Ammar Yaseen Aljobury

lecture 5 projectile motion

https://youtu.be/8NLzuURxFwY?t=24

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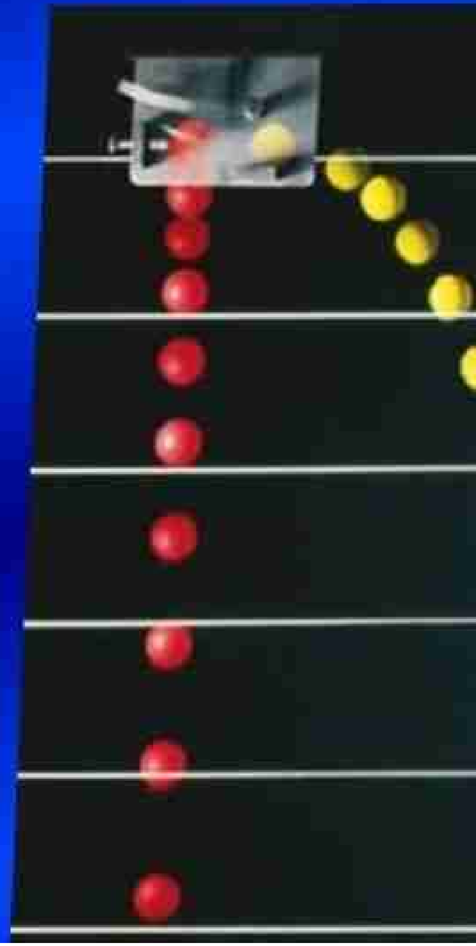
"Projectile Motion"

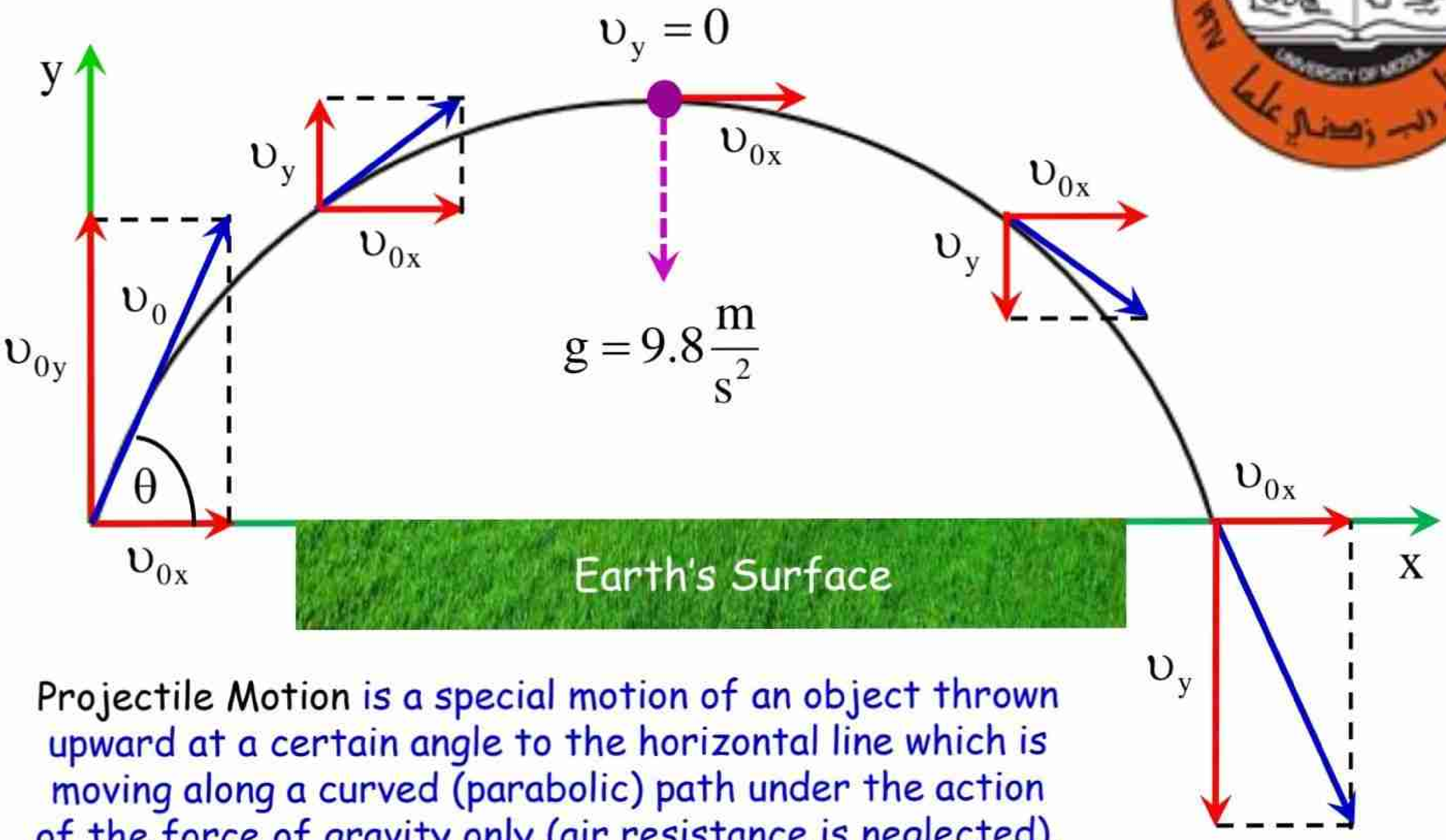
OBJECTIVES

- To discuss projectile motion as a combination of a uniform horizontal motion and a uniformly decelerated/accelerated vertical motion.

