University of Mosul
College of science
Department of Physics
Third Stage
Lecture 6

# Laser

2024-2025

Lecture 5: Types of spectral line shape broadening in the laser

Preparation
Dr. Erada Al- Dabbagh

Erada Al-Dabbagh

**16/10/2024**

## Types of spectral line shape broadening in the laser physics

Types of spectral line shape broadening in the laser physics We've treated the spectra of light emitted by atoms transitioning between states E2 and E1 as having a single frequency

$$\upsilon = (E_{2-}E_1)/h$$

The emission lines of atomic transitions have some finite spectral width, which is called line broadening; this broadening is caused by various broadening mechanisms due to light-matter interaction.

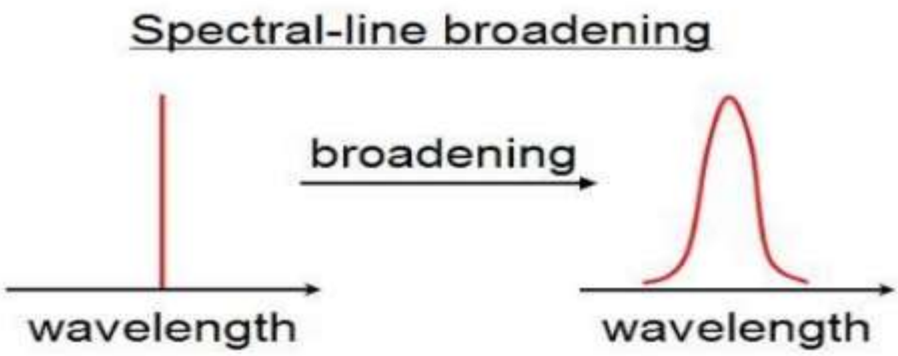


**Figure: left side shows there isn't any kind of line shape broadening, the right side shows the broadening in the spectral line shape**

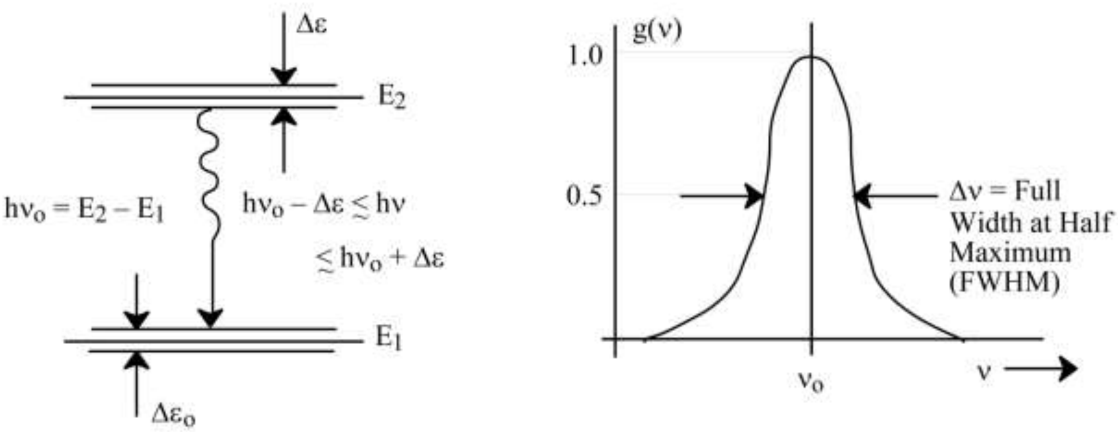


**Figure: the right side shows the spectral line shape broadening. The left side of the figure shows the splitting of the energy levels due to present of EM field which causes the spectral line shape broadening.**

The energy levels of atom or molecule are split due to present of electromagnetic field (EM) field, therefor atoms can absorb photons with a slightly different energy, left side of upper figure.

