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Analytical Mechanics

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Lec.5: General motion of a particle in three dimensions

. General Motion of a partícle in Three Dimensions

4.1 Linear momentum

The general equation of motion is

$$\vec{F} = \frac{d\vec{P}}{dt} \quad \text{or} \quad \vec{F} = \frac{d}{dt}(m\vec{v}) \quad \text{------} (4.1)$$

For cases where $\vec{F}$ is an explicit function of time, P canbe found by finding the impulse

i.e

$$\int F(t)\,dt = p(t) = m\vec{v}(t) \quad \text{--------} (4.2)$$

Similarty

asecond integration will rield the position

$$\int \vec{v}(t)\,dt = \vec{r}(t)$$

4.2 Angular momentum

$$\vec{F} = \frac{d\vec{P}}{dt}$$ general eq. of motion of aparticle

multiply both sides by the operator $\vec{r}$

$$\vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{P}}{dt}$$

L.H.S $\longrightarrow$ moment

R.H.S $\longrightarrow \dfrac{d}{dt}(\vec{r}\times\vec{p})$

$$\frac{d}{dt}(\vec{r}\times\vec{p}) = \vec{v}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt}$$

$$= \vec{v}\times m\dot{v} + \vec{r}\times\frac{d\vec{p}}{dt}$$

$$= 0 \qquad + \vec{r}\times\frac{d\vec{p}}{dt}$$

$$\therefore \quad \vec{r}\times\vec{F} = \frac{d}{dt}(\vec{r}\times\vec{p}) \quad \text{-------} (4.3$$

Where $\vec{r}\times\vec{p} \longrightarrow$ angular momentum of the particle about the origin.

4.3 The work principle

general eq. of motion $\vec{F} = \dfrac{d\vec{p}}{dt}$

Take the dot product of both sides with velocity $\vec{v}$.

$$\vec{F}.\vec{v} = \frac{d\vec{p}}{dt}.\vec{v} = \frac{d}{dt}(m\overset{\text{متغير}}{\vec{v}}).\overset{\text{ثابت}}{\vec{v}} = \frac{1}{2}m\frac{d}{dt}(\vec{v}.\vec{v})$$

From rule of differential $\dfrac{d}{dt}(\vec{v}.\vec{v}) = 2\vec{v}.\dfrac{d\vec{v}}{dt}$

$$F.\vec{v} = \frac{d}{dt}\left(\frac{1}{2}m\vec{v}.\vec{v}\right) = \frac{dT}{dt}$$

$$\vec{F}.\vec{v}\,dt = dT$$