Applications on Maxwell's 3rd Equation

Find the emf induced in a square loop with sides of length a lying in the yz plane in a region in which the magnetic field changes over time as: $\vec{B}(t) = B_o e^{-5t/t_o}\,\hat{\imath}$

$$emf = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da$$

$$\vec{B}(t) = B_0 e^{-5t/t_0}\hat{i}$$

$$emf = -\frac{d}{dt}\int_S B_0 e^{-5t/t_0}\,\hat{i} \circ \hat{i}\, da$$

$$= -\frac{d}{dt}[B_0 e^{-5t/t_0}\int_S da]$$

$$= -\frac{d}{dt}[B_0 e^{-5t/t_0}\,(a^2)]$$

$$= -a^2 B_0 \frac{d}{dt}[e^{-5t/t_0}] = \frac{5a^2 B_0}{t_0}\, e^{-5t/t_0}$$

A square conducting loop with sides of length L rotates so that the angle between the normal to the plane of the loop and a fixed magnetic field varies as θ(t) = θ₀(t/t₀), find the emf induced in the loop.

$$emf = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da$$

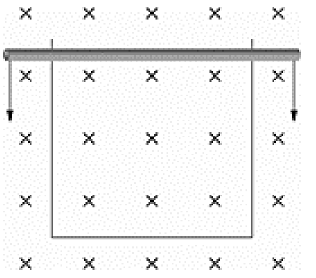
$$= -\frac{d}{dt}\int_S |\vec{B}|\,|\hat{n}|\,\cos[\theta(t)]\, da$$

$$= -\frac{d}{dt}\left\{|\vec{B}|\cos[\frac{\theta_0 t}{t_0}]\int_S da\right\}$$

$$= -|\vec{B}|L^2\,\frac{d[\cos(\frac{\theta_0 t}{t_0})]}{dt}$$

$$= -|\vec{B}|\,L^2[-\sin(\frac{\theta_0 t}{t_0})](\frac{\theta_0}{t_0}) = \frac{|\vec{B}|\,L^2\,\theta_0}{t_0}\sin\left(\frac{\theta_0\, t}{t_0}\right)$$

A conducting bar descends with speed v down conducting rails in the presence of a constant, uniform magnetic field pointing into the page, as shown in the figure. Write an expression for the emf induced in the loop



$$emf = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da$$

$$= -|\vec{B}| \cos(0°) \frac{d}{dt}\int_S da \quad = -|\vec{B}| \frac{dA}{dt}$$

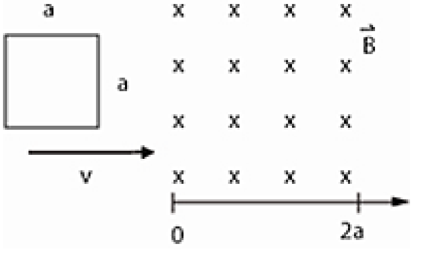
$$emf = -|\vec{B}| \frac{d(wy)}{dt} = -|\vec{B}|\, w \frac{dy}{dt} \quad = -|\vec{B}|\, w\, v$$

A square loop of side s moves with speed v into a region in which a magnetic field of magnitude B exists perpendicular to the plane of the loop, as shown in the figure. Make a plot of the emf induced in the loop as it enters, moves through, and exits the region of the magnetic field.

$$emf = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da$$

$$emf = -\frac{d}{dt}(|\vec{B}| \int_S da) = -\frac{d}{dt}(|\vec{B}|\, A) = -|\vec{B}| \frac{dA}{dt} = -|\vec{B}| \frac{d(ax)}{dt} = -|\vec{B}|\, a \frac{dx}{dt}$$

$$emf = -|\vec{B}|\, a\, v$$



$$emf = -\frac{d\Phi_B}{dt} = 0$$

$$emf = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da = -\frac{d}{dt}(|\vec{B}|\, A)$$

$$= -|\vec{B}| \frac{dA}{dt} = -|\vec{B}| \frac{d}{dt}[(3a - x)a] = -|\vec{B}| \frac{d(-xa)}{dt} = -|\vec{B}|\, a\left(-\frac{dx}{dt}\right) = |\vec{B}|\, a\, v$$