There are two general classification of line broadening:

Homogenous: all atoms behave the same way (i.e., each effectively has the same lineshape function. g(ν)

Inhomogeneous: each atom or molecule has a different lineshape function due to its environment.

i.e. Broadening is Homogeneous when it affects all "atoms" equally and Inhomogeneous when it splits them into sub-groups.

## Classical Emission line width Electron:

The decaying electric filed component of the electromagnetic wave described as:

$$E(t) = \begin{cases} E_0 e^{-\frac{\gamma_0 t}{2}} e^{-i\omega_0 t} & if\ t \geq 0 \\ 0 & if\ t < 0 \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 73$$

where $\gamma_o$ is the proportionality factors "representing the rate at which energy is lost", is not an infinitely long wave and therefore cannot be represented by a single pure frequency $\omega_o$. Rather, it has a finite starting point at t=0 and then decay exponentially with a time constant $\tau_o = 2/\gamma_o$ for the frequency components of the wave represented by (73) taking its Fourier transform as:

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt \qquad \ldots\ldots\ldots\ldots\ldots\ldots 74$$

$$E(\omega) = \frac{E_0}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\left[(\omega-\omega_0)+i\frac{\gamma_0}{2}\right]t} dt \qquad \ldots\ldots\ldots\ldots\ldots\ldots 75$$

$$E(\omega) = \frac{E_0}{\sqrt{2\pi}} \left[\frac{e^{\left(i(\omega-\omega_0)++\frac{\gamma_0}{2}\right)t}}{(\omega-\omega_0)+\frac{\gamma_0}{2}}\right]_0^{\infty} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 76$$

$$E(\omega) = \frac{E_0}{\sqrt{2\pi}} \left[0 - \frac{1}{i\left[(\omega-\omega_0)+i\frac{\gamma_0}{2}\right]}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots 77$$

$$E(\omega) = -\frac{E_0}{\sqrt{2\pi}} \frac{1}{i\left[(\omega-\omega_0)+i\frac{\gamma_0}{2}\right]} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 78$$

Erada Al-Dabbagh

$$E(\omega) = -\frac{E_0}{\sqrt{2\pi}} \frac{1}{i\left[(\omega-\omega_0)+i\frac{\gamma_0}{2}\right]} \times \frac{(\omega-\omega_0)-i\frac{\gamma_0}{2}}{(\omega-\omega_0)-i\frac{\gamma_0}{2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 79$$

$$E(\omega) = -\frac{E_0}{\sqrt{2\pi}} \frac{(\omega-\omega_0)-i\frac{\gamma_0}{2}}{\left[(\omega-\omega_0)^2-\frac{\gamma_0{}^2}{4}\right]} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 80$$

The intensity distribution per unit frequency $I(\omega)$ for this wave is proportional $\tau o \;\; |E(\omega)|^2$ and is given by:

$$I(\omega) = I_0 \frac{(\omega-\omega_0)^2+\frac{\gamma_0{}^2}{4}}{\left((\omega-\omega_0)^2+\frac{\gamma_0{}^2}{4}\right)^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 81$$

Where $I_0 = \frac{E_0^2}{2\pi}$

where $I_o$ represent the total intensity of the emission integrated over the entire frequency width of the emission line.

$$I(\omega) = I_0 \frac{1}{(\omega-\omega_0)^2+\frac{\gamma_0{}^2}{4}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 82$$

