Applications on Maxwell's 2nd Equation

Find the magnetic flux produced by the following magnetic field B through the top, bottom, and side surfaces of the flared cylinder shown in the figure. Given; $\vec{B} = 5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$$\Phi_B = \oint_S \vec{B} \circ \hat{n}\, da = 0$$

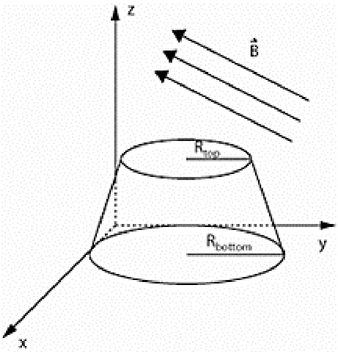
$$\hat{n}_{top} = \hat{k} \quad \text{and} \quad \hat{n}_{bottom} = -\hat{k}$$

$$\Phi_{B,top} = \int_{top} \vec{B} \circ \hat{n}\, da = \int_{top} (5\hat{i} - 3\hat{j} + 4\hat{k}) \circ \hat{k}\, da = 4\pi\, R_{top}^2$$

$$\Phi_{B,bottom} = \int_{bottom} \vec{B} \circ \hat{n}\, da = \int_{bottom} (5\hat{i} - 3\hat{j} + 4\hat{k}) \circ (-\hat{k})\, da = -4\pi\, R_{bottom}^2$$

$$\Phi_{B,top} + \Phi_{B,bottom} + \Phi_{B,sides} = 0$$

$$\Phi_{B,sides} = -(\Phi_{B,top} + \Phi_{B,bottom}) = 4\pi\,(R_{bottom}^2 - R_{top}^2)$$



Find the magnetic flux through all five surfaces of the wedge shown in the figure if the magnetic field in the area is given by: $\vec{B} = 0.002\hat{\imath} - 0.003\hat{\jmath}$ Tesla, and show that the total flux through the wedge is zero.

$$\Phi_B = \int_S \vec{B} \circ \hat{n}\, da. \qquad \hat{n}_A = \hat{i} \qquad \hat{n}_B = -\hat{i} \qquad \hat{n}_C = -\hat{j} \qquad \hat{n}_D = -\hat{k}$$

$$\hat{n}_E = \frac{1}{\sqrt{0.5^2 + 0.7^2}}(0.7\,\hat{j} + 0.5\hat{k}) = 0.814\,\hat{j} + 0.581\hat{k}$$

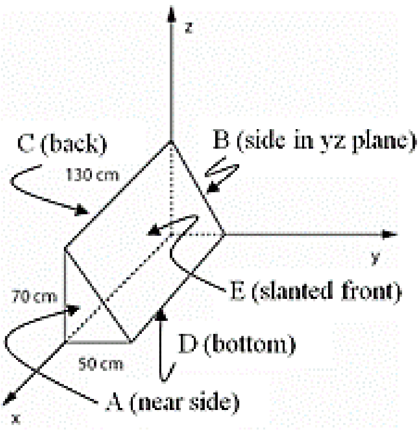
$$\Phi_{B,\text{surface A}} = \int_A [(2\times10^{-3})\hat{i} + (3\times10^{-3})\hat{j}] \circ \hat{i}\, da = (2\times10^{-3})\int_A da$$
$$= [(2\times10^{-3})\text{T}](0.5)(0.7\text{m}\times0.5\text{m}) = 3.5\times10^{-4}\ \text{wb}$$

$$\Phi_{B,\text{surface B}} = \int_B [(2\times10^{-3})\hat{i} + (3\times10^{-3})\hat{j}] \circ (-\hat{i})\, da = -3.5\times10^{-4}\ \text{wb}$$

$$\Phi_{B,\text{surface C}} = \int_C [(2\times10^{-3})\hat{i} + (3\times10^{-3})\hat{j}] \circ (-\hat{j})\, da = -2.73\times10^{-3}\ \text{wb}$$

$$\Phi_{B,\text{surface D}} = \int_D [(2\times10^{-3})\hat{i} + (3\times10^{-3})\hat{j}] \circ (-\hat{k})\, da = 0$$

$$\Phi_{B,\text{surface E}} = \int_E [(2\times10^{-3})\hat{i} + (3\times10^{-3})\hat{j}] \circ (0.814\hat{j} + 0.581\hat{k})\, da$$
$$= [(3\times10^{-3})\text{T}](0.814)(1.3\text{m}\times0.86\text{m}) = 2.73\times10^{-3}\ \text{wb}$$