- To justify a few formulas and draw some general conclusions on the basis of kinematic equations; and to verify them via PhET interactive simulations.



Projectile Motion

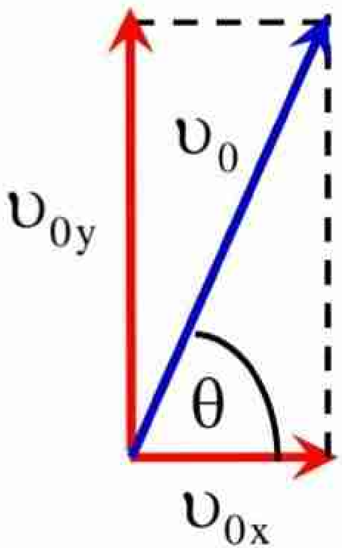


Projectile Motion is a special motion of an object thrown upward at a certain angle to the horizontal line which is moving along a curved (parabolic) path under the action of the force of gravity only (air resistance is neglected).

Projectile Motion: Decomposition

Projectile Motion may be decomposed into two independent types of motion: (1) the uniform (constant-velocity) straight-line motion along the x-axis and (2) uniformly decelerated/accelerated motion along the y-axis (object is affected only by the downward force of gravity, air resistance is ignored).

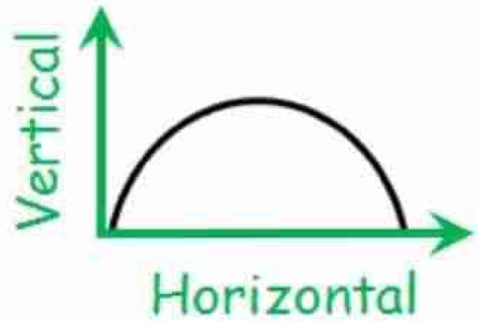
Initial velocity in the projectile motion:



$$\vec{\upsilon}_0 = \upsilon_{0x}\hat{i} + \upsilon_{0y}\hat{j}$$

$$\upsilon_{0x} = \upsilon_0 \cos(\theta)$$

$$\upsilon_{0y} = \upsilon_0 \sin(\theta)$$



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Projectile Motion: Basic Equations

Kinematic equations for horizontal motion:

$$a_x = 0,\quad x_0 = 0 \longrightarrow \begin{cases} \upsilon_x = \upsilon_{0x} = \upsilon_0 \cos(\theta) \\ x = \upsilon_{0x} t = \upsilon_0 t \cos(\theta) \end{cases}$$

(Eq.1)

Kinematic equations for vertical motion:

$$a_y = -g,\quad y_0 = 0 \longrightarrow \begin{cases} \upsilon_y = \upsilon_{0y} - gt = \upsilon_0 \sin(\theta) - gt & \textbf{(Eq.2)} \\ y = \upsilon_{0y} t - \frac{1}{2} g t^2 = \upsilon_0 t \sin(\theta) - \frac{1}{2} g t^2 & \textbf{(Eq.3)} \end{cases}$$

Parabolic Trajectory

Parabolic Trajectory of Projectile Motion

From Eq.1 – kinematic equation for horizontal position:

$$x = \upsilon_0 t \cos(\theta) \xrightarrow{/\upsilon_0 \cos(\theta)} t = \frac{x}{\upsilon_0 \cos(\theta)}$$

From Eq.3 – kinematic equation for vertical position:

$$y = \upsilon_0 t \sin(\theta) - \frac{1}{2} g t^2$$

$$y = \tan(\theta) x - \frac{g}{2\upsilon_0^2 \cos^2(\theta)} x^2 \equiv Bx - Cx^2 = y$$

(Twisted Parabola)

Maximum Height

The Highest Point in Projectile Motion

From Eq.2 – kinematic equation for vertical velocity:

$$\upsilon_y = \upsilon_0 \sin(\theta) - g t_h = 0 \xrightarrow[/g]{+g t_h} t_h = \frac{\upsilon_0 \sin(\theta)}{g}$$

$$\text{for } t = t_h \Rightarrow \quad y = h$$

From Eq.3 – kinematic equation for vertical position:

$$y = \upsilon_0 t \sin(\theta) - \frac{1}{2} g t^2 \longrightarrow h = \upsilon_0 t_h \sin(\theta) - \frac{1}{2} g t_h^2$$

$$h = \frac{\upsilon_0^2 \sin^2(\theta)}{g} - \frac{\upsilon_0^2 \sin^2(\theta)}{2g} = \frac{\upsilon_0^2 \sin^2(\theta)}{2g} = h$$

