Sources of electromagnetic waves

Maxwell's theory of electromagnetic waves

1- Neither stationary charges nor charges in uniform motion (steady currents) can be sources of electromagnetic waves.

2- Accelerated charges radiate electromagnetic waves.

The results of (1) are both (stationary charges and charges in uniform motion) produced electric and magnetic field respectively, while in (2), the accelerated charges radiate electromagnetic waves.

According to Maxwell's equations, electromagnetic waves are produced by changing electric and magnetic fields. Specifically, a time-varying electric field can create a magnetic field, which in turn can create a time-varying electric field, and so on. This process continues, creating a self-sustaining electromagnetic wave that propagates through space at the speed of light.

However, it is important to note that not all sources of electric and magnetic fields can produce electromagnetic waves. In particular, stationary charges and charges in uniform motion, or steady currents, cannot be sources of electromagnetic waves.

This is because stationary charges produce only static electric fields, which do not change with time and therefore cannot create a magnetic field that varies with time. Similarly, steady currents produce only static magnetic fields, which do not change with time and therefore cannot create an electric field that varies with time.

In order for electromagnetic waves to be produced, there must be a changing electric or magnetic field, which can only occur when charges are accelerating or changing direction. For example, when an oscillating current flows through an antenna, it creates a time-varying electric field, which in turn creates a time-varying magnetic field, and so on. This creates an electromagnetic wave that can be transmitted through space and detected by a receiver.

When considering electromagnetic waves in free space, we note that the medium is source less, meaning that $\rho_v = \mathbf{J} = 0$. Under these conditions, Maxwell's equations may be written in terms of **E** and **H** only as:

Source *: 1-Engineering Electromagnetics* by William H. Hayt, Jr. & John A. Buck.

2- College physics by Serway, R. A., & Vuille C.

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (1)$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \quad (2)$$

$$\nabla \cdot \mathbf{E} = 0 \quad (3)$$

$$\nabla \cdot \mathbf{H} = 0 \quad (4)$$

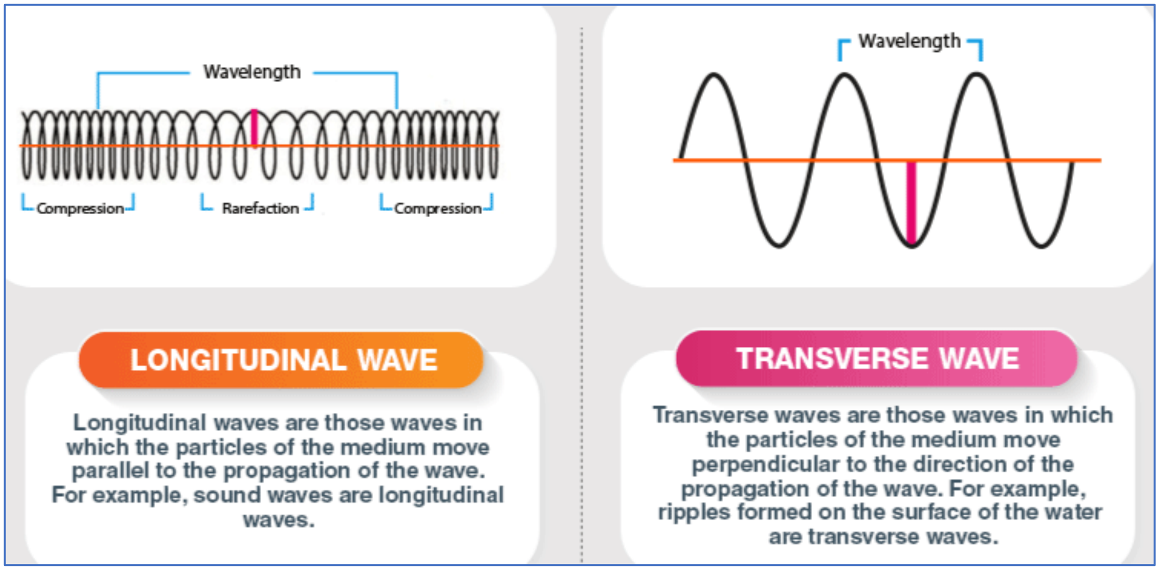
From these equations we can conclude that:

- if electric field **E** is changing with time at some point, then magnetic field **H** has curl at that point, therefore **H** varies spatially in a direction normal to its orientation direction.
- if **E** is changing with time, then **H** will in general also change with time, although not necessarily in the same way.
- a time-varying **H** generates **E**, which, having curl, varies spatially in the direction normal to its orientation.

