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Analytical Mechanics

2023-2024

Lec.14: lagrangian mechanics

Constrained motion of a particle

--- When a moving particle is restricted geometrically in the sense that it must stay on a certain definite surface or curve, the motion is said to be constrain

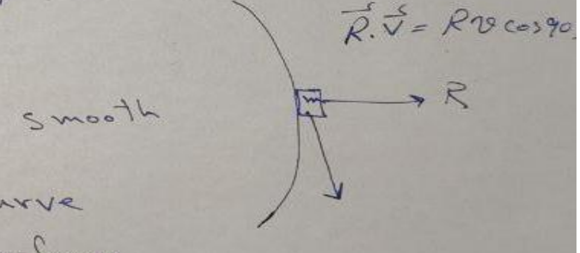
The eq. of motion for a particle moving under constraint maybe written as

$$\underset{\text{total force}}{m\frac{d^2\vec{r}}{dt^2}} = \underset{\substack{\text{external}\\ \text{force}}}{\vec{F}} + \vec{R}$$

← force of constraint. (reaction of the constraining agent upon the particle)

Take the dot product with velocity $\vec{v}$

$$m\frac{d\vec{v}}{dt}\cdot\vec{v} = \vec{F}\cdot\vec{v} + \vec{R}\cdot\vec{v} \quad \text{---} ①$$



R $\perp$ to the surface or curve

& $\vec{v}$ is tangent to the surface

then $\vec{R}\cdot\vec{v} = 0$

Eq. ① reduced to

$$m\frac{d\vec{v}}{dt}\cdot\vec{v} = \vec{F}\cdot\vec{v}$$

$$\frac{d}{dt}\left(\tfrac{1}{2}m\,\vec{v}\cdot\vec{v}\right) = \vec{F}\cdot\vec{v}$$

~~If tt~~

If $\vec{F}$ is conservative, then

$$\cancel{\tfrac{d}{}}\; d\left(\frac{1}{2} m \vec{v}\cdot\vec{v}\right) = \vec{F}\cdot\vec{v}\, dt$$

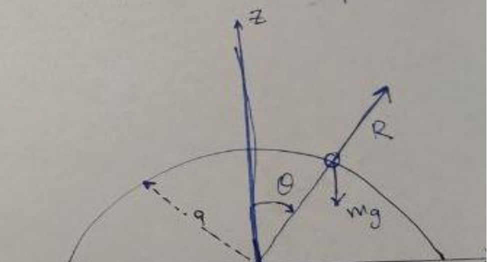
$$\int dT = \int \vec{F}\cdot d\vec{r}$$

$$\int dT = -\int dV$$

$$\frac{1}{2} m v^2 + V(x, y, z) = \text{constant} = E$$

Although the particle remaining moves on the surface or curve, it move in such away that E constant.

Ex/ A particle is placed on top of a smooth sphere of radius a. If the particle is slightly disturbed, at what point will it leave the sphere?



Sol. $F_t = mg + R$

$$m\frac{dv}{dt} = mg + R \quad ---- (1)$$

$$\frac{1}{2} m v^2 + mgz = E \quad ---- (2)$$

In case ($v = 0$ for $z = a$)

$$E = mga \quad ---- (3)$$

sub. eq. (3) in eq. (2, we get

$$\frac{1}{2} m v^2 + mgz = mga \implies v^2 = 2g(a - z)$$

If we take radial components of the equation of motion, we can write the force equation as

$$-\frac{m v^2}{a} = -mg\cos\theta + R$$

تعجيل مركزي

$$= -mg\frac{z}{a} + R$$

$$R = mg\frac{z}{a} - m\frac{v^2}{a} \qquad (3)$$

$$v^2 = 2g(a - z)$$

$$R = mg\frac{z}{a} - \frac{m}{a}2g(a - z)$$

$$R = \frac{mg}{a}(3z - 2a)$$

When $R = 0$ يترك الجسم موضعه

$$0 = \frac{mg}{a}(3z - 2a)$$

$$3z - 2a = 0$$

$$z = \frac{2}{3}a$$