A circular loop of wire of radius 20 cm and resistance of 12 Ω surrounds a 5-turn solenoid of length 38 cm and radius 10 cm as shown in the figure. If the current in the solenoid increases linearly from 80 to 300 mA in 2 seconds, what is the maximum current induced in the circular wire?

$$\left|\vec{B}\right| = \frac{\mu_0 N I}{l}$$

$$emf = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da = -\frac{d}{dt}\int_S \left|\vec{B}\right| \left|\hat{n}\right| da \quad = -\frac{d}{dt}(\left|\vec{B}\right| \pi R^2) = -\pi R^2 \frac{d\left|\vec{B}\right|}{dt}$$

$$= -\pi R^2 \frac{d}{dt}(\frac{\mu_0 N I}{l}) = -\frac{\pi R^2 \mu_0 N}{l}\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{(300-80)\times 10^{-3}\, A}{2\, s} = 0.11 \; A/s$$

$$emf = -\frac{\pi R^2 \mu_0 N}{l}(0.11) = -\frac{\pi (0.1)^2 (4\pi\times 10^{-7})(5)}{0.38}(0.11) \quad = -5.7\times 10^{-8}\, V$$

$$I = \frac{emf}{R} = \frac{-5.7\times 10^{-8}\, V}{12\Omega} = -4.8\times 10^{-9}\; A$$

A 125-turn rectangular coil of wire with sides of 25 and 40 cm rotates about a horizontal axis in a vertical magnetic field of magnitude 3.5 mT. How fast must this coil rotate for the induced emf to reach 5 volts?

$$emf = -\frac{d}{dt}\int_S N\,\vec{B} \circ \hat{n}\, da = -\frac{d}{dt}\int_S N\left|\vec{B}\right|\left|\hat{n}\right| \cos\theta\, da \; = -\frac{d}{dt}[N\left|\vec{B}\right| \cos\theta \int_S da] = -\frac{d}{dt}[N\left|\vec{B}\right| A\cos\theta]$$

$$= -N\left|\vec{B}\right| A \frac{d(\cos\theta)}{dt}$$

$$emf = -N\left|\vec{B}\right| A \frac{d[\cos(\omega t)]}{dt} = -N\left|\vec{B}\right| A\,[-\omega \sin(\omega t)]$$

$$= N\left|\vec{B}\right| A\, \omega \sin(\omega t)$$

$$emf_{Max} = 5\,V = N\left|\vec{B}\right| A\,\omega = 125\,(3.5\times 10^{-3})(0.25)(0.4)\,\omega$$

$$\omega = \frac{5}{125(3.5\times 10^{-3})(0.25)(0.4)} = 114.3 \quad rad/sec$$

The current in a long solenoid varies as $I(t) = I_0 \sin(\omega t)$. Use Faraday's law to find the induced electric field as a function of r both inside and outside the solenoid, where r is the distance from the axis of the solenoid.

Inside the solenoid

$$\oint_C \vec{E} \circ d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da \qquad \oint_C \vec{E} \circ d\vec{l} = \left|\vec{E}\right| \int_C \left|d\vec{l}\right| = \left|\vec{E}\right|(2\pi r)$$

$$-\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da = -\frac{d}{dt}\left(\frac{\mu_0 N I}{l}\pi r^2\right)$$

$$\left|\vec{E}\right|(2\pi r) = -\frac{\mu_0 N(\pi r^2)}{l}\frac{dI}{dt} = -\frac{\mu_0 N(\pi r^2)\omega I_0}{2\pi r l}\cos(\omega t) = -\frac{\mu_0 N r \omega I_0}{2 l}\cos(\omega t)$$

Outside the solenoid

$$\oint_C \vec{E} \circ d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da = -\frac{d}{dt}\left[\frac{\mu_0 N I}{l}\pi R^2\right]$$

$$\left|\vec{E}\right|(2\pi r) = -\frac{\mu_0 N(\pi R^2)}{l}[\omega I_0 \cos(\omega t)]$$

$$\left|\vec{E}\right| = -\frac{\mu_0 N R^2 \omega I_0}{2 r l}\cos(\omega t)$$

The current in a long, straight wire decreases as $I(t) = I_0 e^{-t/\tau}$. Find the induced emf in a square loop of wire of side s lying in the plane of the current-carrying wire at a distance d as shown in the figure.