Transverse electromagnetic (TEM) wave

A transverse electromagnetic (TEM) wave is a type of electromagnetic wave in which the electric and magnetic fields are perpendicular to the direction of wave propagation, meaning they oscillate in a direction perpendicular to the direction the wave is traveling.

In other words, a TEM wave is a type of wave in which the electric and magnetic fields are "transverse" to the direction of the wave's motion. This is in contrast to longitudinal waves, in which the vibrations are parallel to the direction of the wave's motion.



TEM waves are commonly used in transmission lines, such as coaxial cables, which are used to transmit signals in telecommunications and other applications. They are also used in microwave engineering and radar systems.

One of the key features of TEM waves is that they do not have a varying electric or magnetic field component in the direction of propagation, making them useful for applications where minimal interference and noise is desired.

We postulate the existence of a uniform plane wave, in which both fields, **E** and **H**, lie in the transverse plane i.e., the plane who's normal is the direction of propagation. Furthermore, and by definition, both fields are of constant magnitude in the transverse plane. The required spatial variation of both fields in the direction normal to their orientations will therefore occur only in the direction of travel or normal to the transverse plane.

for example;

that $\mathbf{E} = E_x \mathbf{a}_x$, i.e. the electric field is polarized in the x direction. If we further assume that wave travel is in the z direction, we allow spatial variation of **E** only with z. Using Eq. (2), the curl of E is given by:

$$\nabla \times \mathbf{E} = \frac{\partial E_x}{\partial z}\mathbf{a}_y = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} = -\mu_0 \frac{\partial H_y}{\partial t}\mathbf{a}_y \qquad (5)$$

The directions of **E** and **H** and the direction of travel are mutually orthogonal. Using Eq. (1),

Source : *1-Engineering Electromagnetics* by William H. Hayt, Jr. & John A. Buck.
2- College physics by Serway, R. A., & Vuille C.

$$\nabla \times \mathbf{H} = -\frac{\partial H_y}{\partial z}\mathbf{a}_x = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \epsilon_0 \frac{\partial E_x}{\partial t}\mathbf{a}_x \qquad (6)$$

Equations (5) and (6) can be written:

$$\boxed{\begin{aligned} \frac{\partial E_x}{\partial z} &= -\mu_0 \frac{\partial H_y}{\partial t} \qquad (7) \\ \frac{\partial H_y}{\partial z} &= -\epsilon_0 \frac{\partial E_x}{\partial t} \qquad (8) \end{aligned}}$$

Differentiated with respect to *z* & *t* for (7) and (8) Respectively:

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu_0 \frac{\partial^2 H_y}{\partial t \partial z} \qquad (9)$$

$$\frac{\partial^2 H_y}{\partial z \partial t} = -\epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \qquad (10)$$

Equating the similar parts;

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}} \qquad (11)$$

Where

$$\nu = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s} = c \qquad (12)$$

A similar procedure, involving, differentiating (7) with *t* and (8) with *z*, yields the wave equation for the magnetic field;

$$\boxed{\frac{\partial^2 H_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 H_y}{\partial t^2}} \qquad (13)$$

The wave Equation

Considering electromagnetic waves travels in free space, the fact that;.

