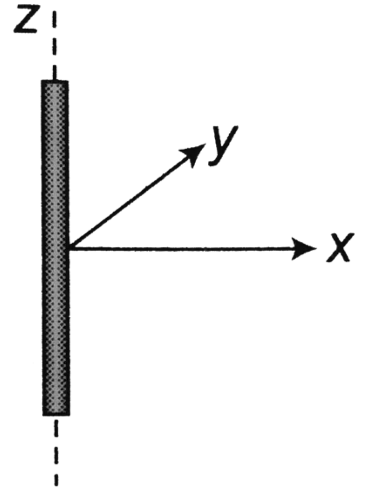
An infinitely long line charge along the z-axis has a linear charge density λ. Find the electric field at a radial distance r from the wire.



Step 1: Choose a Gaussian Surface

- By symmetry, the electric field must be **radial** ($\mathbf{E} = E\hat{\mathbf{r}}$).
- A **cylindrical Gaussian surface** of radius r and length L is chosen.

Step 2: Apply Gauss's Law

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

- The total charge enclosed:

$$Q_{\text{enc}} = \lambda L$$

- The electric field is perpendicular to the curved surface, so

$$E(2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$

- Solving for E:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

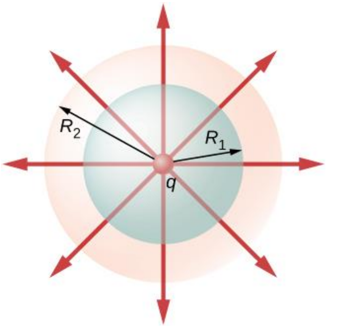
- **Direction: Radially outward** for $\lambda > 0$, inward for $\lambda < 0$.

A thin spherical shell of radius R carries a uniform charge density σ. Find the electric field inside and outside the shell.

Step 1: Use Gauss's Law

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

Case 1: Inside the Shell ($r < R$)



- The **Gaussian surface** is a sphere of radius r.
- The **enclosed charge** is **zero** ($Q_{\text{enc}} = 0$).
- Hence,

$$E = 0$$

Case 2: Outside the Shell ($r > R$)

- The **Gaussian surface** is a sphere of radius r.
- The **enclosed charge** is the **total charge** on the shell:

$$Q_{\text{enc}} = 4\pi R^2 \sigma$$

- Applying Gauss's Law:

$$E(4\pi r^2) = \frac{4\pi R^2 \sigma}{\varepsilon_0}$$

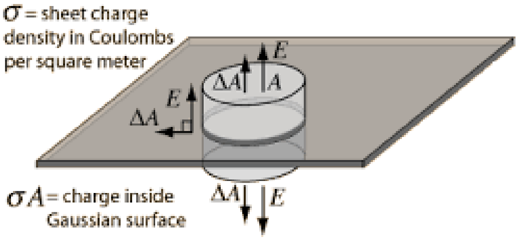
- Solving for E:

$$E = \frac{R^2 \sigma}{\varepsilon_0 r^2}$$

Final Answer:

$$\mathbf{E} = \begin{cases} 0, & r < R \\ \frac{R^2 \sigma}{\varepsilon_0 r^2} \hat{\mathbf{r}}, & r > R \end{cases}$$

An infinite plane carries a uniform surface charge density σ. Find the electric field above and below the plane.



Step 1: Choose a Gaussian Surface

- Use a **cylindrical Gaussian pillbox** of area A, extending equal distances above and below the plane.

Step 2: Apply Gauss's Law

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

- Enclosed charge:

$$Q_{\text{enc}} = \sigma A$$

- The flux through the top and bottom surfaces:

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

- Solving for E:

$$E = \frac{\sigma}{2\varepsilon_0}$$

Final Answer:

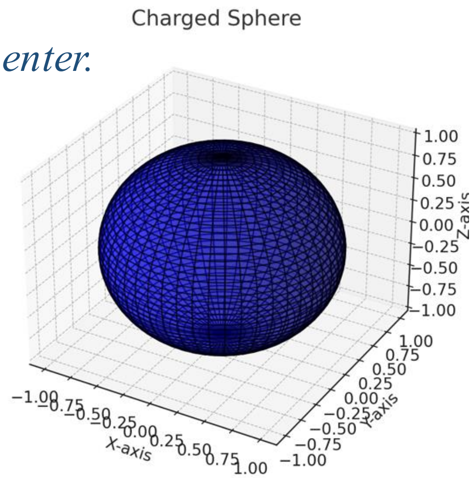
$$\mathbf{E} = \begin{cases} \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{n}}, & \text{above the plane} \\ -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{n}}, & \text{below the plane} \end{cases}$$

(where $\hat{\mathbf{n}}$ is the unit normal to the plane.)

A solid sphere of radius R has a non-uniform charge density given by

$$\rho(r) = \rho_0(1 - r/R)$$

is ρ_0 constant, and r is the radial distance from the center.
Find the electric field inside and outside the sphere.



The total charge enclosed within a sphere of radius r is

$$Q_{\text{enc}} = \int_0^r \rho(r')dV \qquad dV = 4\pi r'^2 dr'$$

$$Q_{\text{enc}} = \int_0^r \rho_0 \left(1 - \frac{r'}{R}\right) 4\pi r'^2 dr'$$

$$Q_{\text{enc}} = 4\pi\rho_0 \int_0^r \left(r'^2 - \frac{r'^3}{R}\right) dr' \qquad \int_0^r r'^2 dr' = \frac{r^3}{3}, \quad \int_0^r r'^3 dr' = \frac{r^4}{4}$$

$$Q_{\text{enc}} = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

Electric Field Inside (r<R) $\qquad E(4\pi r^2) = \frac{Q_{\text{enc}}}{\varepsilon_0}$

$$E(4\pi r^2) = \frac{4\pi\rho_0}{\varepsilon_0}\left(\frac{r^3}{3} - \frac{r^4}{4R}\right) \qquad E = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$$

Electric Field Outside *(r>R)*

$$Q_{\text{total}} = \int_0^R \rho(r')dV \qquad Q_{\text{total}} = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^4}{4R}\right)$$

$$Q_{\text{total}} = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^3}{4}\right) = 4\pi\rho_0 R^3\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$Q_{\text{total}} = 4\pi\rho_0 R^3 \times \frac{1}{12} = \frac{4\pi\rho_0 R^3}{12} = \frac{\pi\rho_0 R^3}{3}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{total}}}{r^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_0 R^3}{3r^2} \qquad E = \frac{\rho_0 R^3}{12\varepsilon_0 r^2}$$

$$\mathbf{E} = \begin{cases} \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)\hat{\mathbf{r}}, & r < R \\ \frac{\rho_0 R^3}{12\varepsilon_0 r^2}\hat{\mathbf{r}}, & r > R \end{cases}$$