$$emf = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da = -\frac{d}{dt}\int_S \left|\vec{B}\right| \left|\hat{n}\right| \cos\theta\, da \qquad \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

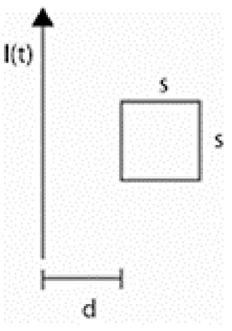
$$emf = -\frac{d}{dt}\int_S \frac{\mu_0 I}{2\pi r} da = -\frac{d}{dt}\int_{y=0}^{s}\int_{x=d}^{d+s} \frac{\mu_0 I}{2\pi x} dx\, dy = -\frac{d}{dt}\left[\frac{\mu_0 I}{2\pi}\int_{y=0}^{s}\int_{x=d}^{d+s}\frac{dx}{x} dy\right] = -\frac{d}{dt}\left[\frac{\mu_0 I}{2\pi} s \ln\left(\frac{d+s}{d}\right)\right]$$

$$emf = -\frac{\mu_0 s}{2\pi}\ln\left(\frac{d+s}{d}\right)\frac{dI}{dt}$$

$$emf = -\frac{\mu_0 s}{2\pi}\ln\left(\frac{d+s}{d}\right)\frac{d}{dt}\left[I_0 e^{-t/\tau}\right] = -\frac{\mu_0 s}{2\pi}\ln\left(\frac{d+s}{d}\right) I_0\left(-\frac{1}{\tau}\right)e^{-t/\tau}$$

$$= \frac{\mu_0 I_0 s}{2\pi\tau}\ln\left(\frac{d+s}{d}\right) e^{-t/\tau}$$

$$= \frac{\mu_0 I_0 s}{2\pi\tau}\ln\left(\frac{d+s}{d}\right) e^{-t/\tau}$$



A circular loop of wire with a radius of 0.2 m is placed in a uniform magnetic field of 0.3 T. The loop is oriented so that its plane is perpendicular to the magnetic field. At time t=0, the magnetic field begins to decrease at a rate of 0.1 T/s. What is the magnitude of the EMF induced in the loop at t=0.5 seconds?

A rectangular loop of wire with dimensions 0.2 m x 0.3 m is placed in a magnetic field that is changing at a rate of 0.1 T/s. The loop is oriented so that its long side is parallel to the magnetic field. At time t=0, the magnetic field begins to decrease at a rate of 0.1 T/s. What is the magnitude of the EMF induced in the loop at t=0.5 seconds?

1.Which of the following is NOT a factor that affects the magnitude of the EMF induced in a coil of wire by a changing magnetic field?

a) The strength of the magnetic field
b) The size of the coil
c) The rate at which the magnetic field changes
d) The resistance of the coil.
e) None

2.Which of the following statements is true regarding Lenz's Law?

a) Lenz's Law describes the direction of the magnetic field generated by a current-carrying wire.
b) Lenz's Law states that the induced EMF in a coil is proportional to the rate of change of magnetic flux.
c) Lenz's Law states that the direction of the induced EMF in a coil is such as to oppose the change that produced it.
d) Lenz's Law states that the magnetic field lines always form closed loops.
e) None

3.A magnetic field is changing at a rate of 0.2 T/s. What is the magnitude of the EMF induced in a circular loop of wire with a radius of 0.5 m if the magnetic field is perpendicular to the plane of the loop?

a) 0.1 V
b) 0.2 V
c) 0.5 V
d) 1.0 V
e) None

4.A long solenoid with N turns per unit length and radius R carries a current I. A second solenoid with N turns per unit length and radius 2R is coaxial with the first solenoid and has a current of 2I. What is the magnitude of the magnetic field inside the first solenoid?

a) $\mu_0 NI$
b) $\mu_0 N(3/2)I$
c) $\mu_0 N(2/3)I$
d) $\mu_0 N(1/2)I$
e) None