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Maximum Height: Typical Problems

An object is thrown vertically upward with an initial velocity 70 m/s. What would be the height (or vertical distance) which this object can reach before start falling down?

$$h = \frac{\upsilon_0^2 \sin^2(\theta)}{2g} = \frac{(70\ \mathrm{m/s})^2 \sin^2(90^o)}{2\ (9.8\ \mathrm{m/s^2})} = 250\ \mathrm{m}$$

A stone is projected at the cliff with an initial speed 42 m/s directed at an angle 60 degrees above the horizontal. How long will it take the stone to reach the maximum height above the ground?

$$t_h = \frac{\upsilon_0 \sin(\theta)}{g} = \frac{(42\ \mathrm{m/s})\ \sin(60^o)}{(9.8\ \mathrm{m/s^2})} = 3.7\ \mathrm{s}$$

Horizontal Range

The Horizontal Range for Projectile Motion

The time travel for horizontal range is twice the time travel for maximum height:

$$t_d = 2t_h = \frac{2\upsilon_0 \sin(\theta)}{g}$$

$$\text{for } t = t_d \Rightarrow \quad x = R$$

From Eq.1 – kinematic equation for horizontal position:

$$x = \upsilon_0 t \cos(\theta) \longrightarrow R = \upsilon_0 t_d \cos(\theta)$$

$$R = \frac{2\upsilon_0^2 \sin(\theta)\cos(\theta)}{g} = \frac{\upsilon_0^2 \sin(2\theta)}{g} = R$$

$$\text{for } 2\theta = 90^{O} \Rightarrow \quad \theta = 45^{O}$$

(Optimal Angle)

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Horizontal Range: Typical Problems

A stone is projected at the cliff with an initial speed 42 m/s directed at an angle 60 degrees above the horizontal. Find the distance at which the stone will hit the ground.

$$R = \frac{\upsilon_0^2 \sin(2\theta)}{g} = \frac{(42\ \mathrm{m/s})^2 \sin(120^{o})}{(9.8\ \mathrm{m/s^2})} = 155.9\ \mathrm{m}$$

An object is thrown upward with an initial speed 35 m/s directed at an angle 50 degrees above the horizontal. How much time will pass until the object hits the ground?

$$t_d = \frac{2\upsilon_0 \sin(\theta)}{g} = \frac{2\ (35\ \mathrm{m/s})\ \sin(50^{o})}{(9.8\ \mathrm{m/s^2})} = 5.5\ \mathrm{s}$$

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Height/Range Ratio

The Height/Range Ratio in Projectile Motion

Maximum Height:

$$h = \frac{\upsilon_0^2 \sin^2(\theta)}{2g}$$

Horizontal Range:

$$R = \frac{2\upsilon_0^2 \sin(\theta)\cos(\theta)}{g}$$

The ratio of maximum height to horizontal range:

$$\frac{h}{R} = \frac{\upsilon_0^2 \sin^2(\theta)}{2g} \frac{g}{2\upsilon_0^2 \sin(\theta)\cos(\theta)} = \frac{\sin(\theta)}{4\cos(\theta)}$$

$$\frac{h}{R} = \frac{\tan(\theta)}{4}$$

$$\text{for } \theta = 45^{o} \Rightarrow \tan(\theta) = 1 \Rightarrow \frac{h}{R} = \frac{1}{4}$$

(Optimal Angle)

Concluding Remarks

Neglecting air resistance, the parabolic trajectory of the object and the time of its travel in the projectile motion are independent of its mass and size!

Hypothesis: the trajectory of the object will become asymmetric and the time travel will increase in the presence of air resistance.

Neglecting air resistance, the maximum height (the highest point) in the projectile motion is independent of object's mass and size!

Hypothesis: the maximum height will decrease when the analyzed object is affected by air resistance.

Neglecting air resistance, the horizontal range is the longest for the optimal angle at which the object is thrown equal to forty-five degrees!

Hypothesis: the optimal angle will decrease when air resistance is included.

Neglecting air resistance, the horizontal range at optimal angle is four times larger than the maximum height (the highest point) in the projectile motion!

Hypothesis: this ratio will change when air resistance is taken into account.

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Drag Force (Air Resistance, Fluid Friction)

Drag force is a force acting opposite to the relative motion of any object moving with respect to a surrounding fluid (air). The ultimate cause of a drag is viscous friction. Drag force depends on the properties of the fluid, and on the size, shape and speed of the object immersed within the fluid (air).

$$F_D = \frac{1}{2}\rho\upsilon^2 C_D A$$

ρ - density of the fluid

υ - speed of the object

C_D - drag coefficient

A - cross-sectional area

In the physics of sports, drag force is necessary to explain the performance of basketball players and runners.

PhET Simulator: https://phet.colorado.edu/en/simulation/projectile-motion