$$

$$= \frac{\rho_0}{a_0} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \int_{r=0}^{a_0} r^3 dr\ sin\theta\, d\theta\, d\phi$$

$$= \frac{\rho_0}{a_0} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \left(\frac{r^4}{4}\right)\Bigg|_0^{a_0} sin\theta\ d\theta\, d\phi$$

$$= \frac{\rho_0}{a_0}(\frac{1}{4}a_0^4)\int_{\theta=0}^{\pi} sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$= \frac{\rho_0 a_0^3}{4}\left(-cos\,\theta\Big|_0^{\pi}\right)\left(\phi\Big|_0^{2\pi}\right)$$

$$= \rho_0 a_0^3 \pi$$

$$\boxed{\Phi_E = q_{enc}/\varepsilon_0 = \frac{\rho_0 a_0^3 \pi}{\varepsilon_0}}$$

A circular disk with surface charge density 2×10^{-10} C/m^2 is surrounded by a sphere with radius of one meter. If the flux through the sphere is 5.2×10^{-2} Vm, what is the diameter of the disk

$$\Phi_E = \oint_S \vec{E}\circ\hat{n}\, da = q_{enc}/\varepsilon_0$$

$$q_{enc} = \varepsilon_0\Phi_E = (8.85\times10^{-12}\, C/Vm)(5.2\times10^{-2}\, Vm) = 4.6\times10^{-13}\, C$$

$$q_{enc} = \sigma A$$

$$q_{enc} = \sigma\pi r^2 = (2\times10^{-10}\, C/m^2)\pi r^2$$

$$r = \left[\frac{4.6\times10^{-13}\, C}{(2\times10^{-10}\, C/m^2)\pi}\right]^{1/2} = 0.027\ m$$

$$d = 2r = 0.054\ m$$

Use a special Gaussian surface around an infinite line charge to find the electric field of the line charge as a function of distance.

The top and bottom surfaces of the cylinder E=0 ?

$$\oint_S \vec{E}\circ\hat{n}\, da = \oint_S |\vec{E}| da = |\vec{E}|\oint_S da = |\vec{E}|(2\pi rL)$$

$$|\vec{E}|(2\pi rL) = q_{enc}/\varepsilon_0$$

$$|\vec{E}|(2\pi rL) = \lambda L/\varepsilon_0$$

$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 r}$$

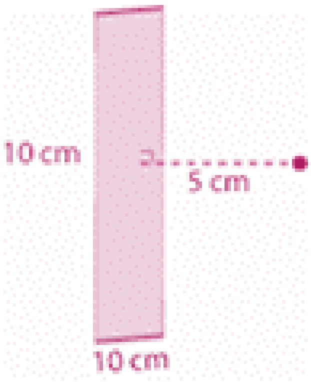
Find the divergence of the field given by A = (1/r)ŕ in spherical coordinates

$$\vec{\nabla}\circ\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\vec{\nabla}\circ\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2(\frac{1}{r})\right] = \frac{1}{r^2}\frac{\partial}{\partial r}(r) = \frac{1}{r^2}$$

A 10 cm x 10 cm flat plate is located 5 cm from a point charge of 10^{-8} C. What is the electric flux through the plate due to the point charge

$$\Phi_E = \oint_S \vec{E} \circ \hat{n} \; da = q_{enc}/\varepsilon_0$$



$$\Phi_E = \frac{1\times10^{-8}\,C}{8.85\times10^{-12}\,C/Vm} = 1.13\times10^{3} \;\; Vm$$

$$\Phi_{E,\,each\,side} = \frac{1}{6}\Phi_E = 188.3 \;\; Vm$$

Find the electric flux through a half-cylinder of height h due to an infinitely long line charge with charge density λ running along the axis of the cylinder.

