University of Mosul
College of science
Department of Physics
Third Stage
Lecture 8

Laser

2024-2025

Lecture 8: Development and Growth of a Laser Beam

Preparation
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Development and Growth of a Laser Beam for a Gain Medium with Homogeneous Broadening:

It would be useful to obtain a value of the gain or the exponent of eq:

$$I = I_o e^{\sigma_{ul}^{H} \Delta N_{ul} z}$$

at which the beam would reach the saturation intensity. We will, obtain an approximation range of values that are dependent on the length and width of the gain medium.

Consider a cylindrical gain medium, as shown in Fig. (25), that has a length L, a cross-sectional area A, and a diameter d_a, we assume a population inversion exist.

$A_{ul} \longrightarrow$ radiative decay rate from u to l.

$\Delta N_{ul} \approx N_u$

For simplicity we will consider the beam as starting at one end of the medium in a region of length l_g ehere l_g as a one gain medium such that

$\sigma_{ul}^{H}(\upsilon) N_{ul} l_{g} = 1$ and $l_g << L$

We will assume that the atoms in level u are radiating at a rate A_{ul} with an energy $h\upsilon_{ul}$. Some of these photons are emitted in the elongated direction of the amplifier and would be enhanced by stimulated emission as they transit through the length L of the medium, so a beam and the intensity grows exponentially.

The energy radiated per unit time into a 4π solid angle from within the volume is Al_g as $\rightarrow$ $N_u(Al_g)A_{ul}h\upsilon_{ul}$

This is multiplied by the fractional portion of the energy radiating within a solid angle as that would reach the opposite end of the medium a $\frac{d\Omega}{4\pi}$ this fraction of the total solid angle can be as:

$$\frac{d\Omega}{4\pi} = \frac{A}{L^2}\frac{1}{4\pi} \Longrightarrow \frac{A}{4\pi L^2} \qquad \text{..................(106)}$$

The energy radiated from that volume element per unit time is amplified by an amount equal to $\sigma_{ul}^{H}(\upsilon)N_u L$ by the time it reaches the other end of the medium. We divide that energy per unit time by the area A to obtain an intensity, thus:

$$(N_u A l_g) A_{ul} h\upsilon_{ul} \frac{A}{4\pi L^2} \frac{e^{\sigma_{ul}^{H}(\upsilon) N_u L}}{A} = I_{sat} = \frac{h\upsilon_{ul}}{\sigma_{ul}^{H}(\upsilon)\tau u} \qquad \text{.................(107)}$$

