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lecture 4 freely falling body

https://youtu.be/bvgemk1y2wa?t=332

Freely falling objects

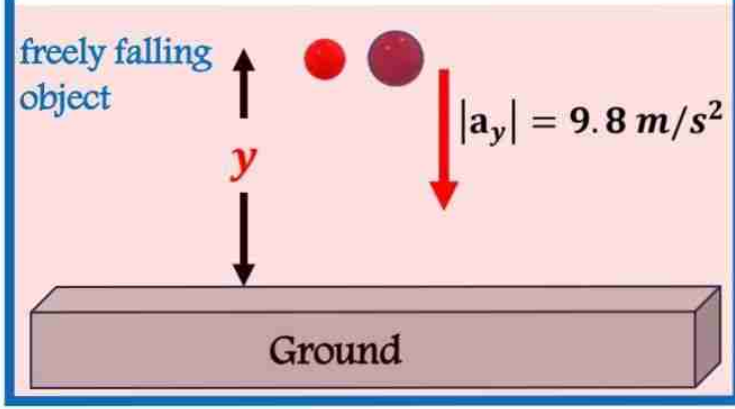
Department of Physics

❖ Definition

An object moving under the influence of gravity alone (upward or downward) is called a freely falling object

Galileo was the first to notice that if we ignore air resistance, then all objects fall with the same constant acceleration

For example, if we drop two objects with different masses from the same height, then the two objects reach the ground at the same time



❖ Free fall acceleration

The magnitude of a free fall acceleration near the Earth's surface was found to be

$$|a_y| = 9.8\ m/s^2$$

The direction of a free fall acceleration is downward toward the ground

If we use the y-axis to describe the motion of a freely falling object with the positive y-axis directed upward and the negative y-axis directed downward, then the free fall acceleration can be written as

$$a_y = -9.8\ m/s^2$$

Where the negative sign means that the direction is downward

❖ The symbol g

We usually use the symbol g instead of a_y to represent the acceleration of a freely falling object

The magnitude of g is given by

$$|g| = 9.8\ m/s^2$$

For simplicity we will stop writing the magnitude sign $\Longrightarrow$

$$g = |g| = 9.8\ m/s^2$$

As a result,

$$a_y = -9.8\ m/s^2$$

$$a_y = -g$$

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❖ The equations of motion for Freely falling objects

To find the equations of motion for a freely falling object, we use the equations of motion along the y-axis, but we replace a_y by $-g$

Remember, the equations of motion along the y-axis are

$$v_{yf} = v_{yi} + a_y\, t$$

$$\Delta y = v_{yi} t + \frac{1}{2} a_y t^2$$

$$v_{yf}^2 = v_{yi}^2 + 2 a_y \Delta y$$

$$v_{y.avg} = \frac{1}{2}(v_{yf} + v_{yi})$$

Where

$$\Delta y = y_f - y_i$$

The equations of motion for a freely falling object become

$$v_{yf} = v_{yi} - g\, t$$

$$\Delta y = v_{yi} t - \frac{1}{2} g t^2$$

$$v_{yf}^2 = v_{yi}^2 - 2 g \Delta y$$

$$v_{y.avg} = \frac{1}{2}(v_{yf} + v_{yi})$$

Where

$$\Delta y = y_f - y_i$$

❖ Example

A ball is thrown upward from the edge of a top of a building. Find the acceleration of the ball at the maximum height the ball can reach

❖ Solution

The acceleration of a freely falling object is constant everywhere and directed downward $\Longrightarrow$

$$a_y = -g = -9.8\ m/s^2$$

❖ Exercise

(a) If the velocity of an object is zero, does it mean that the acceleration is zero?

(b) If the acceleration is zero, does it mean that the velocity is zero?

The answer is

(a) No (b) No

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❖ Example

Find the initial velocity of a ball in the following cases when taking the direction of the positive y-axis to be upward and the direction of the negative y-axis to be downward.

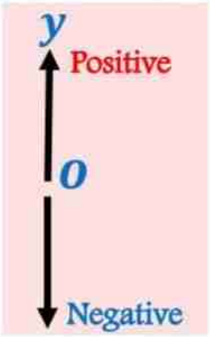
(A) A ball is thrown downward from the edge of a top of a tower with an initial speed of 5 m/s.

(B) A ball is thrown upward from the edge of a top of a tower with an initial speed of 4 m/s.

