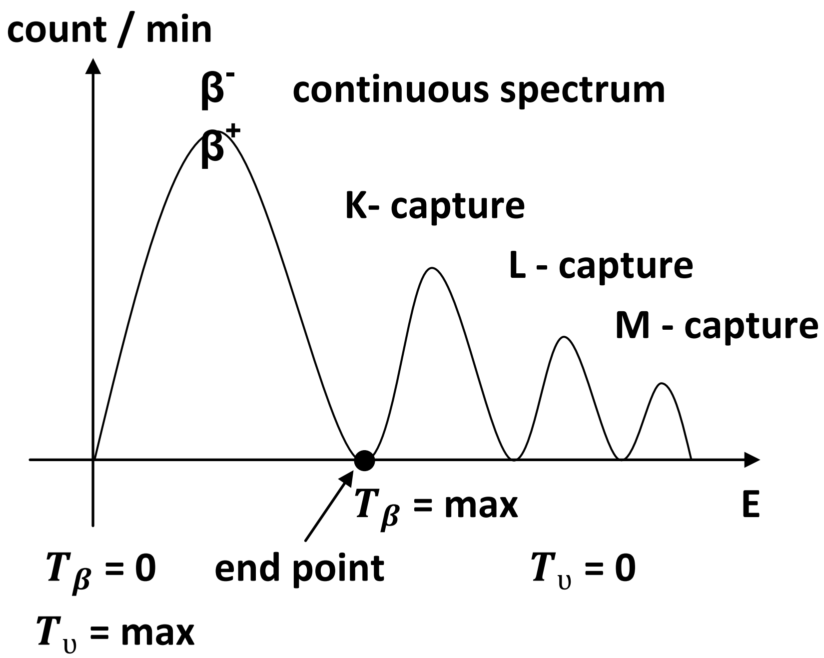
Beta – Decay (β – decay) :

In beta decay electrons ($\boldsymbol{\beta^-}$) positron ($\boldsymbol{\beta^+}$) are emitted from the nucleus with continuous energy distribution (continuous spectrum), the third type of beta decay is the electron capture , the three types of beta decay can be presented as follow :

① ***β^- - decay : $^1_0n \rightarrow {}^1_1P + \beta^- + \bar{\upsilon}$ (antineutrino)***

② ***β^+- decay : $^1_1P \rightarrow {}^1_0n + \beta^+ + \upsilon$ (neutrino)***

③ ***electron capture (E_c) : $^1_1P + \bar{e} \rightarrow {}^1_0n + \upsilon$***



The continuous electron spectrum of
Cs – 137 and K , L , M conversion lines

K – M – L Capture جاءت من ملئ الفجوات عند اقتناص الكترون من مدار ذري ثم يملأ من قبل **K** أو **L** أو **M** ... الخ

* لوحظ ذلك من خلال الأشعة السينية المميزة

Neutrino Hypothesis (1933) :-

The continuous spectrum in **β** – decay cannot be explained without the existence of ***neutrino*** (***υ***)

$$T_{\beta\, max} = [M_P - (M_d + m_e)]c^2 \rightarrow$$ **بدون نيترينيو**

كتلة الأم M_P , كتلة الوليدة M_d , كتلة الإلكترون m_e

There is a violation of the energy conservation law , because of the beta particle is emitted only there will be definite energy by it.

Since the spectrum is continuous this cannot be explained by the equation above.

There must be another particle carries part of the energy which should has Zero charge , Zero mass called the ***Neutrino*** (***υ***)

Therefore the equation above must be :-

$$T_{\beta\, max} = [M_P - (M_d + m_e)]c^2 - T_{\upsilon} \rightarrow$$ **طاقة حركية للنيترينو**

قليلة جدا ما زالت قد البحث $m_{\upsilon} = \left[\frac{1}{2000} - \frac{1}{2500}\right] m_e$

From the conservation of the momentum and according to the (***n – p***) hypothesis and all odd – A nuclei , are expected to have $\frac{1}{2}$ integer spin

If the neutrino is not exist , then , i.e.

$$n \rightarrow p + \beta^-$$

$$\frac{1}{2} = \frac{1}{2} \pm \frac{1}{2}$$

فان العلاقة اعلاه تكون غير صحيحة

$\frac{1}{2} = \frac{1}{2} \pm \frac{1}{2} \rightarrow$ ***1 or 0***

is not correct , therefore must exist a particle with $\frac{1}{2}$ spin and **Z = 0** , **A = 0**

i.e. $\frac{1}{2}$ ***integer*** = $\frac{1}{2}$ ***integer*** $\pm \frac{1}{2} \pm \frac{1}{2}$

