Applications on Maxwell's 4th Equation

Two parallel wires carry currents I_1 and $2I_1$ in opposite directions. Use Ampere's law to find the magnetic field at a point midway between the wires.

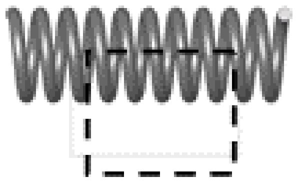
$$\oint_C \bar{B}_1 \circ d\bar{l} = \mu_0 I_{enc} \qquad |\bar{B}_1| \oint_C d\bar{l} = \mu_0 I_1 \qquad |\bar{B}_1|(2\pi r) = \mu_0 I_1$$

$$|\bar{B}_1| = \frac{\mu_0 I_1}{2\pi r}$$

$$|\bar{B}_2| = \frac{\mu_0 (2I_1)}{2\pi r} = \frac{\mu_0 I_1}{\pi r}$$

$$|\bar{B}_{Total}| = |\bar{B}_1| + |\bar{B}_2| = \frac{\mu_0 I_1}{2\pi r} + \frac{\mu_0 I_1}{\pi r} = \frac{3\mu_0 I_1}{2\pi r}$$

Find the magnetic field inside a solenoid (hint: use the Amperian-loop shown in the figure, and use the fact that the field is parallel to the axis of the solenoid and negligible outside).



$$\oint_C \bar{B} \circ d\bar{l} = \int_{side\,1} \bar{B}_1 \circ d\bar{l}_1 + \int_{side\,2} \bar{B}_2 \circ d\bar{l}_2 + \int_{side\,3} \bar{B}_3 \circ d\bar{l}_3 + \int_{side\,4} \bar{B}_4 \circ d\bar{l}_4 = \mu_0 I_{enc}$$

$\bar{B} = |\bar{B}|\,\hat{i}$ within the solenoid $\qquad \bar{B} = 0$ outside the solenoid

$$d\bar{l}_1 = |d\bar{l}_1|\hat{i} \quad d\bar{l}_2 = |d\bar{l}_2|(-\hat{j}) \quad d\bar{l}_3 = |d\bar{l}_3|(-\hat{i}) \quad d\bar{l}_4 = |d\bar{l}_4|\hat{j}$$

$$\int_{side\,1} \bar{B} \circ d\bar{l}_1 = \int_{side\,1} |\bar{B}||d\bar{l}_1| \cos(0^\circ) = |\bar{B}| \int_{side\,1} |d\bar{l}_1| = |\bar{B}|\, l_1$$

$$\int_{side\,2} \bar{B} \circ d\bar{l}_2 = \int_{side\,2} |\bar{B}||d\bar{l}_2| \cos(90^\circ) = 0$$

$$\int_{side\,3} \bar{B} \circ d\bar{l}_3 = \int_{side\,3} 0 \cdot |d\bar{l}_3| = 0$$

$$\int_{side\,4} \bar{B} \circ d\bar{l}_4 = \int_{side\,4} |\bar{B}||d\bar{l}_4| \cos(90^\circ) = 0$$

$$|\bar{B}|\, l_1 = \mu_0 I_{enc}$$

$$|\bar{B}| = \frac{\mu_0 I_{enc}}{l_1}$$

$$|\bar{B}| = \frac{\mu_0 n l_1 I}{l_1} = \mu_0 n I$$

$$|\bar{B}| = \frac{\mu_0 N I}{L}$$

Use the Amperian-loop shown in the figure to find the magnetic field within a torus

$$\oint \vec{B} \circ d\vec{l} = \mu_0 I_{enc} = \mu_0 N I$$

$$I_{enc} = N I$$

$$|\vec{B}| \oint d\vec{l} = \mu_0 N I$$

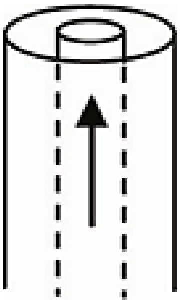
$$|\vec{B}|(2\pi r) = \mu_0 N I$$

$$|\vec{B}| = \frac{\mu_0 N I}{2\pi r}$$

The coaxial cable shown in the figure carries current I_1 in the direction shown on the inner conductor and current I_2 in the opposite direction on the outer conductor. Find the magnetic field in the space between the conductors as well as outside the cable if the magnitudes of I_1 and I_2 are equal.