Take the integral:

$$\int I(\omega) = I_0 \int_{-\infty}^{\infty} \frac{1}{(\omega-\omega_0)^2+\frac{\gamma_0{}^2}{4}} d\omega \qquad \ldots\ldots\ldots\ldots\ldots\ldots 83$$

$$= I_0 \left[\frac{2}{\gamma_0}\left(tan^{-1}\left(\frac{\omega-\omega_0}{\gamma_0/2}\right)\right)\right]_{-\infty}^{\infty} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 84$$

$$= I_0 \frac{2}{\gamma_0}\left(tan^{-1}(\infty) - tan^{-1}(-\infty)\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots 85$$

$$= I_0 \frac{2}{\gamma_0}\left(\frac{\pi}{2} - (-\frac{\pi}{2})\right) = I_0 2\pi/\gamma_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots 86$$

$$\therefore \;\; I\,(\omega) = I_0 \frac{1}{(\omega-\omega_0)^2+\frac{\gamma_0{}^2}{4}} \; = \frac{2\pi}{\gamma_0} I_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots 87$$

$$I(\omega) = I_0 \frac{\gamma_0/2\pi}{(\omega-\omega_0)^2+\frac{\gamma_0{}^2}{4}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 88$$

So, we can get the value of $I(\omega)$ at $\omega = \omega_0$ in eq. 82in the half of the linewidth

$$I(\omega) = \frac{1}{2} I_0 \frac{\gamma_0}{2\pi} \times \frac{4}{\gamma_0{}^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 89$$

Erada Al-Dabbagh

$$I(\omega) = \frac{I_0}{\pi\gamma_0} \qquad \text{..................90}$$

Substitute the eq. 90 in eq. 89, we can then obtain the FWHM emission line width as;

$$\frac{I_o}{\pi\gamma_o} = I_o \frac{\gamma_o/2\pi}{(\omega-\omega_o)^2+\frac{\gamma_o^2}{4}} \qquad \text{..................91}$$

$$\frac{\pi\gamma_o^2}{2\pi} = (\omega-\omega_o)^2 + \frac{\gamma_o^2}{4} \qquad \text{..................92}$$

$$\gamma_o^2 = \gamma_o^2\left(\frac{2(\omega-\omega_o)^2}{\gamma_o^2} + \frac{1}{2}\right) \qquad \text{..................93}$$

$$\frac{1}{2} = \frac{2(\omega-\omega_o)^2}{\gamma_o^2} \rightarrow \frac{\gamma_o^2}{4} = (\omega-\omega_o)^2 \qquad \text{........................94}$$

$$\Delta w^{FWHM} = \gamma_o = 2(\omega-\omega_o) = 2\pi\Delta\upsilon_c \qquad \text{........................95}$$

where $\Delta\upsilon_c$ denotes the classical line width.

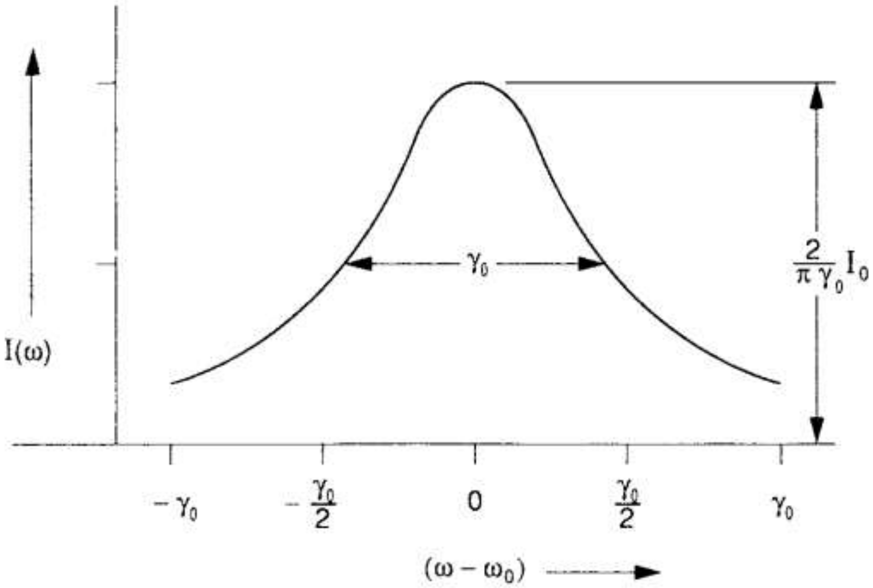


**Figure (22): the classical analysis of a decaying and radiating electron as it makes a transition from one energy level to lower- lying level**

A graph of $I(\omega)$ verses $\omega$-$\omega_o$ is shown in Figure (22). We will refer to this spectral distribution as the **classical theoretical emission shape (or broadening)** of the radiation emitted by the oscillating electron. The form of the line shape (eq. 95) is known as a **Lorentzian distribution** and is symmetrical with respect to $\omega_o$.