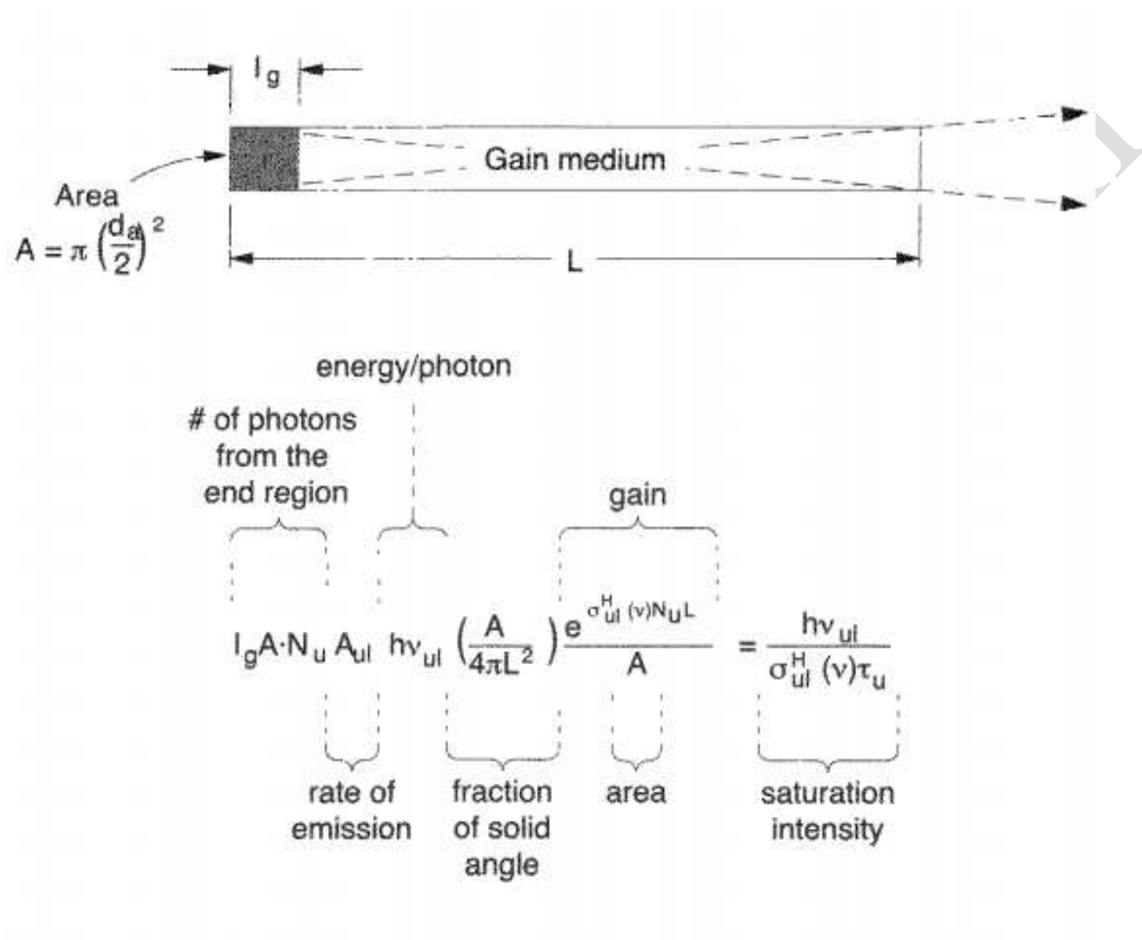

Figure (25): growth and development of a laser beam from an elongated gain medium

$\tau_u = 1/A_{ul}$

$$(N_u A l_g) A_{ul} h\upsilon_{ul} \frac{A}{4\pi L^2} \frac{e^{\sigma_{ul}^{H}(\upsilon) N_u L}}{A} = I_{sat} = \frac{h\upsilon_{ul} A_{ul}}{\sigma_{ul}^{H}(\upsilon)} \qquad \text{.................(108)}$$

$$N_u \left[\pi \left(\frac{d_a}{2}\right)^2\right] \left(\frac{1}{\sigma_{ul}^{H}(\upsilon) N_u}\right) \frac{1}{4\pi L^2} e^{\sigma_{ul}^{H}(\upsilon) N_u L} = \frac{1}{\sigma_{ul}^{H}(\upsilon)} \qquad \text{.................(109)}$$

Where

$$A = \left(\frac{d_a}{2}\right)^2 \pi, \qquad \text{.................(110)}$$

$$l_g = \frac{1}{\sigma_{ul}^H(\upsilon)N_u} \quad \text{……………..(111)}$$

$$e^{\sigma_{ul}^H(\upsilon)N_u L_{sat}} = 16\left(\frac{L_{sat}}{d_a}\right)^2 \quad \text{……………..(112)}$$

If

$$x = \sigma_{ul}^H(\upsilon)N_u L_{sat} \quad \text{……………..(113)}$$

$$e^x = 16\left(\frac{L_{sat}}{d_a}\right)^2 \quad \text{……………..(114)}$$

choosing a length l_g leads to a simplified result. If a very much shorter region than l_g were chosen, then a significant amount of the energy that might eventually be amplified would be left out of the calculation. If a significantly longer region were chosen, a much shorter exponential growth l_g the than I_{sat} would have to be used and the beam would not gain as much energy through amplification.

3/12/2023

Shape or Geometry of Amplifying Medium:

The solution of last equation can be graphed of either L_{sat}/d_a verses x as shown in Fig. (26). The ratio of the length of the amplifier to its diameter is an important factor in *how much gain is needed to reach Saturation*. In most cases, it is difficult to generate a large gain in an amplifying medium of any reasonable length.

We will consider two differently shaped gain media:

1. long Cylinder
2. sphere of diameter d_a

For the long cylinder, say $\frac{lsat}{da} = 100$, spontaneous emission will originate at one end of the medium and will emerge at the other end in an elongated shape, as shown by the dashed lines exiting from the cylinder in Fig. (27). We can see from Figure that the value of $\sigma_{ul}^H(\upsilon)N_u L_{sat} = 12$

$\therefore e^{12} = 1.6\text{x}10^5 \longrightarrow$ this is an extremely large increase resulting in a very intense beam with low divergence.

For the case of sphere, where $\frac{lsat}{da}=1$, First: Figure (26), suggests that the value of the $\sigma_{ul}^{H}(\upsilon)N_u L_{sat} = 2.7$

$e^{2.7} = 15 \longrightarrow$ which is lower than the value of $1.6\text{x}10^5$ obtained for the elongated medium.

First: the radiation originating from different locations within the sphere would cause the beam to diverge rapidly in the same manner as radiation emitting from spherically shaped incoherent source.