(C) A ball is dropped from the edge of a top of a tower

Solution

We are asked to determine the initial velocity



(A)

We are given the speed, which is a positive scalar quantity

$$speed = 5\ m/s$$

The velocity is a vector quantity that can be positive or negative

$$v_{yi} = -5\ m/s$$

(B)

We are given the speed, which is a positive scalar quantity

$$speed = 4\ m/s$$

The velocity is a vector quantity that can be positive or negative

$$v_{yi} = +4\ m/s$$

(C)

Whenever we say "dropped," it means that the initial velocity is zero

$$v_{yi} = 0$$

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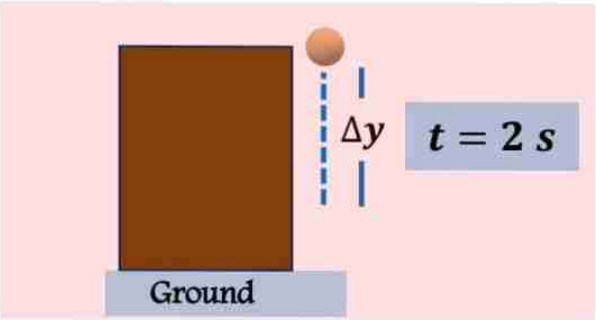
❖ Example

A ball is dropped from the edge of a top of a building. How far will it have fallen after 2 s.

Solution

The word dropped means that the initial velocity is zero

$$\Longrightarrow \quad v_{yi} = 0$$



We are asked to calculate the distance travelled in 2 s

In this case the distance is given by

$$d = |\Delta y|$$

We first calculate Δy

The equations of motion are given by

$$v_{yf} = v_{yi} - g\,t$$

$$\Delta y = v_{yi}t - \frac{1}{2}gt^2$$

Where $\Delta y = y_f - y_i$

$$v_{yf}^2 = v_{yi}^2 - 2g\Delta y$$

$$v_{y.avg} = \frac{1}{2}(v_{yf} + v_{yi})$$

We use the second equation

$$\Delta y = v_{yi}t - \frac{1}{2}gt^2$$

Substitute the numbers $\Longrightarrow$

$$\Delta y = 0 - \frac{1}{2} \times 9.8 \times 2^2 \Longrightarrow$$

$$\Delta y = -19.6\ m \Longrightarrow$$

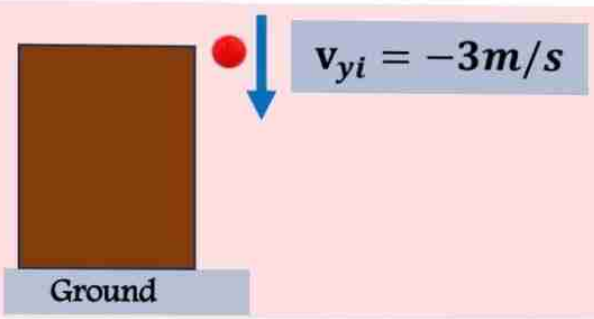
$$d = |\Delta y| = |-19.6|$$

$$= 19.6\ m$$

❖ Example

A ball is thrown downward from the edge of a top of a building with an initial speed of 3 m/s. What would its speed be after 2 s?

