Lecturer: Mohanad Muayad Alyas

Analytical Mechanics

2023-2024

Lec.15: Drill exercises

Ex/1

Given the two-dimensional potential energy function

$$V(r) = V_0 - \tfrac{1}{2} k \delta^2 e^{-r^2/\delta^2}$$

where $r = ix + jy$ and V_0, k, and δ are constants find the force function

solution

$$V(x,y) = V_0 - \tfrac{1}{2} k \delta^2 e^{-(x^2+y^2)/\delta^2}$$

and then apply the gradient operator

$$F = -\nabla V = -\left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} \right] V(x,y)$$

$$= -k(ix+jy)\, e^{-(x^2+y^2)/\delta^2}$$

$$= -k r e^{-r^2/\delta^2}$$

Ex/2

Is the force field $F = ixy + jxz + kyz$ conservative?

The curl of $\vec{F}$ is

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & xz & yz \end{vmatrix} = i(z-x) + 0j + k(z-x)$$

The final expression is not zero for all values of the coordinates; hence, the field is not conservative

~~Ex/3 For what values of the constants a, b and~~

Ex/3

For what values of the constants a, b and c is the force $F = i(ax+by^2) + j\,cxy$ conservative

solution

Taking the

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax+by^2 & cxy & 0 \end{vmatrix} = k(c-2b)y$$

This shows that force is conservative provided

$c = 2b$.