University of Mosul
College of science
Department of Physics
Third Stage
Lecture 4

Laser

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Lecture 1: Population Inversion in Four Levels

Preparation
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Four -Level Lasers:

We consider an arrangement similar to that of the three level system but with the level o added below the lower laser level, see fig. (8) which is typical of many solid state lasers. The level o is the ground state, and the majority of the atoms are initially in that atom before pumping occurs.

γ_{ou}, γ_{oi}, γ_{li} and γ_{lu} $\longrightarrow$ are neglected because of the large separation and according to following equation:

$$\frac{\gamma_{lu}}{\gamma_{ul}} = e^{-(E_u - E_l)/KT}$$

i.e $E_u \gg E_l$ and accordingly $\gamma_{lu} >> \gamma_{ul}$.

Also, γ_{il}, γ_{io} and γ_{uo} $\longrightarrow$ are neglected since they are very small, owing to the large energy separation of the specific levels.

The rate equations are:

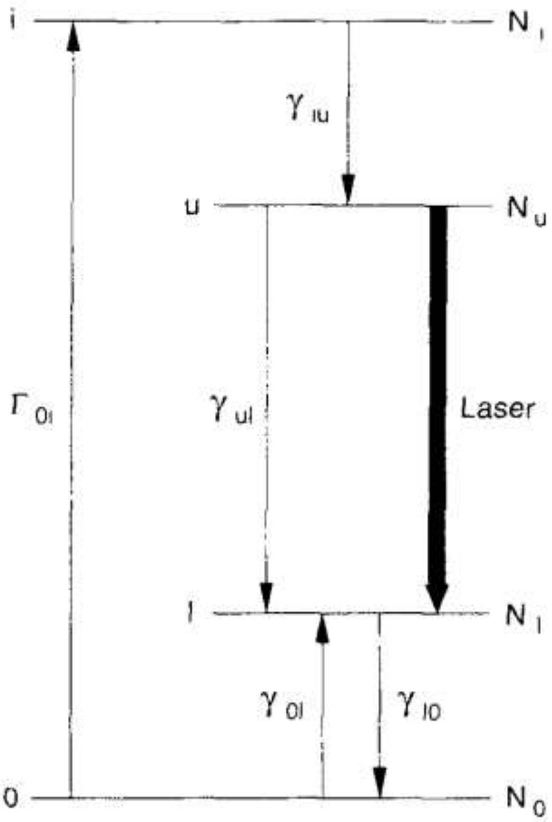

Figure (8): four- level laser

$$\frac{dN_l}{dt} = \gamma_{ol}N_o - \gamma_{lo}N_l + \gamma_{ul}N_u = 0 \qquad \text{……………….65}$$

$$\frac{dN_u}{dt} = -\gamma_{ul}N_u + \gamma_{iu}N_i = 0 \qquad \text{..………………66}$$

$$\frac{dN_i}{dt} = \Gamma_{oi}N_0 - \gamma_{iu}N_i = 0 \qquad \text{…………………67}$$

Sum two last equations, we get:

$$N_u = \frac{\Gamma_{oi}}{\gamma_{ul}} N_o \qquad \text{…………………..68}$$

Also, sum first two equations:

$$\gamma_{ol}N_o - \gamma_{lo}N_l + \gamma_{iu}N_i = 0 \qquad \text{………………..69}$$

$$N_l = \frac{\gamma_{ol}N_o + \gamma_{iu}N_i}{\gamma_{lo}} \qquad \text{………………...70}$$

From eq.(67),

$$N_i = \Gamma_{oi}N_0/\gamma_{iu}$$

$$N_l = \frac{\gamma_{ol}N_o + \frac{\gamma_{iu}\Gamma_{oi}N_o}{\gamma_{iu}}}{\gamma_{lo}} \qquad \text{...........................71}$$

$$N_l = \frac{\gamma_{ol} + \Gamma_{oi}}{\gamma_{lo}} \quad N_o \qquad \text{...........................72}$$

$$\frac{N_u}{N_l} = \frac{\gamma_{lo}\,\Gamma_{oi}}{\gamma_{ul}\ \Gamma_{oi}(\frac{\gamma_{ol}+1)}{\Gamma_{oi}}} > 1 \qquad \text{...………………73}$$

$$\frac{\gamma_{lo}}{\gamma_{ul}} > \frac{\gamma_{ol}}{\Gamma_{oi}} + 1 \qquad \text{...………………74}$$

$$\frac{\gamma_{lo}}{\gamma_{ul}} - 1 > \frac{\gamma_{ol}}{\Gamma_{oi}} \longrightarrow \frac{\gamma_{lo}-\gamma_{ul}}{\gamma_{ul}} > \frac{\gamma_{ol}}{\Gamma_{oi}}$$

$$\Gamma_{oi} = \frac{\gamma_{ol}\gamma_{ul}}{\gamma_{lo}-\gamma_{ul}} \qquad \text{………………..75}$$

Because γ_{lo} is typically much greater than γ_{ul} (owing to the energy – level separation) , we can approximate this equation :

$$\Gamma_{oi} > \frac{\gamma_{ol}\gamma_{ul}}{\gamma_{lo}} = e^{-\Delta E_{lo}/KT}\gamma_{ul} \qquad \text{………………..76}$$

In effect, level i and u as well as l ***and*** o are rapidly brought to thermal distributions in times of the order of 10^{-3} via collisions, thus

$\frac{N_i}{N_u} = e^{-\Delta E_{iu}/KT}$ ..…………………77

$\frac{N_l}{N_0} = e^{-\Delta E_{lo}/KT}$ ………………...78

The pair of levels i&u ⟶ the population in level u since it is the upper laser level.

The pair of levels l&o ⟶ it is desirable to have most of the population reside in level o, but owing to the close energy separation of l and o and to the consequences of Boltzmann enough population can be in level l to affect N_l and then reduce the population inversion ΔNul. We can see that the pumping requirements of the four level system are reduced by the factor $e^{-\Delta E/KT}$ compared to three laser level.

Example: Compare the threshold pumping flux required for the four-level Nd: YAG laser with that of the ruby laser, which uses the three level pumping scheme, where the life time of the upper laser level 4 is 230 µs for the Nd: YAG laser, ΔE_{lo}=0.25 ev at room temperature.

Solutions

$\gamma_{ul} = A_{ul} = \frac{1}{\tau u} = (\frac{1}{2.3x10^{-4}}) = 4.35x10^3\ S^{-1}$

$\Gamma_{oi} > e^{-\Delta E_{lo}/KT}\ \gamma_{ul} = \exp(-0.25/(8.6x10^{-5}x300))\ (4.35x10^3)$.

$= e^{-9.7}\ (4.35x\ 10^3) \cong 0.265$.

If we compare this pumping rate to that obtained for ruby in the previous example, we see that the pumping rate is reduced by a factor:

333/0.265=1257

This even though the transition probability of the Nd:YAG is significantly higher than that of the ruby laser and would therefore increase the pumping thresholds the reduction due the exponential factor far outweigh this increase. Hence, we have a much lower threshold pumping rate for a four-level system than for a three-level system.

https://www.youtube.com/watch?v=eEjbVU5FirQ