in the space between the conductors

$$\oint \vec{B} \circ d\vec{l} = \mu_0 I_{enc} = \mu_0 I_1$$



$$|\vec{B}| \oint d\vec{l} = \mu_0 I_1$$

$$|\vec{B}|(2\pi r) = \mu_0 I_1 \qquad |\vec{B}| = \frac{\mu_0 I_1}{2\pi r}$$

outside the cable

$$I_{enc} = I_1 + I_2 = I_1 + (-I_1) = 0$$

$$\oint \vec{B} \circ d\vec{l} = \mu_0 I_{enc} = 0 \qquad |\vec{B}| = 0$$

Find the displacement current produced between the plates of a discharging capacitor for which the charge varies as: $Q(t) = Q_o\, e^{-t/RC}$, where Q_0 is the initial charge, C is the capacitance of the capacitor, and R is the resistance of the circuit through which the capacitor is discharging.

$$I_d = \varepsilon_0 \frac{d}{dt}\int_S \vec{E} \circ \hat{n}\; da \qquad |\vec{E}| = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$I_d = \varepsilon_0 \frac{d}{dt}\int_S \frac{Q}{\varepsilon_0 A}\; da = \varepsilon_0 \frac{d}{dt}\left[\frac{Q}{\varepsilon_0 A}\int_S da\right] = \frac{dQ}{dt}$$

$$I_d = \frac{dQ}{dt} = \frac{d}{dt}\left[Q_0\, e^{-t/RC}\right] = Q_0 \frac{d}{dt}\left[e^{-t/RC}\right]$$

$$I_d = -\frac{Q_0}{RC}\, e^{-t/RC}$$

A magnetic field of: $B = a\sin(by)\, e^{bx}\,\hat{z}$, is produced by an electric current. What is the density of that current?

$$\bar{\nabla}\times\bar{B} = \mu_0\,\bar{J}$$

$$\bar{\nabla}\times\bar{B} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\hat{i} + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\hat{j} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\hat{k}$$

$$\bar{\nabla}\times\bar{B} = \frac{\partial B_z}{\partial y}\,\hat{i} - \frac{\partial B_z}{\partial x}\,\hat{j} = \mu_0\,\bar{J}$$

$$\frac{\partial}{\partial y}\left[a\sin(by)\,e^{bx}\right]\hat{i} - \frac{\partial}{\partial x}\left[a\sin(by)\,e^{bx}\right]\hat{j} = \mu_0\,\bar{J}$$

$$ab\cos(by)\,e^{bx}\,\hat{i} - a\sin(by)\,b\,e^{bx}\,\hat{j} = \mu_0\,\bar{J}$$

$$\bar{J} = \frac{ab\,e^{bx}}{\mu_0}\left[\cos(by)\,\hat{i} - \sin(by)\,\hat{j}\right]$$

Find the electric current density that produces a magnetic field given by:
$B = B_o\,(e^{-2r}\sin\varphi)\ \hat{z}$, in cylindrical coordinates.

$$\bar{\nabla}\times\bar{B} = \left(\frac{1}{r}\frac{\partial B_z}{\partial\phi} - \frac{\partial B_\phi}{\partial z}\right)\hat{r} + \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial(rB_\phi)}{\partial r} - \frac{\partial B_r}{\partial\phi}\right)\hat{z}$$

$$\bar{\nabla}\times\bar{B} = \frac{1}{r}\frac{\partial B_z}{\partial\phi}\,\hat{r} - \frac{\partial B_z}{\partial r}\,\hat{\phi} = \mu_0\,\bar{J}$$

$$= \frac{1}{r}\frac{\partial}{\partial\phi}\left[B_o\,e^{-2r}\sin(\phi)\right]\hat{r} - \frac{\partial}{\partial r}\left[B_o\,e^{-2r}\sin(\phi)\right]\hat{\phi} = \mu_0\,\bar{J}$$

$$= \frac{1}{r}\left[B_o\,e^{-2r}\cos(\phi)\right]\hat{r} + B_o\left[2e^{-2r}\right]\sin(\phi)\,\hat{\phi} = \mu_0\,\bar{J}$$

$$\bar{J} = \frac{B_o\,e^{-2r}}{\mu_o}\left[\frac{1}{r}\cos(\phi)\,\hat{r} + 2\sin(\phi)\,\hat{\phi}\right]$$

