Mathematics and Simulation

Silibus: المفردات

1) Matrices and systems of linear equations.
   - Some special matrices.
   - Determinants. محددات
   - The inverse and related matrices
   - Matrices algebra.
   - Diagonalization of matrices.
   - Transformations and mapping.
   - Systems of linear equ.s
   - Eigenvalue and eigenvector.
2) Complex Analysis.
   - Complex numbers (review).
   - Complex functions.
   - Limits, Continuity and derivatives.
   - Chachy - Riemann equations.
   - Series of complex numbers.
   - Power series representation (Taylor series and Laurant expansio
   - Singularities.
   - The residue theorem.
   - Some applications of residue theorem.

3) Ordinary differential equ.s.

- Laplace transform (solution of initial value problem, Shifting theorem).
- Power series using recurrence relation. تكرار، معاودة
- Power series solution of initial value problem.

4) Special functions (some functions).

- Gamma function Γ.
- Beta function β.
- Relation between Γ and β functions.

5) Fourier series.

6) Numerical analysis. عملي

References:

1) Mathmatics of physics and modering engineering by: Sokolnikoff.

2) An introduction to matrices, set and groups. by: Stephenson 1967.

3) Calculus and analytical geometry by: Thomas.

4) Advanced engineering Mathmatics 5th edition by: Peter O'Neil 2003.

5) Mathematical physics
H.K. Dass 2009

6) Mathematical method
by: Redteffer 2008
J. chand.

7) الدوال المعقدة
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Matrices :-

Introduction:

Matrix is an inevitable tool in the study of man subjects like, physics, Mechanics, statistics, electronic circuit and computers.
يتطلب وجوده

Matrices and determinants are discovered and developed in the eighteenth and nineteenth centrui
محددات

There development with the study of system of simultaneous linear equ.s.
متزامن، آني

To illustrate the idea of a matrix, consider the system of linear equ.s.
توضيح

$$x_1 + 2x_2 - x_3 + 4x_4 = 0 \quad ---- (1)$$

$$3x_1 - 4x_2 + 3x_3 - 6x_4 = 0 \quad ---- (2)$$

$$x_1 - 3x_2 - 2x_3 + x_4 = 0 \quad ----- (3)$$

All the information needed to solve this system lies in its coefficient. سواء Whether the first unknown is called x_1 or y_1 or some other name, is unimporta it is important that the coefficient of the first unknown in the second equ. is 3.

If we change this number we may change the solution of the system. We can therefore work with such a system by storing its coefficients in an array called a matrix.

$$\begin{pmatrix} 1 & 2 & -1 & 4 \\ 3 & -4 & 3 & -6 \\ 1 & -3 & -2 & 1 \end{pmatrix}$$

This matrix display the coefficients in the pattern in which they appear in the system of equs. The coefficient of the ith equ. are in row i.

And the coefficients of the jth unknown x_j are in column j.

The number in row i column j is the coefficient of x_j in equ. i.

<u>Note:</u> The matrix is a kind of what is called tensor of rank 2, and the vector is a kind of tensor of rank 1, while the scalar is kind of tensor of rank 0. Therefore, matrix can be written in more general form (rectangular array)

$$A = a_{ij} = \begin{array}{c} \text{n columns j changes} \\ \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & \cdots & \cdots & a_{mn} \end{pmatrix} \\ \text{m by n matrix} \end{array} \begin{array}{l} \text{m row} \\ \text{i changes} \\ \\ n \neq m \end{array}$$

Specific entries of a matrix are often غالبا referenced by using pairs of subscript, for the numbers at each of the rows and columns.

as follows

$$A(i, j)$$

for rows — for column

subscript الوسم

$\therefore$ ① If $i=1$ and j any integer (say $j=3$), then A is a row matrix

$$A(1,3) = [2 \ 5 \ 3]$$

② If $j=1$ and i any integer (say $i=3$)

$$A(3,1) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

③ If $i=j$, then A is a square matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

and the elements of any matrix may be real, imaginary or trigonometric functions.

