University of Mosul College of science Department of Physics Third Stage Lecture 9

# Laser

2024-2025

Lecture 9: Threshold Requirements for a Lasers

Preparation
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### 3/11/2024

# **Threshold Requirements for a Lasers**

Most lasers have limitations of population density that can be achieved in any particular energy level. Also, the stimulated emission cross section is essentially a constant. Therefore, the only factor of the gain that can be increased is the *effective length* of the amplifier since it is not practical to make long lasers. This increased effective length can be realized with the use of mirrors. We will determine the threshold gain requirements for producing a laser with no mirrors, with either one or two mirrors located at the ends of the gain medium The threshold gain conditions are defined as the necessary requirements for the beam to grow to the point at which it reaches the saturation intensity  $I_{\text{sat}}$ .

### Laser with no Mirrors

A diagram of the amplifiers is shown in Figures (19), along with an outline of the beam envelope that would emerge if the length L were sufficient for the beam to reach  $I_{\text{sat}}$  at the end of the amplifier.

$$g_{th}L_{sat} = \sigma_{ul}^H(v)\Delta N_{ul}L_{sat} = 12 \pm 5$$

# Example

A laser amplifier has a gain coefficient of  $60\text{cm}^{-1}$ , and an amplifier length of 0.2m, for what diameter  $d_a$  of the gain medium would the saturation intensity be reached in a single pass through the amplifier?

#### **Solution:**

g=60 cm<sup>-1</sup> & L=0.2 m

the exponential growth factor (x)=60x0.2=12

$$e^x = 16\left(\frac{L}{d_a}\right)^2 = 16\left(\frac{0.2}{d_a}\right)^2 = e^{12}$$

SO

$$\frac{L_{sat}}{d_a} = 100 \Rightarrow d_a = 2 \ mm$$

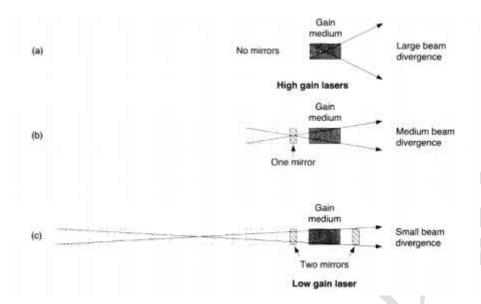


Figure (19): Laser beam divergence for an amplifier with, (a) no mirror, (b) one mirror, (c) two mirrors

# **Laser with one Mirror:**

If the length L were not sufficient to reach saturation, we could add a mirror behind the amplifier as shown in Fig. (20, b). adding a mirror can be thought of as adding a second amplifier behind the first one. We assume that the mirror reflectivity is 100% at the laser wavelength and that the mirror is placed directly behind the amplifier. assuming that the beam just reaches  $I_{\text{sat}}$  after two passes through the amplifier. We can see that the beam is narrow, as it emerges from the end of the amplifier when compared to the case with no mirrors. The gain medium would meet the threshold gain if

$$g_{th}L_{sat} = g_{th}(2L) = g_{th}L_{eff} = \sigma_{ul}N_{ul}(2L) = 12 \pm 5$$

Where  $L_{\text{eff}}$  is  $\rightarrow$  effective saturation length.

# Example:

A laser amplifier with a length of 0.12 m and again coefficient of 60 m<sup>-1</sup> has a 100% reflecting mirror on one end of the laser rod. For a rod diameter of 5 mm, would the beam reach the saturation intensity as it emerges from the rod after having made a double pass through the rod?

#### **Solution:**

The effective saturation length  $L_{\text{eff}}$ =2L

1)

$$e^{x} = 16 \left(\frac{L_{eff}}{d_{a}}\right)^{2} = 16 \left(\frac{2x0.12}{0.005}\right)^{2} \to x = 10.5$$
$$x = g(v_{o})L_{eff} = g(v_{o})2L$$
$$L = \frac{x}{2g(v_{o})} = \frac{10.5}{2x60} = 0.087 m$$

Thus the 0.12 m rod will be more than sufficient to reach the saturation intensity.