What density of current would produce a magnetic field given by:
$B = (\frac{a}{r} + \frac{b}{r}\, e^{-r} + c\, e^{-r})\ \widehat{\emptyset}$, in cylindrical coordinates

$$\bar{\nabla}\times\bar{B} = \mu_0 \bar{J} \qquad \bar{\nabla}\times\bar{B} = \left(\frac{1}{r}\frac{\partial B_z}{\partial\phi} - \frac{\partial B_\phi}{\partial z}\right)\hat{r} + \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial(rB_\phi)}{\partial r} - \frac{\partial B_r}{\partial\phi}\right)\hat{z}$$

$$\bar{\nabla}\times\bar{B} = -\frac{\partial B_\phi}{\partial z}\,\hat{r} + \frac{1}{r}\frac{\partial}{\partial r}(rB_\phi)\,\hat{z}$$

$$= -\frac{\partial\left(\frac{a}{r} + \frac{b}{r}e^{-r} + ce^{-r}\right)}{\partial z}\,\hat{r} + \frac{1}{r}\frac{\partial}{\partial r}\left(a + be^{-r} + cre^{-r}\right)\hat{z}$$

$$= 0 + \frac{1}{r}(-be^{-r} + ce^{-r} - cre^{-r})\,\hat{z}$$

$$= e^{-r}\left(-\frac{b}{r} + \frac{c}{r} - c\right)\hat{z}$$

$$\bar{J} = \frac{\bar{\nabla}\times\bar{B}}{\mu_0} = \frac{e^{-r}}{\mu_0}\left(-\frac{b}{r} + \frac{c}{r} - c\right)\hat{z}$$

To directly measure the displacement current, researchers use a time-varying voltage to charge and discharge a circular parallel-plate capacitor. Find the displacement current density and electric field as a function of time that would produce a magnetic field given by: $\vec{B} = \frac{r\,\omega\,\Delta V\cos(\omega t)}{2d(c^2)}\,\widehat{\emptyset}$.
where r is the distance from the center of the capacitor, ω is the angular frequency of the applied voltage ΔV, d is the plate spacing, and c is the speed of light.

the displacement current density

$$\bar{\nabla}\times\bar{B} = \mu_0\left(\bar{J} + \varepsilon_0\frac{\partial\bar{E}}{\partial t}\right) \qquad \bar{\nabla}\times\bar{B} = \mu_0\,\varepsilon_0\frac{\partial\bar{E}}{\partial t} \qquad \varepsilon_0\frac{\partial\bar{E}}{\partial t} = \frac{1}{\mu_0}(\bar{\nabla}\times\bar{B})$$

$$\varepsilon_0\frac{\partial\bar{E}}{\partial t} = \frac{1}{\mu_0 r}\frac{\partial}{\partial r}\left[r\,\frac{r\,\omega\,\Delta V\cos(\omega t)}{2d(c^2)}\right]\hat{z} = \frac{1}{\mu_0 r}\left[\frac{\omega\,\Delta V\cos(\omega t)}{2d(c^2)}\right]\frac{\partial(r^2)}{\partial r}\,\hat{z} = \frac{\omega\,\Delta V\cos(\omega t)}{\mu_0\, d(c^2)}\,\hat{z}$$

the electric field

$$\bar{E}(t) = \int_0^t \frac{\omega\,\Delta V\cos(\omega t)}{\mu_0\,\varepsilon_0\, d(c^2)}\,\hat{z}\,dt$$

$$\bar{E}(t) = \frac{1}{\omega}\left[\frac{\omega\,\Delta V\sin(\omega t)}{\mu_0\,\varepsilon_0\, d(c^2)}\right]\hat{z} = \frac{\Delta V\ \sin(\omega t)}{\mu_0\,\varepsilon_0\, d(c^2)}\,\hat{z}$$

1-Which of the following is a statement of Ampere-Maxwell's equation?
a) $\nabla \cdot E = \rho/\varepsilon_0$
b) $\nabla \times E = -\partial B/\partial t$
c) $\nabla \times H = J/\sigma$
d) $\nabla \cdot B = 0$
c) None

2-Ampere-Maxwell's equation relates which two quantities?
a) Electric field and magnetic field
b) Electric field and charge density
c) Magnetic field and current density
d) Electric field and current density
c) None

3-What is the significance of Ampere-Maxwell's equation in electromagnetic theory?
a) It relates the electric field and magnetic field to the sources of these fields.
b) It describes the behavior of electric and magnetic fields in the presence of conductors.
c) It explains the phenomenon of electromagnetic radiation.
d) It is used to derive the equations governing the behavior of electromagnetic waves.
c) None