example: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ real matrix

$\begin{pmatrix} i & -i \\ i & i \end{pmatrix}$ imaginary

مثلثية

$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ trigonometric matrix

Some properties of a matrix:

1. Equality: for the matrices A and B; A = B if and only if $a_{ij} = b_{ij}$

$$a_{ij} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$b_{ij} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

2. Addition and subtraction: if $a_{ij} \mp b_{ij} = c_{ij}$ for all values of i and j.

3. Multiplication:

for the matrices A and B

$AB = C$

if and only if the columns of A is equal the rows of B.

$$\underbrace{A_{1\times 2} \times B_{2\times 1}} = C_{11}$$

example: Let $A = \begin{pmatrix} 1 & 2 \end{pmatrix}$

and $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1 + 4 = 5$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

We conclude that:

$$AB \neq BA$$

it means that A and B are not commute.

But if $AB = BA$, then A and B are commute.

④ Associative:

In addition

e.g. $A + (B + C) = (A + B) + C$

In multiplication

$$(AB)C = A(BC)$$

اثبات

Proof:

$$\because A = (a_{ij})_{m,n} \quad \begin{matrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{matrix}$$

i.e. the subscripts of a start from i and j to its final values n and m

and $B = (b_{jk})_{n,p}$

and $C = (c_{kr})_{p,q}$

~~[struck]~~ now $(AB) = \left(\sum_{j=1}^{n} a_{ij} b_{jk}\right)_{m,p}$

then $(AB)C = \left(\sum_{k=1}^{p} \sum_{j=1}^{n} a_{ij} b_{jk} c_{kr}\right)_{m,q}$

$= L.H.S$

H.W : Prove $L.H.S = R.H.S$

⑤ $A.A.A \ldots \binom{\text{for } n}{\text{matrices}} = A^n$

$A^n A^m = A^{n+m}$

e.g. $A^2 . A^3 = A^5$

1-2 : Special cases

A number of matrices are of special interest:

- the unit matrix :-

The unit matrix I has elements δ_{ij} (Kronical delta) such that

$$\delta_{ij} = \begin{cases} 0 & \text{when } i \neq j \\ 1 & \text{when } i = j \end{cases}$$

or $I_{n \times n} = (\delta_{ij})_{n \times n}$

Let $n=3$

$$I = \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

note that

$$AI = IA = IAI$$
$$= A$$

note that I is a unit matrix as well as قطرية diagonal.

- The null matrix :- المصفوفة الصفرية

If all the elements of a given matrix are ~~Zeros~~ Zero then we call it a null matrix

$$O = \begin{matrix} 0 & 0 & 0 & \cdots & 0 \\ \vdots & 0 & & & 0 \\ \vdots & & 0 & 0 \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 \end{matrix}$$

Such that

$$OA = AO = O$$

- Diagonal matrix :-

Is the matrix in which all the non-diagonal elements are Zero except that elements on the diagonal are not zer

e.g.

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

i.e. a_{ii} : non Zero elements.

مثلثة درج

- Triangular matrix (Echolen matrix) :-

A square matrix, all of whose elements below the leading diagonal are Zero, is called an upper triangular matrix. e.g.,

$$\begin{array}{cl} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 6 \end{bmatrix} & \begin{array}{l} \leftarrow \text{upper triangle} \\ \\ \leftarrow \text{leading diagonal} \end{array} \end{array}$$

While a square matrix, all of whose elements, above the leading diagonal are Zero, is called a lower triangula matrix.

e.g.,
$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 6 & 7 \end{bmatrix}$$
lower triangular matrix

- Transpose of a matrix:-

تبديل

If in a given matrix A, we interchanged the rows and the corresponding columns, the new matrix obtained is called the transpose A and is denoted A^T, $\tilde{A}$ or A'

i.e. $A=(a_{ij})_{m,n} \longrightarrow A^T = (a_{ji})_{n,m}$

observe that if A is an $n \times m$ matrix, then A^T is an $m \times n$ matrix.