Solution



$\mathbf{v}_{yi} = -3m/s$

We are asked to calculate the speed of the ball after 2 s

$$t = 2\ s$$

The speed is a positive scalar quantity

In this case, the speed is given by

$$S = |\mathbf{v}_{yf}|$$

We first calculate $\mathbf{v}_{yf}$

The equations of motion are given by

$$\mathbf{v}_{yf} = \mathbf{v}_{yi} - g\,t$$

$$\Delta y = \mathbf{v}_{yi}t - \frac{1}{2}gt^2$$

Where $\Delta y = y_f - y_i$

$$\mathbf{v}_{yf}^2 = \mathbf{v}_{yi}^2 - 2g\Delta y$$

$$\mathbf{v}_{y.avg} = \frac{1}{2}(\mathbf{v}_{yf} + \mathbf{v}_{yi})$$

We use the first equation

$$\mathbf{v}_{yf} = \mathbf{v}_{yi} - g\,t$$

Substitute the numbers $\Longrightarrow$

$$\mathbf{v}_{yf} = -3 - 9.8 \times 2 \Longrightarrow$$

$$\mathbf{v}_{yf} = -22.6\ m/s \Longrightarrow$$

$$S = |\mathbf{v}_{yf}| = |-22.6|$$

$$= 22.6\ m/s$$

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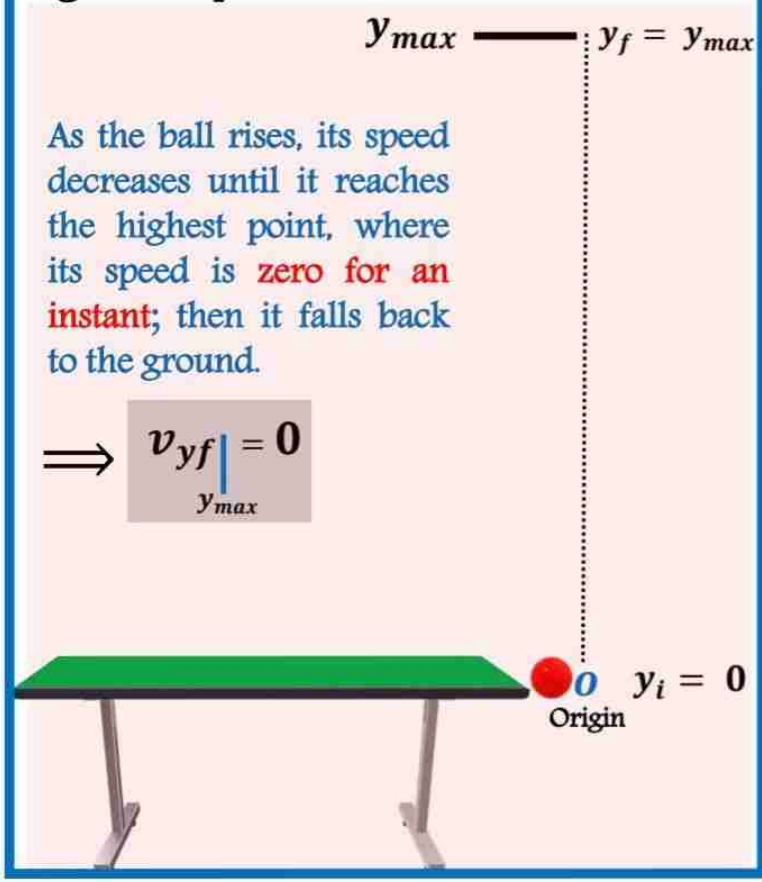
❖ The velocity at the maximum height

Consider a ball thrown upward from the edge of a top of a table.



❖ Calculating the maximum height

To find the maximum height, we first choose the origin at the point of projection $\Longrightarrow$

The initial position of the ball is

$$y_i = 0$$

The final position of the ball is at the maximum height

$$y_f = y_{max}$$

$$\Longrightarrow \Delta y = y_f - y_i \quad = y_{max} - 0$$

$$\Longrightarrow \Delta y = y_{max}$$

We also, use the fact that the velocity at the maximum height is zero

$$v_{yf}\big|_{y_{max}} = 0$$

We now choose the suitable equation of motion to calculate the maximum height

The equations of motion are given by

$$v_{yf} = v_{yi} - g\,t \qquad v_{y.avg} = \frac{1}{2}(v_{yf} + v_{yi})$$

$$\Delta y = v_{yi}t - \frac{1}{2}gt^2$$

Where $\Delta y = y_f - y_i$

$$v_{yf}^2 = v_{yi}^2 - 2g\Delta y$$

We use the third equation

$$v_{yf}^2 = v_{yi}^2 - 2g\Delta y$$

Substitute for $\Delta y = y_{max}$ and $v_{yf} = 0$ leads to

$$0 = v_{yi}^2 - 2gy_{max}$$

$$\Longrightarrow y_{max} = \frac{v_{yi}^2}{2g}$$

Which is the maximum height with respect to the point of projection

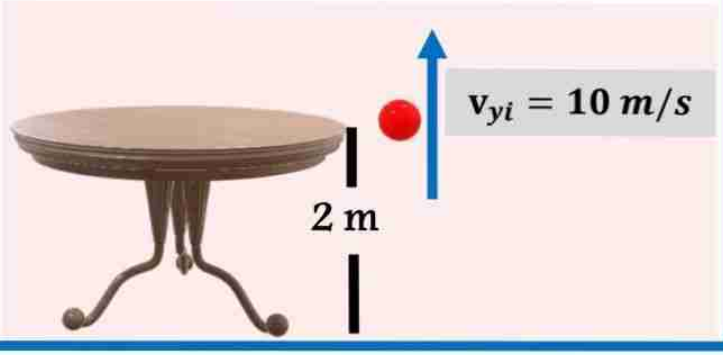
❖ Example

A ball is thrown upward from the edge of a top of a table with an initial speed of 10 m/s. if the height of the table is 2 m.

(A) Find the maximum height the ball can reach with respect to the table.