$\therefore \frac{1}{2}$ ***integer*** = $\frac{1}{2}$ ***integer***

$*$ ***Energetic in Beta – decay*** :-

The conservation law of energy in **β** – decay can be written as follows :

$$T_{\beta\, max} = [M_P - M_d - M_e]c^2 - T_\upsilon$$

$$\therefore T_\beta + T_\upsilon = [M_P - M_d - M_e]c^2$$

$$Q = [M_P - M_d - M_e]c^2 \Rightarrow Q = T_\beta + T_\upsilon \Rightarrow \textbf{energy – decay}$$

معادلة عامة طاقة الانحلال (***Q***)

$$Q = T_\beta + T_\upsilon = [M_P - M_d - M_e]c^2$$

$$T_d \ll M_d c^2$$

① β^- - *decay*

$$\therefore\ n \rightarrow P + \beta^- + \bar{\upsilon}$$

$$^{3}_{1}H_2 \rightarrow {}^{3}_{2}He_1 + \beta^- + \bar{\upsilon}$$

$$\text{(Parent) الأم}\ M(A,Z) \rightarrow M_d(A,Z+1) + m_e + Q_{\beta^-} \ \ldots\ldots *$$

$Q_{\beta^-} = T_{\beta^-} + T_{\bar{\upsilon}}$

الطاقة الناتجة يتقاسمها الالكترون و النيترينيو و لهذا تكون **مستمرة**

توضيح لمعادلة ($*$) عندما تنحل النواة الأم تلقائيا ($^{3}_{1}H_2$) الى هليوم ($^{3}_{2}He_1$) فضلا عن تكون او انتاج اشعاع بيتا السالب (β^-) (على شكل الكترون سالب) له كتلة (M_e) بالإضافة الى طاقة متحررة Q_{β^-} ($Q_{\beta^-} = T_{\beta^-} + T_{\bar{\upsilon}}$)

فلو فرضنا اضافة Zm_e الى طرفي المعادلة ($*$) ينتج

$$M(A,Z) + Zm_e = M_d(A,Z+1) + Zm_e + m_e + \frac{Q_{\beta^-}}{c^2}$$

$$M(A,Z) + Zm_e = M_d(A,Z+1) + m_e(Z+1) + \frac{Q_{\beta^-}}{c^2}$$

$$M(A,Z) = M_d(A,Z+1) + \frac{Q_{\beta^-}}{c^2}$$

$$\therefore\ Q_{\beta^-} = [M_P - M_d]c^2$$

② β^+ - decay

$$\therefore\ {}^{1}_{1}P \rightarrow {}^{1}_{0}n + \beta^+ + \upsilon$$

$${}^{11}_{6}C \rightarrow {}^{11}_{5}B + \beta^+ + \upsilon$$

نلاحظ خلال انحلال بيتا الموجب (β^+) ان شحنة النواة الوليدة اقل من شحنة النواة الأم **بوحدة واحدة** و لذلك فان

$$M(A,Z) = \bar{M}_D(A,Z-1) + m_e + \frac{Q_{\beta^+}}{c^2}$$

و لتحويل الكتل النووية الى كتل ذرية نضيف Zm_e الى الطرف الايسر و نضيف المقدار $Zm_e + m_e - m_e$ الى الطرف الايمن نحصل على

$$M(A,Z) + Zm_e = \bar{M}_D(A,Z-1) + Zm_e + m_e - m_e + m_e + \frac{Q_{\beta^+}}{c^2}$$