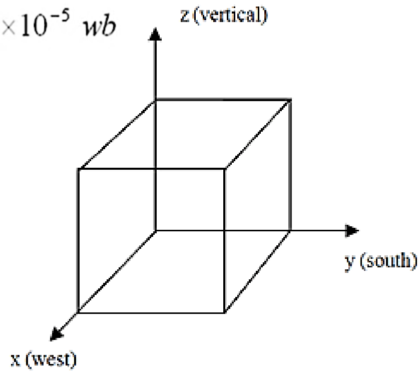


$$\Phi_{B,\text{Total}} = (3.5\times10^{-4}) + (-3.5\times10^{-4}) + (-2.73\times10^{-3}) + 0 + (2.73\times10^{-3}) = 0$$

Find the flux of the Earth's magnetic field through each face of a cube with 1-meter sides, and show that the total flux through the cube is zero. Assume that at the location of the cube, the Earth's magnetic field has amplitude of 4 x 10^{-5} T and points upward at an angle of 30° with respect to the horizontal. You may orient the cube in any way you choose.

$$\hat{n}_{Top} = \hat{k} \qquad \hat{n}_{Bottom} = -\hat{k} \qquad \hat{n}_{West} = \hat{i} \qquad \hat{n}_{East} = -\hat{i} \qquad \hat{n}_{South} = \hat{j} \qquad \hat{n}_{North} = -\hat{j}$$

$$\bar{B} = (4\times10^{-5}\,T)(\cos 30^\circ\,\hat{j} + \sin 30^\circ\,\hat{k}) = (3.46\times10^{-5})T\,\hat{j} + (2\times10^{-5})T\,\hat{k}$$



$$\Phi_{B,Top} = \int_{Top}[(3.46\times10^{-5})T\,\hat{j} + (2\times10^{-5})T\,\hat{k}]\circ\hat{k}\,da = (2\times10^{-5}T)\int_{Top} da = 2\times10^{-5}\ wb$$

$$\Phi_{B,Bottom} = \int_{Bottom}[(3.46\times10^{-5})T\,\hat{j} + (2\times10^{-5})T\,\hat{k}]\circ(-\hat{k})\,da = -2\times10^{-5}\ wb$$

$$\Phi_{B,West} = \int_{West\,side}[(3.46\times10^{-5})T\,\hat{j} + (2\times10^{-5})T\,\hat{k}]\circ\hat{i}\,da = 0$$

$$\Phi_{B,East} = \int_{East\,side}[(3.46\times10^{-5})T\,\hat{j} + (2\times10^{-5})T\,\hat{k}]\circ(-\hat{i})\,da = 0$$

$$\Phi_{B,South} = \int_{Southside}[(3.46\times10^{-5})T\,\hat{j} + (2\times10^{-5})T\,\hat{k}]\circ\hat{j}\,da = 3.46\times10^{-5}\ wb$$

$$\Phi_{B,North} = \int_{Northside}[(3.46\times10^{-5})T\,\hat{j} + (2\times10^{-5})T\,\hat{k}]\circ(-\hat{j})\,da = -3.46\times10^{-5}\ wb$$

$$\Phi_{B,Total} = (2\times10^{-5}\,wb) + (-2\times10^{-5}\,wb) + 0 + 0 + (3.46\times10^{-5}\,wb) + (-3.46\times10^{-5}\,wb) = 0$$

A cylinder of radius r_0 and height h is placed inside a solenoid with the cylinder's axis parallel to the axis of the solenoid. Find the flux through the top, bottom, and curved surfaces of the cylinder, and show that the total flux through the cylinder is zero.