(B) Find the maximum height the ball can reach with respect to the ground

❖ Solution



(A)

The maximum height with respect to the point of projection is given by

$$y_{max} = \frac{v_{yi}^2}{2g}$$

Substitute for $v_{yi} = 10\ m/s$

$$\Longrightarrow y_{max} = \frac{10^2}{2 \times 9.8} = 5.1\ m$$

(B)

The maximum height with respect to the ground is given by

$$y_{max/ground} = 5.1 + 2 = 7.1\ m$$

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❖ Exercise

A stone is thrown vertically upward with a given initial velocity. It reaches a maximum height of 100 m. If, on a second shot, the initial velocity is doubled then the stone will reach a maximum height of:

A. 70.7 m
B. 141.4 m
C. 200 m
D. 241 m
E. 400 m

Hint $y_{max} = \frac{v_{yi}^2}{2g}$

If the velocity is doubled, then

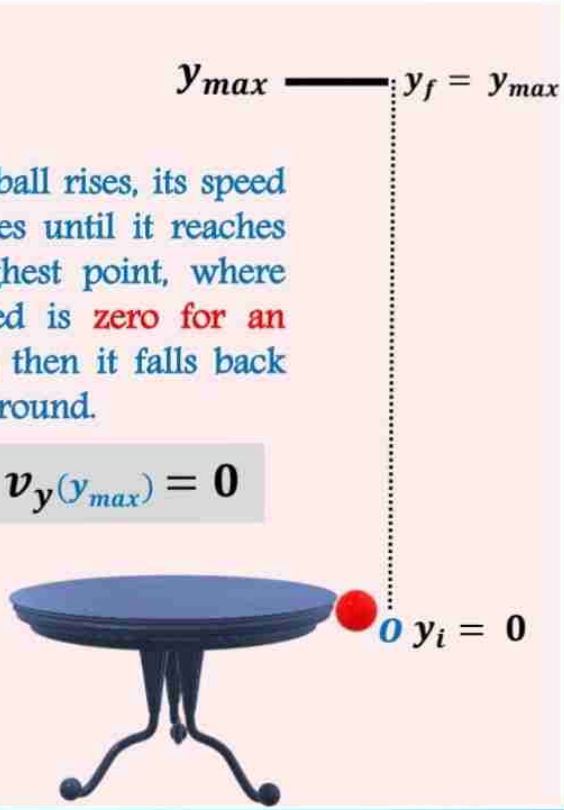
$$y_{max/new} = \frac{(2v_{yi})^2}{2g} = \frac{4\ v_{yi}^2}{2g}$$

$$= 4y_{max}$$

Freely falling objects

❖ Calculating the time required to reach the maximum height

Consider a ball thrown upward from the edge of a top of a table.



As the ball rises, its speed decreases until it reaches the highest point, where its speed is zero for an instant; then it falls back to the ground.

$$\Longrightarrow \quad v_y(y_{max}) = 0$$

To find the time required to reach the maximum height, we use the fact that the velocity at the maximum height is zero

$$v_{yf}\Big|_{y_{max}} = 0$$

We now choose the suitable equation of motion to calculate the time required to reach the maximum height

The equations of motion are given by

$$v_{yf} = v_{yi} - g\,t$$

$$\Delta y = v_{yi}t - \frac{1}{2}gt^2$$

$$v_{yf}^2 = v_{yi}^2 - 2g\Delta y$$

Where $\Delta y = y_f - y_i$

We use the first equation

$$v_{yf} = v_{yi} - g\,t$$

Substitute for $v_{yf} = 0$ leads to

$$0 = v_{yi} - g\,t \quad \Longrightarrow \quad t = \frac{v_{yi}}{g}$$

Which is the time required to reach the maximum height and can be written as

$$\Longrightarrow \quad t_{max-height} = \frac{v_{yi}}{g}$$

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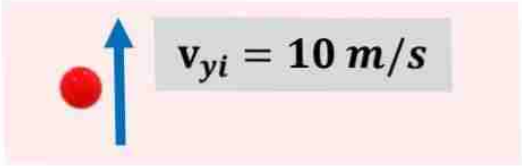
❖ Example

An object is thrown upwards with a speed of 10 m/s. Find the time required for the object to reach its maximum height

Solution



$$v_{yi} = 10\ m/s$$

The time required to reach the maximum height is given by

$$t_{max-height} = \frac{v_{yi}}{g}$$

Substitute for $v_{yi} = 10\ m/s$

$$\Longrightarrow t_{max-height} = \frac{10}{9.8} = 1.02\ s$$

❖ Exercise

A ball is hit straight up into the air. The ball is observed to reach its maximum height 3.1 s after being hit. What is its speed after 4.2 s?

The answer is

$$\text{speed} = 10.8\ m/s$$

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