$$A^3 = \frac{-210}{3}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{117}{3}\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

$$= -\begin{pmatrix} 70 & 0 \\ 0 & 70 \end{pmatrix} + 39\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

$$= -\begin{pmatrix} 70 & 0 \\ 0 & 70 \end{pmatrix} + \begin{pmatrix} 156 & 39 \\ 78 & 117 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 86 & 39 \\ 8 & 47 \end{pmatrix}$$

Note: ① A square matrix A which satisfy the relation $A^2 = A$ is called idempotent. جامدة

Proof: If $AB = A$ & $BA = B$

for them $ABA = (AB)A$

$$= A.A = A^2 \text{ ------ } ①$$

$$ABA = A(BA) = AB = A \text{ ---- } ②$$

from equ.s ① & ②

$A = A^2$

(2) A square matrix which satisfy the relation $A^K = 0$ where K is a positive integer, is said to be nilpotent matrix.

– Two Dimensional Transformation:

1– Homogeneous Coordinate:

In mathematics homogeneous coordinates or (projection coordinates) are a system of coordinates used in projection geometry like cartesian coordinates as used in Euclidean (الهندسة الاقليدية) geometry. They have an advantage that the coordinates of points including points at infinity can be projected using infinite coordinate.

في الهندسة الاقليدية يكون الشكل فيها مستوي.

2- The transformations:

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2a. Translation:

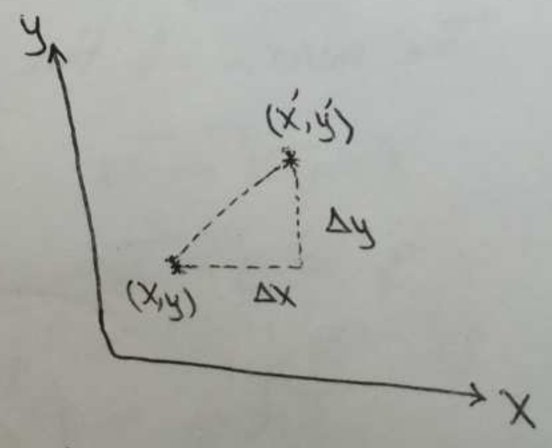
In Euclidean geometry, a translation is a function that move any point a const. distance, in specified direction.

A translation can be also interpreted as the addition of a const. vector to every point, or as shifting the origin of the coordinates system. A translation operator is an operator such that

$$T_{\delta} f(x_i) = f(x_i + \delta)$$

The translation from the point (x, y) to the point (x', y'), as shown in the fig., then

$$x' = x + \Delta x$$
$$y' = y + \Delta y$$



Let $T_x = \Delta x$, & $T_y = \Delta y$

then

$$\boxed{\begin{aligned} x' &= x + T_x \\ y' &= y + T_y \end{aligned}}$$

Translation

In the projection coordinates (homogeneous coordinates) in a matrix form.

$$(x \;\; y \;\; 1) \longrightarrow (x' \;\; y' \;\; 1)$$

$$(x' \;\; y' \;\; 1) = (x \;\; y \;\; 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{pmatrix}$$

Therefore the translation matrix $T_{(T_x, T_y)}$

$$T_{(T_x, T_y)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{pmatrix}$$

$$(x' \;\; y' \;\; 1) = (x \;\; y \;\; 1) \; (T_{(T_x, T_y)})$$

The inverse of this transformations is

$$x = x' - T_x$$

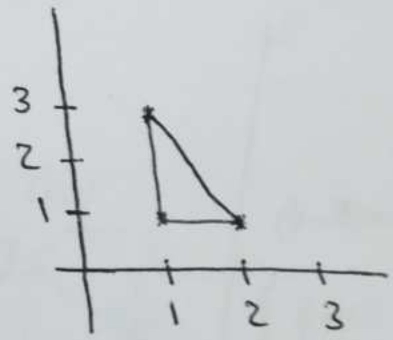
$$y = y' - T_y$$

&

$$T^{-1}_{(T_x, T_y)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -T_x & -T_y & 1 \end{pmatrix}$$ H.W.

Example: If the triangle A(1,1) B(2,1) C(1,3) translate the triangle by a const. 2 for both x & y dimension.