Ex.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \longrightarrow A^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

note: If A^T and B^T be the transposes of A and B respectively, then

1. $(A^T)^T = A$
2. $(A+B)^T = A^T + B^T$
3. $(KA)^T = KA^T$ where K is a scalar.
4. $(AB)^T = B^T A^T$

- Conjugate of a matrix:

The matrix obtained from any given matrix A, on replacing its elements by the Corresponding conjugate complex numbers, is called the conjugate of A and is denoted by A^* or $\bar{A}$

Thus if $A = [a_{ij}]_{m,n}$

then $\bar{A} = [\bar{a}_{ij}] = [b_{ij}]_{m,n}$

or $A^* = [a^*_{ij}] = [b_{ij}]_{m,n}$

the conjugate complex of a_{ij}

Ex. $A = \begin{bmatrix} 2+3i & 1+i \\ 2-i & 4i \end{bmatrix}_{m,n}$

then $A^* = \begin{bmatrix} 2-3i & 1-i \\ 2+i & -4i \end{bmatrix}_{m,n}$

Note: If A^* and B^* be the conjugate of A and B respectively, then

1. $(A^*)^* = A$
2. $(A+B)^* = A^* + B^*$
3. $(KA)^* = KA^*$ K is a real
   $(KA)^* = K^* A^*$ K is imaginary
4. $(AB)^* = A^* B^*$

- The conjugate transpose of a matrix:

The conjugate of transpose of a matrix A is denoted b

$$A^{\theta} = A^{\dagger} = A^{*T} = \bar{A}^{\sim}$$

Where θ means conjugate transpose

i.e. $\theta \equiv *T$

or $\dagger$ dagger (سيف) $\equiv *T$
or adjoint

$$A^{\dagger} = [a_{ij}^{*}]^{T}$$
$$= a_{ji}^{*}$$

Ex.

$$A = \begin{pmatrix} 2+3i & 1+i \\ 2-i & 4i \end{pmatrix}$$

then

$$A^{\theta} = A^{\dagger} = A^{*T} = \begin{pmatrix} 2-3i & 1-i \\ 2+i & -4i \end{pmatrix}^{T}$$

$$= \begin{pmatrix} 2-3i & 2+i \\ 1-i & -4i \end{pmatrix}$$

متناظرة

- Symmetric matrix:

A square matrix $A = [a_{ij}]$ is said to be symmetric if $a_{ij} = a_{ji}$ for every i and j

Ex. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ symmetric

$B = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$ symmetric

غير متناظرة

- Skew symmetric matrix:

A square matrix is called skew symmetric (non symmetric, antisymmetric, asymmetric) matrix, if

1. $a_{ij} = -a_{ji}$ for all values of i and j

or $A^T = -A$

Ex. $\begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -5 \\ 3 & 5 & 0 \end{bmatrix}$ Skew-symmetric matrix

$\begin{bmatrix} 0 & b & c \\ b & 0 & e \\ c & e & 0 \end{bmatrix}$

2. All diagonal elements are Zero.

H.W.

① Show that $A+B = B+A$

where

$$A = [a_{ij}]_{m,n} , \quad B = [b_{ij}]_{m,n}$$

② Find the values of x, y, z and w

if
$$3\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ 2+w & 3 \end{pmatrix}$$

- Some properties of symmetric and skew-symmetric matrices:

a- symmetric matrix:

1. It is $n \times n$ matrix

example:
$$A = \begin{pmatrix} 1 & x & y \\ x & 3 & z \\ y & z & 4 \end{pmatrix}$$

i.e. $a_{ij} = a_{ji}$

and the leading diagonal is non zero elements.

اختيارية عام

For arbitrary square matrix of $n \times n$, there are $n \times n = n^2$ elements independent elements, since $a_{ij} = a_{ji}$, the