$\nabla \cdot \mathbf{B} = 0$, $\nabla \cdot \mathbf{E} = 0$, $\mathbf{J}=0$ and $\rho=0$

Also $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$

For **E** :.

$$\nabla \times \mathbf{E} = -\partial/\partial t(\nabla \times \mathbf{B})$$

$$\nabla \times (\nabla \times \mathbf{E}) = -\partial/\partial t(\nabla \times \mathbf{B}) = -\mu_o\partial/\partial t(\mathbf{J} + \varepsilon_o\partial\mathbf{E}/\partial t)$$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E} = -\mu_o\partial/\partial t(\mathbf{J} + \varepsilon_o\partial\mathbf{E}/\partial t)$$

$$\nabla^2\mathbf{E} - (1/c^2)\partial^2\mathbf{E}/\partial t^2 = 0$$

For **B**:.

$$\nabla \times \mathbf{B} = \mu_o(\mathbf{J} + \varepsilon_o\partial\mathbf{E}/\partial t)$$

$$\nabla \times (\nabla \times \mathbf{B}) = \mu_o(\partial/\partial t(\nabla \times \mathbf{E}) - \nabla \times \mathbf{J})$$

$$\nabla(\nabla \cdot \mathbf{B}) - \nabla^2\mathbf{B} = \mu_o\varepsilon_o\partial^2\mathbf{B}/\partial t^2 + \mu_o\partial\mathbf{J}/\partial t$$

$$\nabla^2\mathbf{B} - (1/c^2)\partial^2\mathbf{B}/\partial t^2 = 0$$

Where; $\mu_o\varepsilon_o = 1/c^2$

The general solution to the wave equation depends on the boundary conditions and the nature of the sources of the wave. However, I can give you the general form of the solution for a homogeneous wave equation, which is given by:

$$E_x(z,t) = f_1(t - z/\nu) + f_2(t + z/\nu) \qquad (14)$$

where again f_1 amd f_2 can be any function whose argument is of the form ($t \pm z/v$) represent sinusoidal functions of a specified frequency in the form of forward- and backward-propagating cosines. And because the waves are sinusoidal, we denote their velocity as the phase velocity, v_p. The waves are formulated as;

$$\begin{aligned} E_x(z,t) &= \mathcal{E}_x(z,t) + \mathcal{E}'_x(z,t) \\ &= |E_{x0}|\cos[\omega(t - z/\nu_p) + \phi_1] + |E'_{x0}|\cos[\omega(t + z/\nu_p) + \phi_2] \\ &= \underbrace{|E_{x0}|\cos[\omega t - k_0 z + \phi_1]}_{\text{forward } z \text{ travel}} + \underbrace{|E'_{x0}|\cos[\omega t + k_0 z + \phi_2]}_{\text{backward } z \text{ travel}} \qquad (15) \end{aligned}$$

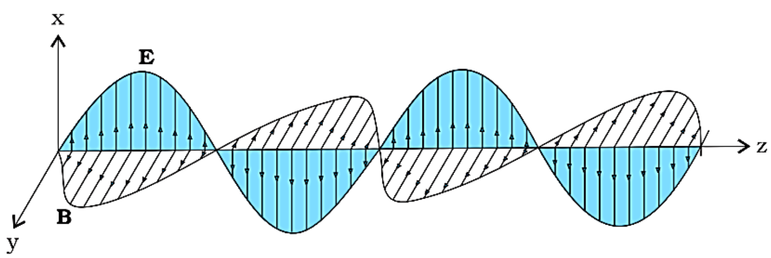
the waves are traveling in free space, in which case the phase velocity, $v_p = c$. Additionally, the wavenumber in free space is defined as $k_o\, z = k_o\lambda = 2\pi$;

$$\boxed{\lambda = \frac{2\pi}{k_0} \quad \text{(free space)}} \tag{16}$$

The typical example of a plane electromagnetic wave propagating along the *z*-direction (the fields are shown as a function of the *z* coordinate, at a given time *t*). The electric field $\boldsymbol{E_x}$ is along the *x*-axis, and varies sinusoidally with *z*, at a given time. The magnetic field $\boldsymbol{B_y}$ is along the *y*-axis, and again varies sinusoidally with *z*. The electric and magnetic fields $\boldsymbol{E_x}$ and $\boldsymbol{B_y}$ are perpendicular to each other, and to the direction *z* of propagation. We can write $\boldsymbol{E_x}$ and $\boldsymbol{B_y}$ as follows:

$\boldsymbol{E_x} = E_0 \sin(kz - \omega t)$

$\boldsymbol{B_y} = B_0 \sin(kz - \omega t)$



$\boldsymbol{k}$ the wave vector (or propagation vector) and its direction describes the direction of propagation of the wave. The speed of propagation of the wave is ($\omega/\boldsymbol{k}$).

for Ex and By and Maxwell's equations

$\omega = ck$

$k = 2\pi\, \nu$

$\nu\lambda = c$

also from Maxwell's equations, we can prove that, the magnitude of the electric and the magnetic fields in an electromagnetic wave are related as;

$B_0 = E_0/c$

Example: A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, $\mathbf{E} = 6.3\,\hat{\jmath}$ V/m. What is B at this point?

Solution:

$$B = \frac{E}{c} = \frac{6.3\ \text{V/m}}{3\times10^{8}\ \text{m/s}} = 2.1\times10^{-8}\ \text{T}$$

To find the direction, we note that E is along y-direction and the wave propagates along x-axis. Therefore, B should be in a direction perpendicular to both x- and y-axes. Using vector algebra, E × B should be along x-direction. Since, (+ ˆj) × (+kˆ) = ˆi , B is along the z-direction.

Thus, B = 2.1 × 10–8 kˆ T

Example: The magnetic field in a plane electromagnetic wave is given by

$B_y = (2 \times 10^{-7}) \sin (0.5\times10^{3} x+1.5\times10^{11}t)$ T.

(a) What is the wavelength and frequency of the wave? (b) Write an expression for the electric field.

Solution:

(a) Comparing the given equation with

$$B_y = B_0 \sin\left[2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)\right]$$

We get, $\lambda = \dfrac{2\pi}{0.5\times10^{3}}$ m = 1.26 cm,

and $\dfrac{1}{T} = \nu = \left(1.5\times10^{11}\right)/2\pi = 23.9$ GHz

(b) $E_0 = B_0 c = 2\times10^{-7}$ (T) × 3 × 10^8 (m/s) = 6 × 10^1 (V/m).

The electric field component is perpendicular to the direction of propagation and the direction of magnetic field. Therefore, the electric field component along the z-axis is obtained as $E_z = 60 \sin (0.5 \times 10^3\, x + 1.5 \times 10^{11}\, t)$ V/m

Example: A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

Solution:

to find the corresponding wavelength band, we can use the formula: wavelength (λ) = speed of light (c) / frequency (f)

where the speed of light (c) is approximately 3×10^8 m/sec.

The lowest frequency in the given band is 7.5 MHz, which is equivalent to 7.5×10^6(Hz). The corresponding wavelength can be found as:

$\lambda = c / f = 3\times10^8 / 7.5\times10^6 = 40$ m

Similarly, the highest frequency in the band is 12 MHz, which is equivalent to 12 x 10^6 Hz. The corresponding wavelength can be found as:

$\lambda = c / f = 3\times10^8 / 12\times10^6 = 25$ m

Therefore, the wavelength band for the radio's frequency range is approximately 25 meters to 40 meters.

What is the relationship between the electric and magnetic fields in an electromagnetic wave?

a) E and B are perpendicular to each other and to the direction of propagation
b) E and B are parallel to each other and to the direction of propagation
c) E and B are perpendicular to each other and parallel to the direction of propagation
d) E and B are parallel to each other and perpendicular to the direction of propagation

Which of the following is true for a transverse wave?

a) The wave motion is perpendicular to the direction of wave propagation
b) The wave motion is parallel to the direction of wave propagation
c) The wave motion is both parallel and perpendicular to the direction of wave propagation
d) The wave motion is circular in nature

What is the relationship between the frequency and wavelength of an electromagnetic wave in vacuum?

a) They are inversely proportional
b) They are directly proportional
c) They are unrelated
d) They are proportional to the square of each other