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$



Then the scaling (translation) matrix is

$$T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

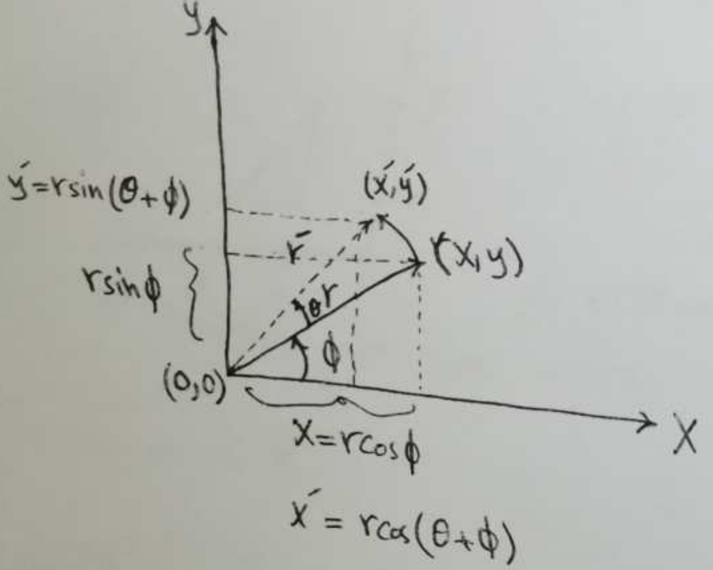
$$\therefore \underline{P' = PT}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 1 \\ 4 & 2 & 1 \\ 2 & 6 & 1 \end{bmatrix}$$

- Rotation:

1) Consider the rotation about Z-axis which passes through the origin (0, 0, 0).



from the fig.

$$\left.\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \end{aligned}\right\} \text{------ (1)}$$

then the rotated coordinates x', y' are

$$\left.\begin{aligned} x' &= r\cos(\theta+\phi) \\ y' &= r\sin(\theta+\phi) \end{aligned}\right\} \text{------ (2)}$$

Using the trigonometric

$$\left.\begin{aligned} x' &= r(\cos\theta \, \underline{\cos\phi} - \sin\theta \, \underline{\sin\phi}) \\ y' &= r(\sin\theta \, \underline{\cos\phi} + \cos\theta \, \underline{\sin\phi}) \end{aligned}\right\} \text{---- (3)}$$

$$\therefore \left.\begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= x\sin\theta + y\cos\theta \end{aligned}\right\} \text{------ (4)}$$

The rotation about the origin.

In the matrix form equ. (4) can be written as

$$(x' \quad y') = (x \quad y)\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

or simply we can write

$$X' = XR$$

$\therefore$ The matrix of rotation about the origin is

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

The other matrix of rotation about the origin in the homogeneous coordinates is

$$R_{(\theta)} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then the transformation (rotation)

$$(x' \; y' \; 1) = (x \; y \; 1) \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Properties of $R(\theta)$: $= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

1. $R^{-1}(\theta) = R^{T}(\theta) = R(\theta)$

   $R^{-1}(\theta) = \dfrac{adj(R)}{|R|} = R^{T}$ H.W.

2. $R(\theta_1 + \theta_2) = R(\theta_1) \cdot R(\theta_2)$ H.W.

3. $|R(\theta)| = 1$ H.W.

4. $R^{n}(\theta) = R(n\theta)$ H.W.

3. Scaling:

The equ.s which describe the scaling are

$$x' = S_x \; x$$
$$y' = S_y \; y$$

Therefore

$$\therefore (x' \; y' \; 1) = (x \; y \; 1) \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ in homogenous coordinates

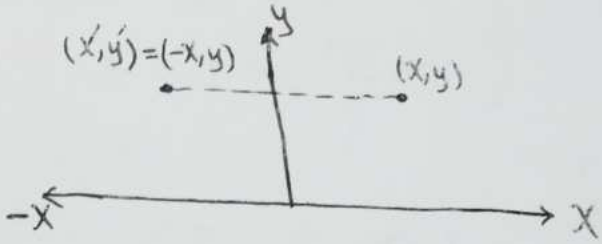
If $\left\{ \begin{matrix} S_x > 1 \\ S_y > 1 \end{matrix} \right\}$ applification & $\left. \begin{matrix} S_x < 1 \\ S_y < 1 \end{matrix} \right\}$ minimization

$$S = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

find the S^{-1} H.W.

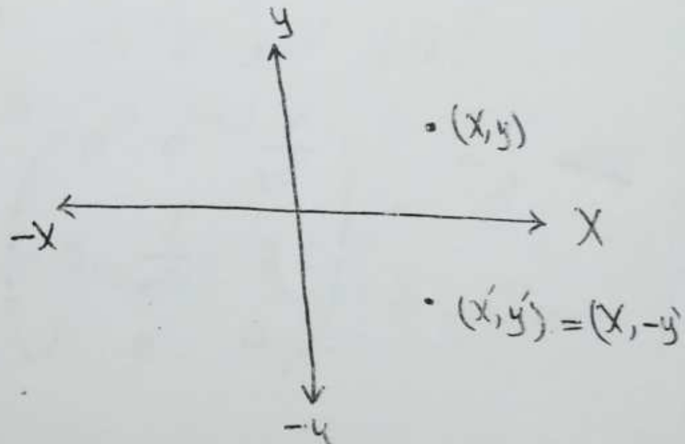
ans. $$S^{-1} = \begin{pmatrix} \frac{1}{S_x} & 0 & 0 \\ 0 & \frac{1}{S_y} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Reflection:



$$\left.\begin{array}{l} x' \longrightarrow -x \\ y' \longrightarrow y \end{array}\right\} \text{reflection about } y\text{-axis}$$

$$\left.\begin{array}{l} x' \longrightarrow x \\ y' \longrightarrow -y \end{array}\right\} \text{reflection about } x\text{-axis}$$



Therefore the reflection can be written in the matrix form in homogenous coordinates

a. Reflection about y-axis:

$$(x' \quad y' \quad 1) = (x \quad y \quad 1) \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b. Reflection about x-axis:

$$(x' \quad y' \quad 1) = (x \quad y \quad 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

C. Reflection about the origin:

$$(x' \quad y' \quad 1) = (x \quad y \quad 1) \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{pmatrix}$$

H.W: Write down the inverse matrices of the reflections in homogenous coordinates

i.e.

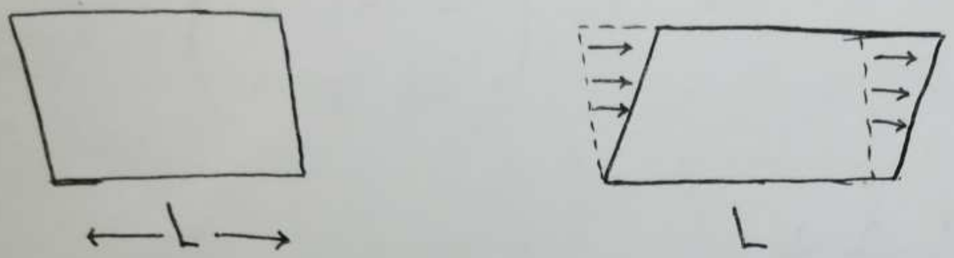
$$Ref(y) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow Ref^{-1}(y)$$

$$Ref(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow Ref^{-1}(x)$$

$$Ref(o) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow Ref^{-1}(o)$$

القص، التشويه

5. Shearing:



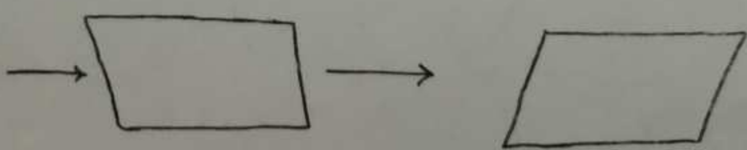
A transformation in which all points along a given line L remain fixed, while other points are shifted parallel to L, by a distance proportional to their perpendicular distance from L.

Shearing a plane fig. does not change it area.

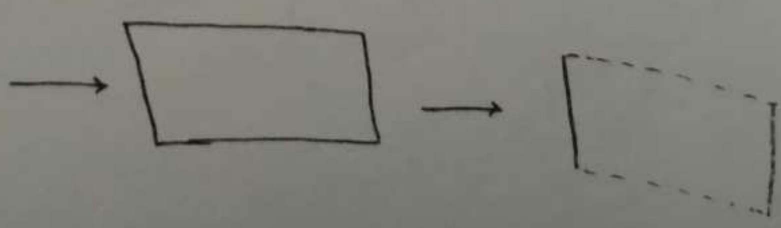
The shear can also be generalized to three dimensional in which plane are translated instead of lines.

Therefore the shearing transformations are

$x' = x + ay$



$y' = y + bx$



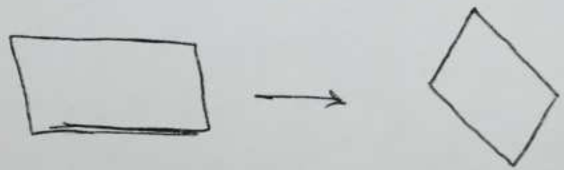
$$(x' \quad y' \quad 1) = (x \quad y \quad 1)\begin{pmatrix} 1 & b & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

i.e.

$$Sh_x = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Sh_y = \begin{pmatrix} 1 & b & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Sh_{xy} = \begin{pmatrix} 1 & b & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



<u>Note:</u> All the previous transformations are at the origin (0,0).

2. Transformation about general point (fixed point) (X_0, y_0):

1. Scaling about general point

$$S' = T^{-1}(x_0, y_0)\, S\, T(x_0, y_0)$$