4-Which of the following is a consequence of Ampere-Maxwell's equation?
a) The electric field can induce a magnetic field.
b) The magnetic field can induce an electric field.
c) A changing electric field can induce a changing magnetic field.
d) All of the above.
c) None

5-In which form is Ampere-Maxwell's equation often written in electromagnetism textbooks?
a) $\nabla \cdot E = \rho/\varepsilon_0$
b) $\nabla \times E = -\partial B/\partial t$
c) $\nabla \times H = J/\sigma + \partial D/\partial t$
d) $\nabla \cdot B = 0$
c) None

6-Which of the following is a consequence of Ampere-Maxwell's equation for a time-independent current density?
a) The magnetic field is proportional to the current density.
b) The electric field is proportional to the current density.
c) The magnetic field is proportional to the curl of the current density.
d) The electric field is proportional to the gradient of the current density.
c) None

Which of the following is a mathematical consequence of Ampere-Maxwell's equation?
a) The divergence of the magnetic field is zero.
b) The curl of the electric field is proportional to the time derivative of the magnetic field.
c) The curl of the magnetic field is proportional to the sum of the current density and the displacement current density.
d) The divergence of the electric field is proportional to the charge density.

How does Ampere-Maxwell's equation relate to the phenomenon of electromagnetic waves?
a) It shows that a time-varying magnetic field can induce an electric field, and vice versa, leading to the creation of electromagnetic waves.
b) It describes the way that electromagnetic waves propagate through space, and relates their electric and magnetic field strengths.
c) It explains the polarization of electromagnetic waves, and how they interact with matter.
d) It provides the mathematical framework for describing the interference and diffraction of electromagnetic waves.

What is the displacement current density?
a) The current density due to the motion of charges in a conductor.
b) The current density due to the motion of charges in a dielectric.
c) The current density that arises from time-varying electric fields in free space.
d) The current density that arises from the magnetic fields produced by moving charges.

In what form can Ampere-Maxwell's equation be written in the absence of a time-varying electric field?
a) $\nabla \cdot E = \rho/\varepsilon_0$
b) $\nabla \times E = -\partial B/\partial t$
c) $\nabla \times H = J$
d) $\nabla \cdot B = 0$

What does the integral form of Ampere-Maxwell's equation relate to?
a) The electric field generated by a time-varying magnetic field.
b) The magnetic field generated by a time-varying electric field.
c) The relationship between the current density and the magnetic field.
d) The behavior of electromagnetic waves in free space.
e) None

In the integral form of Ampere-Maxwell's equation, what does the surface integral of the displacement current density represent?
a) The total charge enclosed by a closed surface.
b) The total current enclosed by a closed surface.
c) The change in electric flux through a closed surface.
d) The change in magnetic flux through a closed surface.
e) None

What is the integral form of Ampere-Maxwell's equation?
a) $\nabla \cdot E = \rho/\varepsilon_0$
b) $\nabla \times E = -\partial B/\partial t$
c) $\nabla \times H = J + \partial D/\partial t$
d) $\oint H \cdot dl = I_{enc}/\varepsilon_0 + \partial/\partial t \iint D \cdot dS$
e) None

Which of the following is a consequence of the integral form of Ampere-Maxwell's equation for a steady-state current?
a) The magnetic field is proportional to the current density.
b) The electric field is proportional to the current density.
c) The magnetic field is proportional to the curl of the current density.
d) The electric field is proportional to the gradient of the current density.
e) None

How is the integral form of Ampere-Maxwell's equation related to the differential form?
a) The differential form is derived from the integral form using the divergence theorem.
b) The integral form is derived from the differential form using the divergence theorem.
c) The differential form is derived from the integral form using Stokes' theorem.
d) The integral form is derived from the differential form using Stokes' theorem.
e) None

Consider a region of space where the magnetic field is given by $B = B_o\, e^{(-z/a)}\, \hat{\imath}$. Find the electric field in this region, assuming there are no charges or currents present.

Consider a cylindrical wire of radius R carrying a current I. Find the magnetic field both inside and outside the wire, using Ampere-Maxwell's equation in differential form.

A parallel-plate capacitor consists of two plates of area A separated by a distance d. The plates are connected to a voltage source of V volts. Find the displacement current density between the plates as a function of time, assuming the voltage is time-varying.

A square loop of side L is located in the xy-plane with one corner at the origin. A uniform magnetic field B is directed in the positive z-direction. Find the magnetic flux through the loop as a function of time, assuming the magnetic field is time-varying.