$$M(A,Z) + Zm_e = \bar{M}_D(A,Z-1) + (Z-1)m_e + 2m_e + \frac{Q_{\beta^+}}{c^2}$$

$$M_p = M_D + 2m_e + \frac{Q_{\beta^+}}{c^2}$$

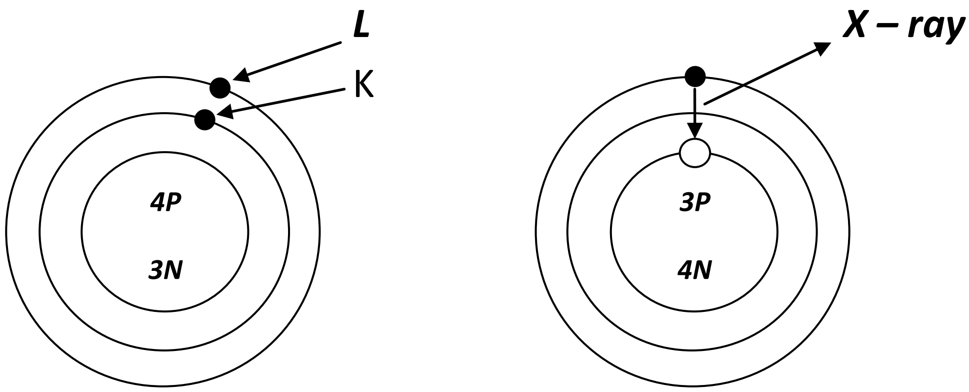
$$Q_{\beta^+} = [M_P - M_D - 2m_e]c^2$$

$$m_e c^2 = 0.511\ MeV$$

$$2m_e c^2 = 1.022\ MeV$$

و اذا كان $(M_P - M_D)c^2 < 1.022$ لا يحدث الانحلال

③ ***Electron Capture*** :-



$${}^{7}_{4}Be_{3} \xrightarrow[53.6\ d]{e.c} {}^{7}_{3}Li_{4} + \upsilon$$

$$M_P + m_e \rightarrow M_d + \frac{Q}{c^2}$$

Or

$$M_P(A,Z) \rightarrow M_d(A,Z-1) + \frac{Q}{c^2}$$

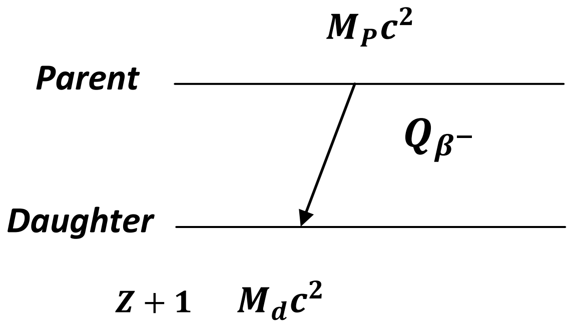
$$Q_{e.c} = [M_P - M_d(Z-1)]\,c^2 - E_e$$

Where E_e is the binding energy of electron in its atomic shell capture

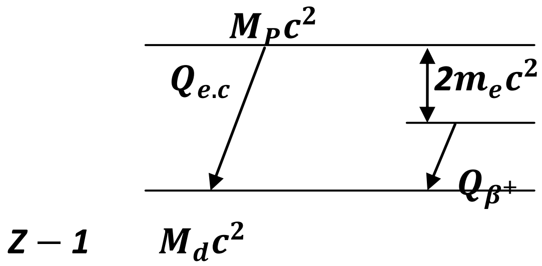
* ان عملية اقتناص الالكترون من القشرة الذرية القريبة (الداخلية) يتبعه عملية ثانوية هي انبعاث اشعة (***X – ray***)

** ان طاقة ***X – ray*** المنبعثة يقدر بطاقة ارتباط الالكترون المقتنص.

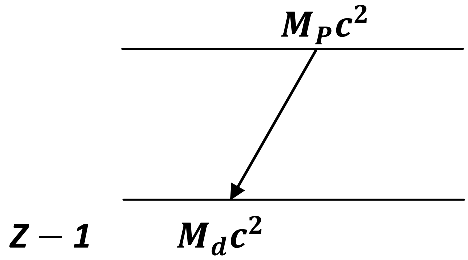
*** يمكن التعبير عن الانحلالات لبيتا كما يلي



$M_P c^2$
Parent
Q_{β^-}
Daughter
$Z+1$ $M_d c^2$



$M_P c^2$
$Q_{e.c}$
$2m_e c^2$
Q_{β^+}
$Z-1$ $M_d c^2$



$M_P c^2$
$Z-1$ $M_d c^2$

Decay constant for Beta Decay :-

The half life for beta decay vary approximately from $\mathbf{10^{-13}}$ sec up to $\mathbf{10^{16}}$ year.

The various types of beta decay can be classified by

① The orbital angular momentum (ℓ) carried by electron and neutrino ***(e - υ pair)***.

