Lecturer: Mohanad Muayad Alyas

Analytical Mechanics

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Lec.9: Conditions for the existence of a potential function

4.7 Conditions for the existance of a potential function

The condition that a force be conservative can be written as :-

$$\because \quad \nabla \times F = i\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{j}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) = 0$$

$\therefore$ if $\nabla \times F = 0$, then F can be derived from scalar function V by the operation $F = -\nabla V$, since $\nabla \times \nabla V \equiv 0$, or the curl of any gradient is طابقاً identically 0.

We are now able to generalize the conservation of energy principle to three dimensions. The work done by a conservative ~~can be written~~ force in moving a particle from point A to point B can be written as

$$\int_A^B F \cdot dr = -\int_A^B \nabla V(r) \cdot dr = -\int_{A_x}^{B_x} \frac{\partial V}{\partial x}\, dx - \int_{A_y}^{B_y} \frac{\partial V}{\partial y}\, dy - \int_{A_z}^{B_z} \frac{\partial V}{\partial z}\, dz$$

$$= -\int_A^B dV(r) = -\nabla V = V(A) - V(B)$$

The Last step illustrates the fact that $\nabla V \cdot d\vec{r}$ is an exact differential equal to dV. The work done by any net force is always equal to the change

Meaning of curl $F = 0$

$\vec{F} \cdot d\vec{r} \longrightarrow$ exact differential

$\int \vec{F} \cdot d\vec{r} \longrightarrow$ independent on the Path of integration.

physically

* F is conservative
* the work done by F on a moving particle is independent of the path of the particle in going from one given point to another
* The sum of $T + V = E$ constant.

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divergence of $\vec{F}$, $\vec{\nabla} \cdot \vec{F}$

The divergence $\vec{\nabla} \cdot \vec{F}$ gives a measure of the density of the sources of the field at a given point.

The divergence is of a particular importance in the theory of electricity and magnetism.