The full width of this emission of half maximum intensity $\Delta W^{FWHM}$ is obtained by setting (eq. 95) equal to half the value of (ω) at $\omega = \omega_o$.

Although the line shape function of (95) was derived for a single electron, it can be shown that an identical line shape function will result for an assembly of N atoms in the same upper level, all radiating on the same transition with random phase, with the intensity of the emission increased by a factor of N. This type of emission broadening occurs when every atom of the same species making the same transition produces an identical emission line shape and width. Such a situation leads to Lorentzian line shape function of, and is referred to as homogeneous broadening.

## Doppler broadening in Gases:

We are all familiar with the Doppler effect associated with sound waves, as the train approaches, the sound is first heard at a higher frequency and then changes to a lower frequency as the train passes.

This same frequency shift is characteristic of the emission of light waves from moving atoms. Consider light of frequency $\upsilon_o$ and velocity c being emitted over a time interval $\Delta t$ from an atom moving toward you with a velocity v. At the end of the period of

(c-v) Δt ⟶ distance in your direction.

cΔt ⟶ distance if the source had not been moving with respect to you

The separation between wave peaks is compressed for waves traveling in your direction, which thereby raises their frequency in order to satisfy $c=\lambda\upsilon$.

The value of this frequency shifts for source moving toward the observer, can be obtained by letting the observed frequency υ shift with respect to the original frequency υo as:

$$\upsilon = \left(\frac{c\Delta t}{(c-\upsilon)\Delta t}\right)\upsilon_o = \left(\frac{1}{1-\upsilon/c}\right)\upsilon_o \cong \left[1 + \frac{\upsilon}{c} + \left(\frac{\upsilon}{c}\right)^2 + \cdots\ldots\right]\upsilon_o \qquad \ldots\ldots\ldots\ldots\ldots 95$$

The same argument for the source moving away from the observer, thus the frequency shift is:

$$\upsilon = \left(\frac{c\Delta t}{(c+\upsilon)\Delta t}\right)\upsilon_o = \left(\frac{1}{1+\upsilon/c}\right)\upsilon_o \cong \left[1 - \frac{\upsilon}{c} + \left(\frac{\upsilon}{c}\right)^2 - \ldots\ldots\right]\upsilon_o \qquad \ldots\ldots\ldots\ldots\ldots 96$$

Erada Al-Dabbagh

For non-relativistic velocities, where the ratio $\frac{v}{c}$ is very small, terms involving powers of $\frac{v}{c}$ i.e $(\frac{v}{c})^2$ or higher can be neglected, thus

$$\upsilon = \left(1 + \frac{v}{c}\right)\upsilon o \qquad \text{moving toward the observer} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 97$$

$$\upsilon = \left(1 - \frac{v}{c}\right)\upsilon o \qquad \text{moving away from the observer} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 98$$

This Doppler shift can be significant when observing radiation from atoms in the gaseous state. In such a case the atoms are in thermal equilibrium, and hence they are moving randomly in all directions with a range of velocity:

$$\overline{V} = \sqrt{\frac{8KT}{M\pi}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 99$$

Where T → atom temperature, k → Boltzmann's constant, and M → mass of the atom, thus v range from $10^2$ to $10^3$ m/s with lighter atoms having the higher velocities.