② The parity change **(Δπ)** which occurs

③ The direction of spins of electron and neutrino (↑↓) or (↑↑)

a) If the directions of spins of electron and neutrino are parallel (↑↑) or (↓↓) the decay is called ***Gammo – Teller***

$$S_e \,||\, S_\upsilon$$

$$\therefore\ S_\beta = \frac{1}{2} + \frac{1}{2} = 1$$

$$\therefore\ S_\beta = 1 \quad \Rightarrow \quad \textit{(G – T) decay mode}$$

b) If the directions of spins are anti parallel (↑↓) , then the decay called ***Fermi – decay***

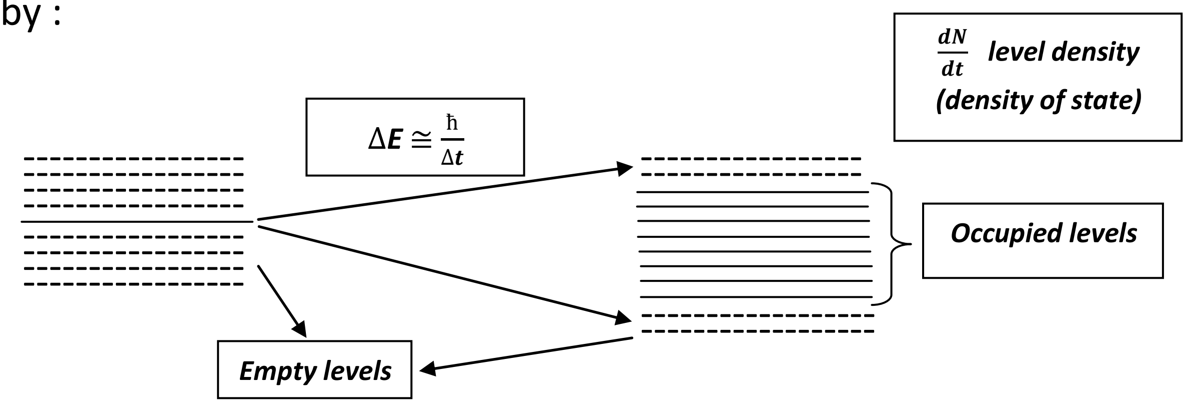
$$S_e \nparallel S_\upsilon$$

$$S_\beta = \frac{1}{2} - \frac{1}{2} = 0$$

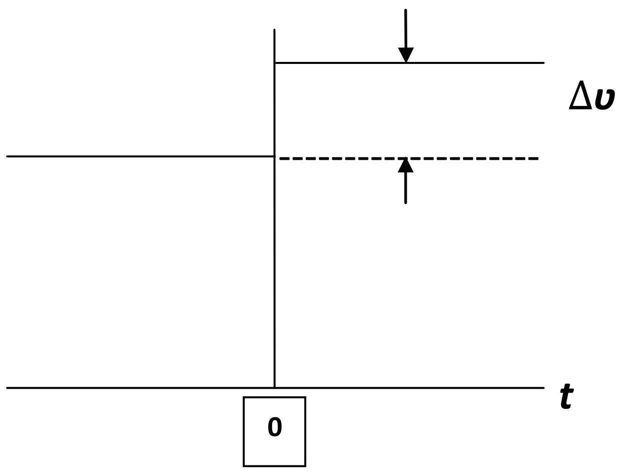
$$S_\beta = 0 \quad \Rightarrow \quad \textit{Fermi decay mode}$$

∗ ***Fermi Theory of Beta Decay*** :-

Our treatment of beta decay based on a fundamental theorem of quantum – mechanics which state that the transition rate λ between an initial (**i**) and final (**f**) states is given by :



Quantum mechanical treatment of transition probability ***υ(t)***



Δυ (is perturbation Potential by transition)

The decay constant of states (or the transition probability per unit time) is proportional to :

$\lambda \propto \frac{dN}{dE}$ $\Rightarrow$ level density or **ρ**(**E**) كثافة الحالات النهائية

λ = **const.** $\frac{dN}{dE}$

$\lambda = \frac{2\pi}{\hbar} |\int \Psi_f^* \, \Delta\upsilon \, \Psi_i \, d\upsilon|^2 \, \frac{dN}{dE}$

Where

Ψ_i (***system***) = Ψ_p (***parent nucleus***)

Ψ_f(***system***) = Ψ_D(***daughter***) Ψ_{β^-}(***electron***) $\Psi_{\bar{\upsilon}}$(***antineutrino***)

