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Analytical Mechanics

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Lec.8: Gradient and Del operator in mechanics

4.6 Gradient and Del operator in Mechanics

$$F.dr = F_x\,dx + F_y\,dy + F_z\,dz$$

$$= -\frac{\partial V}{\partial x}\,dx + \frac{\partial V}{\partial y}\,dy - \frac{\partial V}{\partial z}\,dz$$

this implies

$$F_x = -\frac{\partial V}{\partial x} \quad \Rightarrow \quad F_y = -\frac{\partial V}{\partial y} \quad \Rightarrow \quad F_z = -\frac{\partial V}{\partial z}$$

In words

If the force field is conservative, then the components of the force are given by the -ve partial derivatives of a potential energy function ~~that is basically how we can express~~

Vectorially, we can express

$$\vec{F} = -i\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$$

or

$$\vec{F} = -\vec{\nabla} V$$

where

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

The expression $\vec{\nabla} V \equiv \text{grad } V$ and same time called gradient of V

Mathematically

The gradient of a function is a vector that represents the spatial derivative of the function in direction and magnitude.

physically

The negative gradient of the potential energy function gives the direction and magnitude of the force that act on a particle located in a field created by other particles.

- The meaning of -ve sign is that the particle is tried to move in the direction of decreasing potential energy rather than in the opposite direction as "V".