$$\Phi_B = \int_S \bar{B}\circ\hat{n}\,da \qquad \bar{B} = \frac{\mu_0 N I}{l}\,\hat{x}$$

$$\hat{n}_{Top} = \hat{i} \qquad \hat{n}_{Bottom} = -\hat{i}$$

$$\Phi_{B,Top} = \int_{Top}\bar{B}\circ\hat{n}_{Top}\,da = \int_{Top}\frac{\mu_0 N I}{l}\,\hat{i}\circ\hat{i}\,da = \frac{\mu_0 N I}{l}\int_{Top} da = \frac{\mu_0 N I}{l}(\pi r_0^2)$$

$$\Phi_{B,Bottom} = \int_{Bottom}\bar{B}\circ\hat{n}_{Bottom}\,da = \int_{Bottom}\frac{\mu_0 N I}{l}\,\hat{i}\circ(-\hat{i})\,da$$

$$= -\frac{\mu_0 N I}{l}\int_{Bottom} da = -\frac{\mu_0 N I}{l}(\pi r_0^2)$$

$$\Phi_{B,Total} = \Phi_{B,Top} + \Phi_{B,Bottom} + \Phi_{B,Side}$$

$$= \frac{\mu_0 N I}{l}(\pi r_0^2) - \frac{\mu_0 N I}{l}(\pi r_0^2) + 0 = 0$$

Determine whether the vector fields given by the following expressions in cylindrical coordinates could be magnetic fields:

$$\bar{A}(r,\varphi,z) = \frac{a}{r}\cos^2(\varphi)\hat{r}$$

$$\bar{\nabla}\circ\bar{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\bar{\nabla}\circ\bar{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) = \frac{1}{r}\frac{\partial}{\partial r}[r\,\frac{a}{r}\,\cos^2(\phi)]$$

$$= \frac{1}{r}\frac{\partial}{\partial r}[a\cos^2(\phi)] = 0$$

The vector is magnetic field. Why?

Problem: A long, straight wire carries a current of 4 A. What is the magnetic field at a distance of 5 cm from the wire?
Solution: is 2.54 * 10^-5 T.

Problem: A long straight wire carrying a current of 5 A is surrounded by a cylindrical surface of radius 4 cm and height 10 cm, with the wire passing through the center of the cylinder. Determine the magnetic flux through the surface.
Solution: 1.005 * 10^-2 Wb.

Problem: A magnetic field is directed perpendicular to the surface of a circular loop of wire with radius 10 cm. The field varies with time according to the equation B(t) = 0.2t + 0.1 T. What is the magnitude of the induced emf in the loop when t = 2 s?
Solution: 0.00628 V/s.

1.What is Gauss's law for magnetism?
A. The total magnetic flux through a closed surface is equal to zero.
B. The magnetic field at a point due to a current-carrying conductor is proportional to the current and inversely proportional to the distance from the conductor.
C. The magnetic field at a point due to a current-carrying conductor is proportional to the length of the conductor and inversely proportional to the distance from the conductor.
D. The magnetic field at a point due to a current-carrying conductor is proportional to the current and directly proportional to the distance from the conductor.
E. None

2.A current-carrying wire is bent into a circular loop. What is the direction of the magnetic field inside the loop?
A. Clockwise
B. Counterclockwise
C. Zero
D. Cannot be determined
E. None

3. A long, straight wire carries a current of 5 A. What is the magnetic field at a distance of 10 cm from the wire?
A. 1.26 × 10^-6 T
B. 3.14 × 10^-5 T
C. 5.64 × 10^-5 T
D. 1.13 × 10^-4 T
E. None

4. A square loop of side 10 cm is placed with its plane perpendicular to a uniform magnetic field of magnitude 0.2 T. What is the magnetic flux through the loop?
A. 0.02 Wb
B. 0.04 Wb
C. 0.08 Wb
D. 0.16 Wb
E. None

5. A long, straight wire carries a current of 2 A. A rectangular loop of width 5 cm and length 10 cm is placed parallel to the wire with its center at a distance of 8 cm from the wire. What is the magnetic flux through the loop?
A. 1.26 × 10^-6 Wb
B. 6.28 × 10^-6 Wb
C. 3.14 × 10^-5 Wb
D. 1.57 × 10^-4 Wb
E. None