Therefore

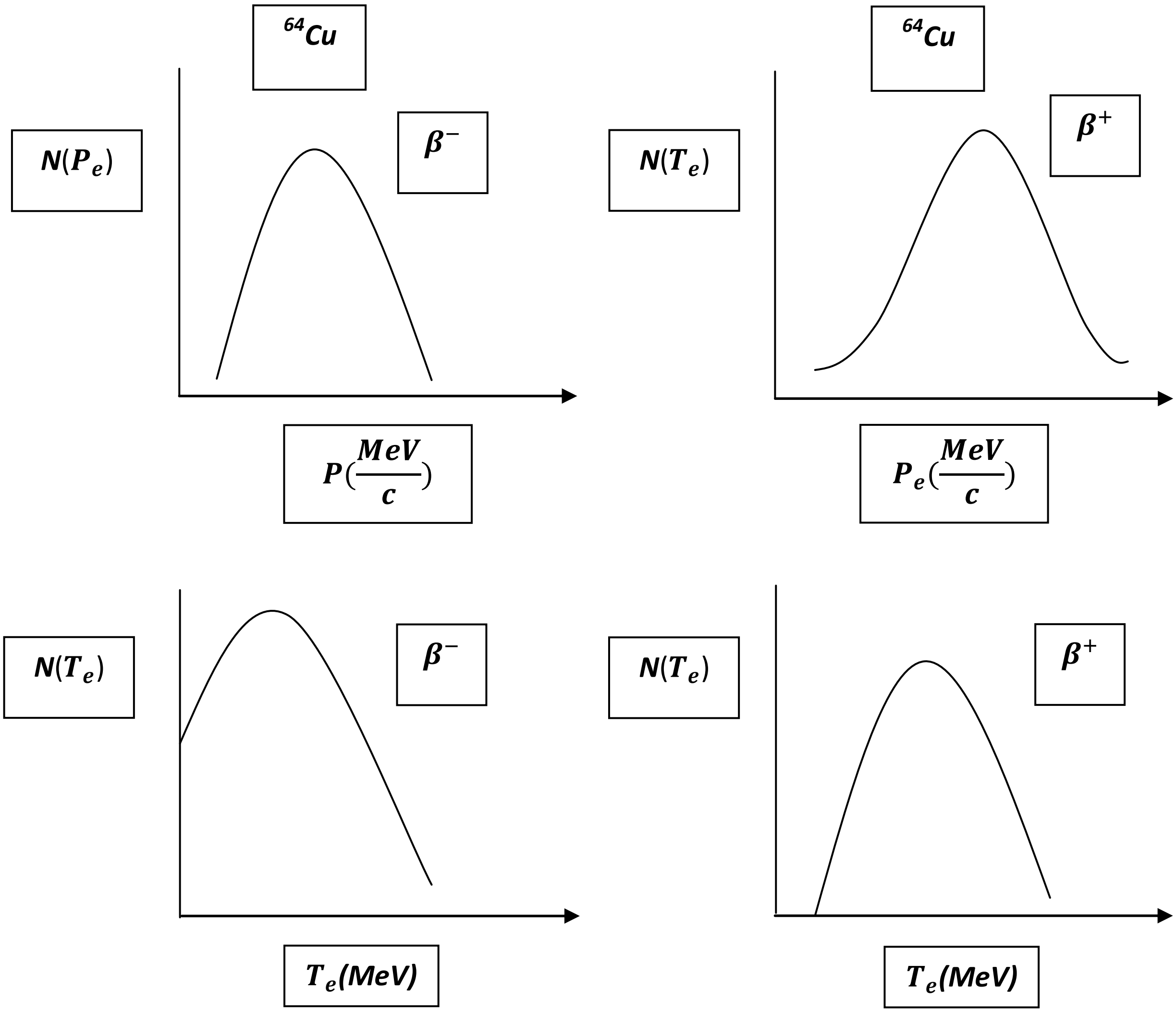
$\lambda = \frac{2\pi}{\hbar} |\int \Psi_D^* \, \Psi_{\beta^-}^* \, \Psi_{\bar{\upsilon}}^* \, \Delta\upsilon \, \Psi_p \, d\upsilon|^2 \, \frac{dN_{tot.}}{dQ_{\beta^-}}$

$\lambda = \frac{2\pi}{\hbar} |M|^2 \, \frac{dN_{tot.}}{dQ_{\beta^-}}$

Where ***M*** $\Rightarrow$ is the matrix element of transition

$M = \int \Psi_D^* \, \Psi_{\beta^-}^* \, \Psi_{\bar{\upsilon}}^* \, \Delta\upsilon \, \Psi_p \, d\upsilon$