$$\Phi_E = \oint_S \vec{E} \circ \hat{n} \; da = q_{enc}/\varepsilon_0$$

$$q_{enc} = \lambda\, h$$

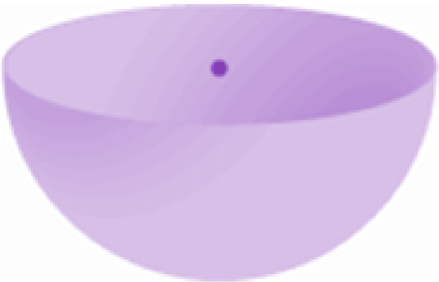
$$\Phi_E = \lambda\, h/\varepsilon_0$$

$$\Phi_{E,half-cylinder} = \Phi_E/2 = \lambda\, h/2\varepsilon_0$$

A proton rests at the center of the rim of a hemispherical bowl of radius R. What is the electric flux through the surface of the bowl.

$$\Phi_{E,half-sphere} = \frac{1}{2}\Phi_{E,full-sphere} = \frac{1}{2} q_{enc}/\varepsilon_0$$

$$\Phi_{E,half-sphere} = \frac{1}{2}\left(\frac{1.6\times10^{-19}\;C}{8.85\times10^{-12}\,C/Vm}\right) = 9.04\times10^{-9}\;Vm$$



Another Method

$$\Phi_E = \oint_S \vec{E} \circ \hat{n} \; da = \int_{\theta=\pi/2}^{\pi}\int_{\phi=0}^{2\pi} \frac{q_{proton}}{4\pi\varepsilon_0 R^2} R^2 \sin\theta \; d\theta \; d\phi$$

$$= \frac{q_{proton}}{4\pi\;\varepsilon_0}\left[-\cos\theta\Big|_{\pi/2}^{\pi}\right](2\pi) = \frac{q_{proton}}{2\varepsilon_0} = 9.04 \text{ x } 10^{-9}\; Vm$$

Use a special Gaussian surface to prove that the magnitude of the electric field of an infinite flat plane with surface charge density σ is: $E= \sigma/2\varepsilon_0$.

$$\oint_S \vec{E} \circ \hat{n}\, da = \left|\vec{E}\right| \int_{Top\ \&\ Bottom} da = q_{enc}/\varepsilon_0$$

$$\left|\vec{E}\right|(s^2 + s^2) = \sigma s^2/\varepsilon_0 \quad \Rightarrow \quad \boxed{\left|\vec{E}\right| = \sigma/(2\varepsilon_0)}$$

Find the charge density in a region for which the electric field in cylindrical coordinates is given by: $\vec{E} = \frac{az}{r}\hat{r} + br\hat{\phi} + cr^2 z^2 \hat{z}$

$$\vec{\nabla} \circ \vec{E} = \rho/\varepsilon_0$$

$$\vec{\nabla} \circ \vec{E} = \frac{1}{r}\frac{\partial}{\partial r}(rE_r) + \frac{1}{r}\frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z}$$

$$\vec{\nabla} \circ \vec{E} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(\frac{az}{r})\right] + \frac{1}{r}\frac{\partial}{\partial \phi}(br) + \frac{\partial}{\partial z}(cr^2 z^2) = \rho/\varepsilon_0$$

$$\vec{\nabla} \circ \vec{E} = \frac{1}{r}(0) + \frac{1}{r}(0) + 2zcr^2 = \rho/\varepsilon_0 \quad \Rightarrow \quad \boxed{\rho = 2zcr^2\varepsilon_0}$$

Find the charge density in a region for which the electric field in spherical coordinates is given by:

$$\vec{E} = ar^2\hat{r} + \frac{b\cos(\theta)}{r}\hat{\theta} + c\hat{\phi}$$

$$\vec{\nabla} \circ \vec{E} = \rho/\varepsilon_0$$

$$\vec{\nabla} \circ \vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(E_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial \phi}$$

$$\vec{\nabla} \circ \vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 a r^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\frac{b\cos(\theta)\sin(\theta)}{r}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(c)$$

$$\vec{\nabla} \circ \vec{E} = 4ar + \frac{b}{r^2\sin\theta}(-\sin^2\theta + \cos^2\theta) + 0$$

$$\cos^2\theta = 1 - \sin^2\theta,$$

$$4ar + \frac{b}{r^2}(\frac{1 - 2\sin^2\theta}{\sin\theta}) = \rho/\varepsilon_0$$

$$\rho = 4ar\varepsilon_0 + \frac{b\varepsilon_0}{r^2}(\frac{1}{\sin\theta} - 2\sin\theta)$$

1.What is Maxwell's 1st equation?
a. $\nabla \cdot E = 0$
b. $\nabla \times E = -\partial B/\partial t$
c. $\nabla \cdot B = 0$
d. $\nabla \times B = \mu 0j$
e. None.