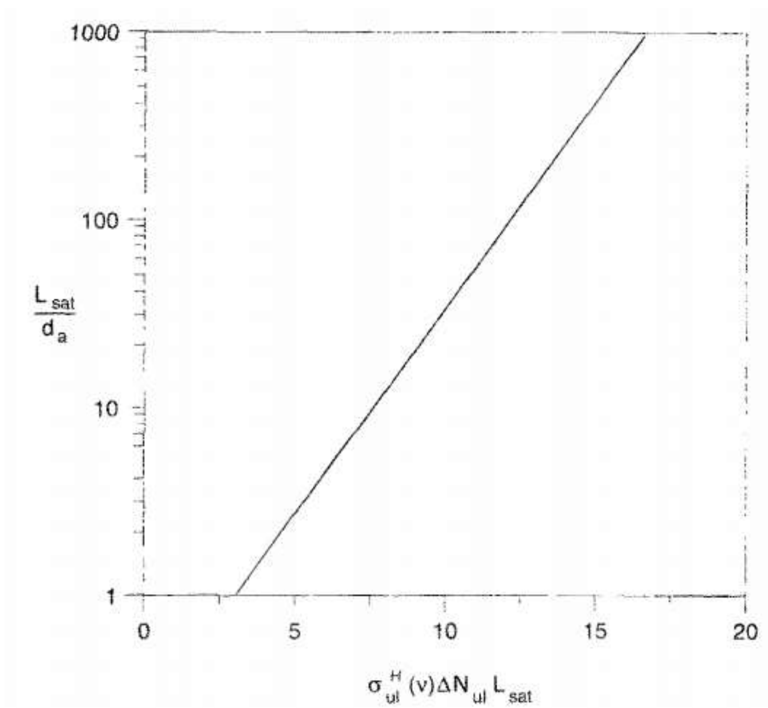

Figure (26): graph of the saturated gain length L_{sat} diameter d_a verses the exponential gain coefficient.

Second: the radiation emitted from the entire sphere will be emitted equally in all directions since the sphere is completely symmetrical. Therefore, the maximum amount of energy E_{ul} that could be radiated on that transition from u to l is

$E_{ul} = N_u V h\upsilon_{ul}$ within a time $1/A_{ul}$

where V is the volume of the sphere.

This is also the same amount that would be radiated by spontaneous emission at a rate A_{ul}. Because E_{ul} represents all of the energy stored in level u during the level lifetime, there is no additional energy available to be radiated by stimulated emission. Thus, the same amount of energy would emerge from the sphere under the presence of gain as would occur with no gain in the medium.

Thus, it makes no sense to construct a laser that radiates in all directions, since the purpose of having a laser that of concentrating the available energy in both direction and frequency is thereby defeated. It was mentioned before that all a laser does is take energy that would normally be radiated in all directions and concentrate it into a specific direction.

