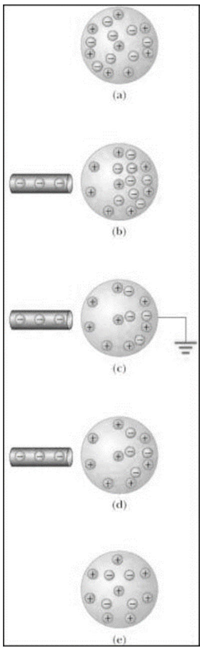



## Charging by induction

(a) We have a neutrally charged conductor.

(b) Negatively charged rod polarizes the sphere. The charge in the rod repels electrons to the opposite side of the sphere.

(c) Then we ground the sphere and some part of electrons is repelled into the Earth. There is **induced** positive charge near the rod.

(d) Then ground connection is removed.

(e) Eventually, we get positively charged sphere.

## The Law of Conservation of Charge

- Charge of an isolated system is conserved.
- This law is a fundamental physical law: net charge is the same before and after any interaction.

## Elementary charges

| | Mass (kg) | Charge (C) |
|---|---|---|
| Neutron, $n$ | $1.675 \times 10^{-27}$ | 0 |
| Proton, $p$ | $1.673 \times 10^{-27}$ | $1.602 \times 10^{-19}$ |
| Electron, $e^-$ | $9.11 \times 10^{-31}$ | $-1.602 \times 10^{-19}$ |

- Elementary charges are electrons and protons. Usually only electrons can be free and take part in electrical processes.
- Excess of electrons causes negative charge and deficiency of electrons causes positive charge of a body.

## Coulomb's law

- From Coulomb's experiments, we can generalize the following properties of the electric force between two stationary point charges:
  - is inversely proportional to the square of the separation $r$ between the particles and directed along the line joining them;
  - is proportional to the product of the charges $q_1$ and $q_2$ on the two particles;
  - is attractive if the charges are of opposite sign and repulsive if the charges have the same sign;
  - is a conservative force.

## Coulomb's Law

- The magnitude of the electric force is

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

- $k_e = 8.9875 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$ is the Coulomb constant, it can be written in the following form:

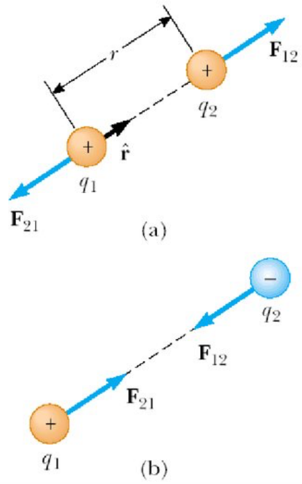
$$k_e = \frac{1}{4\pi\epsilon_0}$$

- where $\epsilon_0 = 8.8542 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$ is the electric permittivity of free space.

In a vector form, the force exerted by charge $q_1$ on $q_2$ is:

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

Where $\hat{r}$ is a unit vector directed from $q_1$ to $q_2$.



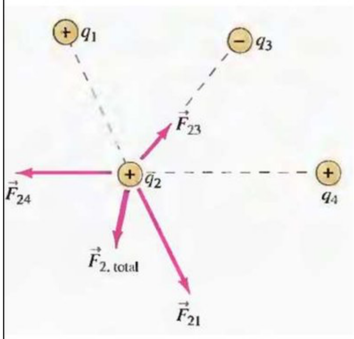

(a) two similar charges repels

(b) two different charges attracts

## Forces of Multiple Charges

Electrostatic force is a vector quantity, so in the case of multiple charges the principle of superposition is applicable:

$$\vec{F} = \sum_{i=1}^{N} \vec{F}_i = \frac{q}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i$$




The total force on charge $q_2$ is the **vector sum** of all forces:

$$\vec{F}_{2,\text{ total}} = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24}.$$

## Electric Field

- In general: field forces can act through space, producing an effect even when no physical contact occurs between interacting objects.
- Charges gives rise to an *electric field.*
- The electric field can be detected at any particular point by a small test positive charge $q_o$ and observing if it experiences a force. Then the electric field vector is:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0}$$

- Note: force $F_e$ and field E are not produced by the test charge $q_o$ .