For such a thermal distribution of velocities, some of atoms are moving

1. Directly toward you.
2. Away from you.
3. Partially toward you.
4. Partially away from you.

These groups of atoms will appear to have shifted frequencies, another group of atoms will be moving directly to one side or the other with respect to the observer and will therefore have no component of their velocity in a direction toward or away from you. These atoms will observed to have frequencies radiating at the center frequency $\upsilon_o$, according to (85) & (86), since v represents the component of velocity moving either toward or away from the observer and for this state v=0.

All radiating transitions have a natural emission line width, so we would observe the Doppler broadening only if it were significantly larger than the natural broadening. We will therefore calculate the Doppler width of an emission line if the natural emission line width is narrower than the Doppler width and thus can be ignored.



Eiada Al-Dabbagh

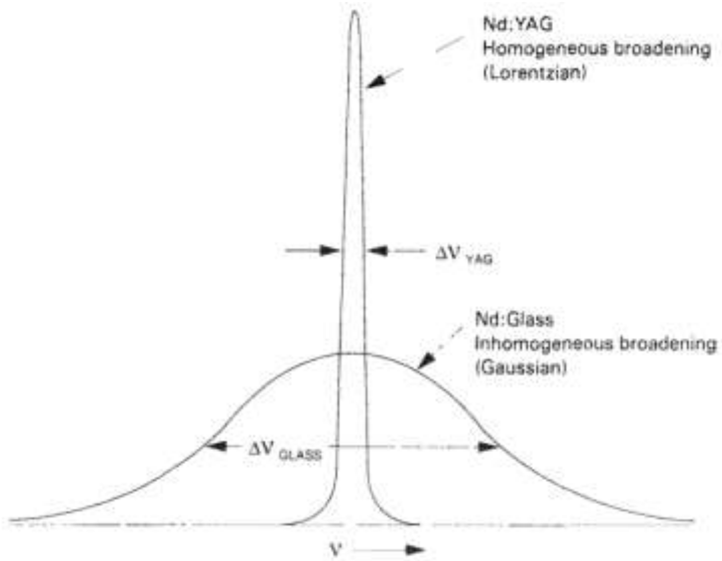


**Figure (23): Relative emission linewidth.**

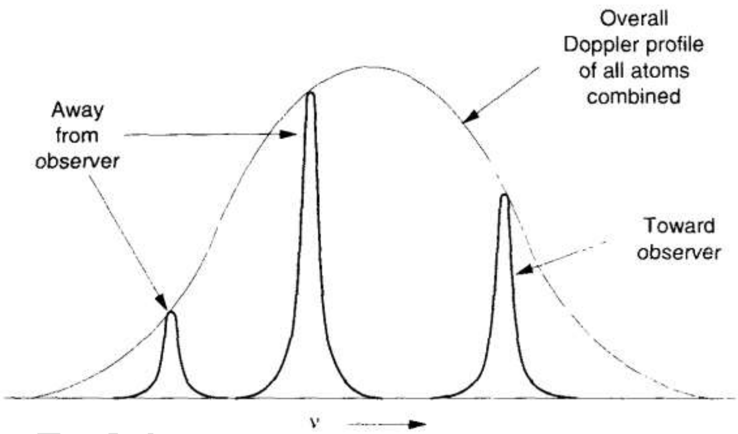


**Figure (24):shape of a Doppler-broadened emission line, indicating the natural emission linewidths of individual atoms radiating while traveling in various directions.**

**https://www.google.com/search?q=Types+of+spectral+line+shape+broadening+in+the+laser%2C+vedio&oq=Types+of+spectral+line+shape+broadening+in+the+laser%2C+vedio&gs_lcrp=EgZjaHJvbWUyBggAEEUYOTIJCAEQIRgKGKABMgkIAhAhGAoYoAHSAQg1NDA1ajBqN6gCCLACAQ&sourceid=chrome&ie=UTF-8#fpstate=ive&vld=cid:e97399a2,vid:lOZCC6R2f6U,st:0**