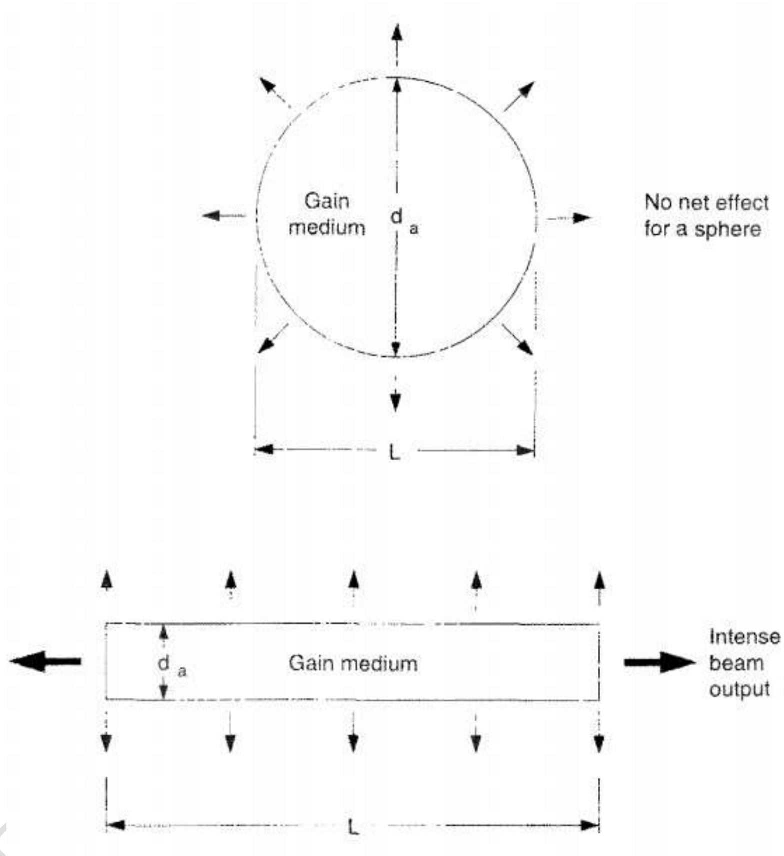

Figure (27): two possible types of gain media

Figure (28) shows how a beam would grow as length is added to the amplifier: it grows exponentially over the length L_{sat}. Then, as it approaches the saturation intensity, it can no longer grow at that rate. It will then begin to extract significant energy from the amplifiers because the stimulated emission rate will exceed the spontaneous emission rate. But observe that before I_{sat} is reached the intensity is so low that the developing beam has very little effect on the gain medium since N_u is not significantly altered.

Typical lasers require an L_{sat}/d_a ratio ranging from about 10 to 1000. Which suggests a desirable range of gain values from 7 to 17 in order to reach Saturation:

$$g_{th}L_{sat} = \sigma_{ul}^{H}(\upsilon)\Delta N_{ul}L_{sat} = 12 \pm 5 \quad \text{..................(115)}$$

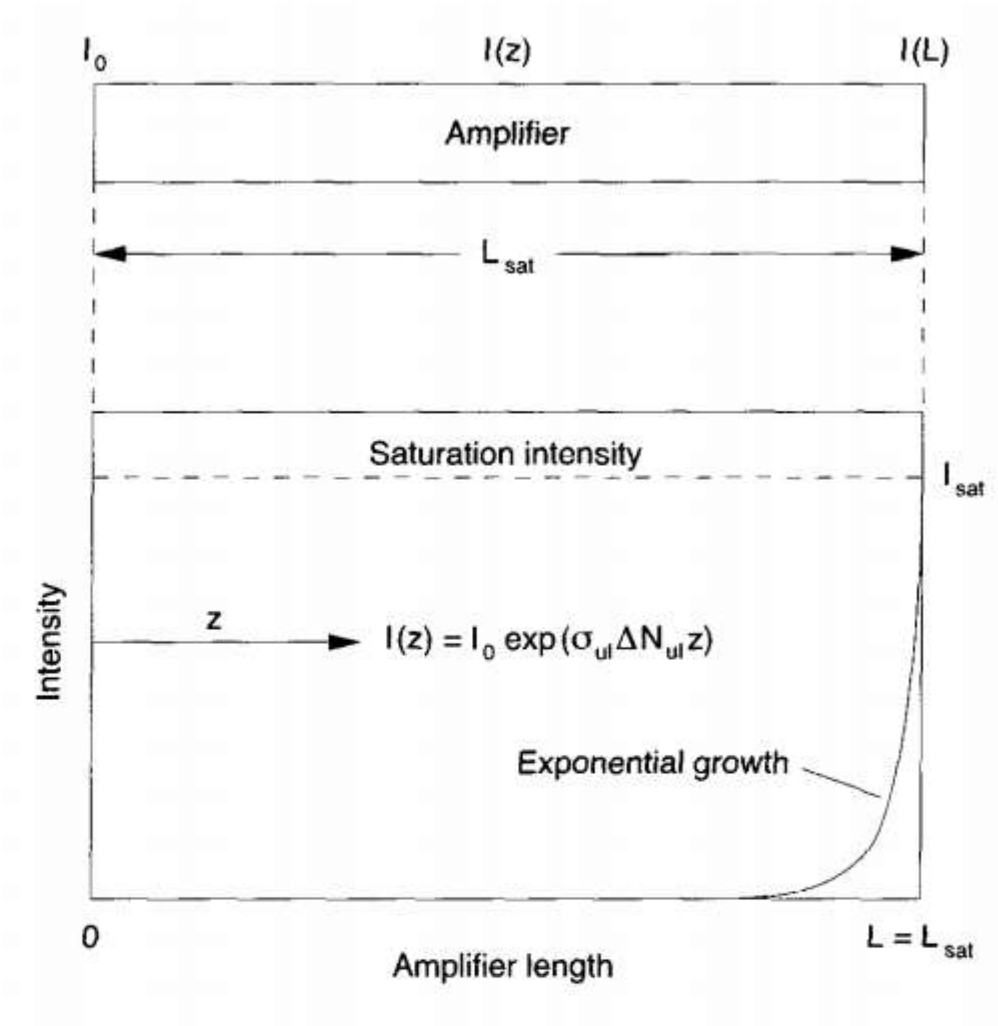

Figure (28): Exponential growth and saturation of a laser beam as a function of amplifier length.

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