And $\frac{dN_{tot.}}{dQ_{\beta^-}}$ $\Rightarrow$ the density of states (***levels***) which **mainly determined the shape** of beta spectrum



Spectrum of momentum and kinetic energy for β^+ and β^- emitted from ^{64}Cu

The electron ($\boldsymbol{\beta^-}$) is hold back by the positive electric field of the nucleus (***+Ze***) , while the positron ($\boldsymbol{\beta^+}$) is repelled by the nucleus ; therefore , the quantity $|\mathbf{M}|^2$ is found to contain a coulomb penetration factor

$$|M|^2 = F(Z_D, P_e)\, |\bar{M}|^2$$

Where $|\bar{M}|$ $\Rightarrow$ is the matrix element without penetration factor.

$F(Z_D, P_e)$ $\Rightarrow$ is the penetration effected or ***Fermi – Function***.

(مثال محلول صفحة 193 في كتاب الفيزياء النووية)

Example :-

Determine the energy release due to free neutron decay ($t_{\frac{1}{2}}$ = 10 min) , if m_n = **1.008665 amu ,**
m_P **= 1.007825 amu ,** m_e **= 0.0005498 amu and**
M_H **= 1.007825 amu ?**

Solution

$\therefore\ n \rightarrow p + \beta^- + \bar{\upsilon}$

$$Q = \left(M_n - M_p - M_{\beta^-} - M_{\bar{\upsilon}}\right)c^2$$

$$M_{\bar{\upsilon}} \cong 0 \rightarrow \text{تهمل}$$

$$Q = (1.008665 - 1.007825 - 0.0005498)c^2$$

$$= (0.000841\ amu) \times 931.5 = 0.782\ MeV$$

Example :- Giving the following nuclei

$^{64}_{28}Ni \rightarrow$ **63.94813 amu** , $^{64}_{29}Cu \rightarrow$ **63.9494 amu**
$^{64}_{30}Zn \rightarrow$ **63.94932 amu** , $^{64}_{31}Ga \rightarrow$ **63.95710 amu**

① which one of them is stable? Which one is radioactive?

② Which one can decay by β - emission , What are the end points of the spectra and Q – value of the decay?

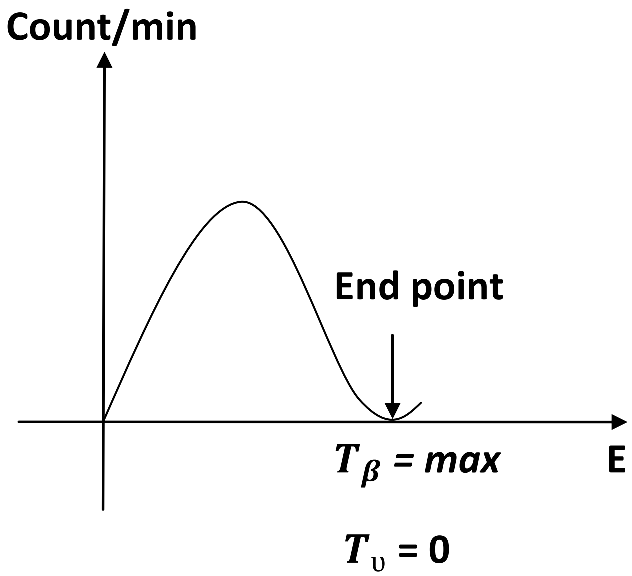
$* \ ^{64}_{29}Cu \rightarrow \ ^{64}_{28}Ni + \beta^+ + \upsilon$ نيترينو

$$Q_{\beta^+} = [M_P - M_d - 2m_e]c^2 \qquad 2m_e = 2(0.51\ MeV)$$
$$= 1.02\ MeV$$

$* \ {}^{64}_{29}Cu \rightarrow {}^{64}_{30}Zn + \beta^{-} + \bar{\upsilon}$ نيترينيو مضاد

$$Q_{\beta^-} = [M_P - M_d]c^2$$

$* \ {}^{64}_{31}Ga \rightarrow {}^{64}_{30}Zn + \beta^{+} + \upsilon$ $\qquad Q_{\beta^+} = ?$



(سؤال 12-6 صفحة 213 في كتاب الفيزياء النووية)