## Electric Field Vector

- The force exerted by $q$ on the test charge $q_0$ is:

$$\mathbf{F}_e = k_e \frac{q q_0}{r^2} \hat{\mathbf{r}}$$

- Then dividing it by $q_0$ we get the electric field vector:

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

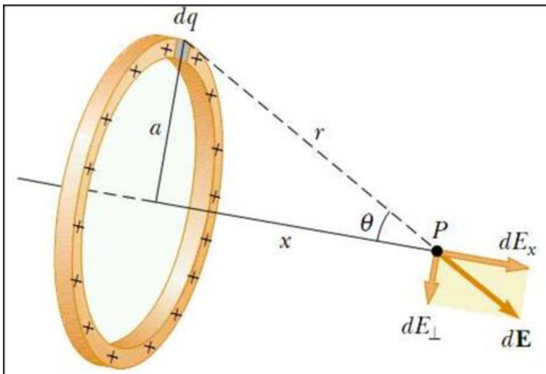
- Electric field is created by a charge.
- If a charge is positive then the electric field vector is directed away from the source charge.
- If a charge is negative then the electric field vector is directed to the source charge.

## Continuous Charge Distribution

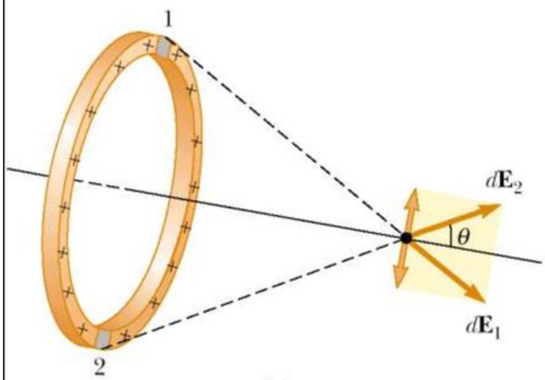
- Volume charge density $\rho \equiv \dfrac{Q}{V} \quad dq = \rho\, dV$
- Surface charge density $\sigma \equiv \dfrac{Q}{A} \quad dq = \sigma\, dA$
- Linear charge density $\lambda \equiv \dfrac{Q}{\ell} \quad dq = \lambda\, d\ell$

## Electric Field of a Uniformly Charged ring

- A ring of radius *a* carries a uniformly distributed positive total charge Q. Let's find the electric field due to the ring along the central axis perpendicular to the plane of the ring.

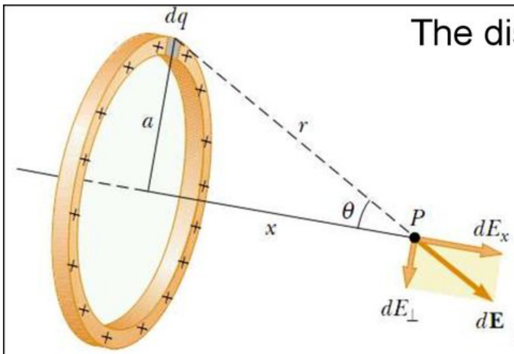



dE is the field at point P on the x axis due to an element of charge dq. dE has two perpendicular components: $E_x$ and $E_\perp$.




(b)

Using the symmetry:
The perpendicular component the field at *P* due to segment 1 is canceled by the perpendicular component due to segment 2.
Thus the total E is directed along x axis.




The distance from a charge $dq$ to point P:

$$r = (x^2 + a^2)^{1/2}$$

$$\cos\theta = x/r,$$

Then the contribution of a charge $dq$ to electric field $E$ at point P is:

$$dE_x = dE\cos\theta = \left(k_e\frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2+a^2)^{3/2}}\,dq$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Thus, we can integrate to obtain the total field at P:

$$E_x = \int \frac{k_e x}{(x^2+a^2)^{3/2}}\,dq = \frac{k_e x}{(x^2+a^2)^{3/2}}\int dq = \frac{k_e x}{(x^2+a^2)^{3/2}}\,Q$$

# Extreme Case Analysis

- So we found the electric field of a uniformly charged ring along its symmetry axis at distance x from the centre of a ring:

$$E_x = \frac{k_e x}{(x^2+a^2)^{3/2}}\,Q$$

$k_e$ is the Coulomb constant, $a$ – the ring's radius, $Q$ – the charge of the ring.

- Let's analyze the obtained result for extreme cases:

1. If x=0, then *E=0.*
2. If x>>a, then we get the Coulomb formula for a point charge:

$$E = k_e\frac{Q}{r^2}$$

- Look more examples of calculating electric field for continuous charge distribution:
  - in Serway p.721-723,
  - Fishbane 642-647.