2.What does Maxwell's 1st equation describe?
a. The relationship between the curl of electric field and magnetic field.
b. The relationship between the divergence of electric field and magnetic field.
c. The relationship between the curl of magnetic field and electric field.
d. The relationship between the divergence of magnetic field and electric field.
e. None.

3.Which of the following is true for a region with no charges or currents?
a. $\nabla \cdot E = 0$ and $\nabla \cdot B = 0$
b. $\nabla \cdot E = 0$ and $\nabla \times B = 0$
c. $\nabla \times E = 0$ and $\nabla \cdot B = 0$
d. $\nabla \times E = 0$ and $\nabla \times B = 0$
e. None.

4.Which of the following is a consequence of Maxwell's 1st equation?
a. Electric charges always move in closed loops.
b. Magnetic monopoles exist.
c. Electromagnetic waves can propagate in vacuum.
d. Electric fields always point toward negative charges.
e. None.

5. How does the divergence of the electric field affect the behavior of the electric field near charges?
a. It causes the electric field to point towards charges.
b. It causes the electric field to point away from charges.
c. It causes the electric field to circulate around charges.
d. It does not affect the behavior of the electric field near charges.
e. None.

6. What is the relationship between the divergence of the electric field and the presence of electric charges?
a. The divergence of the electric field is proportional to the density of electric charges.
b. The divergence of the electric field is proportional to the magnitude of electric charges.
c. The divergence of the electric field is zero in the absence of electric charges.
d. The divergence of the electric field is infinite in the presence of electric charges.
e. None.

7. How does Maxwell's 1st equation relate to the continuity equation for electric charge?
a. It is equivalent to the continuity equation for electric charge.
b. It is a consequence of the continuity equation for electric charge.
c. It is independent of the continuity equation for electric charge.
d. It contradicts the continuity equation for electric charge.
e. None.

8. How does the electric potential relate to the divergence of the electric field in the solution to Maxwell's 1^{st} equation?
a. The electric potential is proportional to the divergence of the electric field.
b. The electric potential is equal to the divergence of the electric field.
c. The electric potential is related to the gradient of the electric field.
d. The electric potential is not related to the divergence of the electric field.
e. None.

Unsolved Applications on Maxwell's 1st Equation

A point charge of +4 μC is located at the center of a spherical surface with a radius of 0.2 m. Calculate the electric flux through the surface.
Answer : the electric flux through the surface of the sphere is 4.52×10^4 *N·m²/C.*

A long straight wire carries a uniform charge density of 2 μC/m. Find the electric field at a distance of 5 cm from the wire.
Answer : the electric field at a distance of 5 cm from the wire is 3.6×10^5 *N/C.*

A closed surface encloses a total charge of 4 μC. The surface consists of two concentric spheres of radii 5 cm and 10 cm, respectively. Find the electric flux through each sphere.
Answer : the electric flux through both spheres is the same, and it is 4.514×10^5 Nm^2/C.

A charged non-conducting sphere of radius R has a charge density ρ = k/r, where k is a constant and r is the distance from the center of the sphere. Find the total charge enclosed within a sphere of radius 2R centered at the origin.
Answer : the total charge enclosed within the sphere of radius 2R is 16 times the charge enclosed within the sphere of radius R.

A charge q is located at the origin, and another charge Q is located on the x-axis at a distance d from the origin. Find the electric flux through a sphere of radius r centered at the origin, where r > d.
Answer : the electric flux through the sphere of radius r is given by: $\Phi = q/\varepsilon_0$ *for* $r > d$ $\Phi = (q+Q)/\varepsilon_0$ *for* $r < d$ *Note that the electric flux is discontinuous at* $r = d$*, where the charge Q is located. This is because the electric field changes abruptly at this point due to the presence of the charge Q.*