# **Laser with Two Mirrors:**

If the length L were still not sufficient for the beam to reach  $I_{\rm sat}$  after two passes through the amplifier, we could add a second mirror in front of the amplifier, we will allow a slight amount of light to leak out of the end by using a mirror with only 99% reflectivity, so that a portion of the beam can escape and provide an observable signal placing a mirror at each end of the amplifier effectively adds an infinite series of amplifiers behind the original amplifiers as shown in Fig. (19), this allows the amplifier to have as much length as necessary to reach  $I_{\rm sat}$ , provided that the mirror reflectivity's are sufficiently high. For such an arrangement the beam emerges with a very narrow angular divergence. This results in a very low-divergence laser beam and i's therefore the arrangement used for most laser.

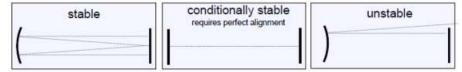
### **Stable Curved Mirror Cavities**

**Longitudinal modes:** only specific frequencies are possible inside the optical cavity of a laser, according to standing wave condition.

**Transverse modes:** are created in cross section of the beam, perpendicular to the optical axis of the laser.

#### Resonator Stability

Need the resonator to be "stable", i.e. the light stays in the cavity

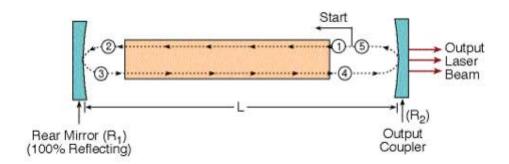


The total losses of the laser system is due to a number of different processes these are:

- 1. Transmission at the mirrors
- 2. Absorption and scattering by the mirrors
- 3. Diffraction losses at the mirrors
- 4. Absorption in the laser medium

All these losses will contribute to reduce the effective gain coefficient

Round trip gain Figure below show the round trip path of the radiation through the laser cavity. The path is divided to sections numbered by 1-5, while point "5" is the same point as "1".



# **Optical cavities**

The optical cavities (also known as optical resonators) are made to amplify the light within the cavity, so the mirrors used are highly reflective. Essentially, light enters the cavity through one mirror, reflects off the opposite mirror, and returns to the first mirror, while some of it is transmitted (exits the cavity) through each mirror. This light transmitted through the first mirror in each arm is the light that interferes at the beam splitter to form the signal.

# **Curved Mirror Cavities**

The curved mirrors have lower diffraction losses than plane parallel mirrors, where the radius of curvature of the mirrors is equal to the spacing between the mirrors.

There are a number of different types of curved mirror laser cavities, as shown in Fig. (30).

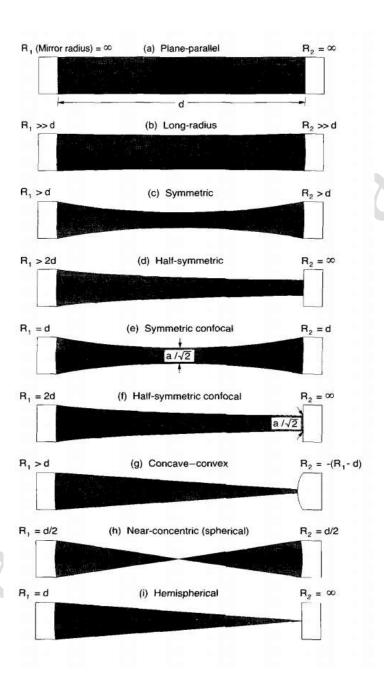


Figure (30): Two mirror laser cavity

As indicated in the figure, the cavities are distinguished from each other in terms of the relative value of the radius of curvature of the mirror in comparison to the separation distance d between the mirrors. However, using these definitions does not tell us whether or not the cavities will function as useful, stable laser cavities.

Thus, in order for a beam to evolve into a stable-steady-state beams the beam profile at the mirrors *must duplicate* itself after successive passes through the amplifier.

Our approach will therefore be to analyze the round trip propagation of a beam from one mirror to the other many successive times and then to determine the conditions for which the beam remains concentrated within the cavity as opposed to diverging out of the cavity. Those conditions for which it converges will then be designated as the stable conditions. In Summary for spherical mirrors of radius *R* such that:

$$R=2f$$
 where  $0 < d < 2R$ 

For two mirrors of unequal curvature (f1  $\neq$  f2) separated by distance d, we have:

$$0 < \alpha_1 \alpha_2 < 1$$
 .....(120)

Where

$$\alpha_1 = 1 - \frac{d}{2f_1} = 1 - \frac{d}{R_1} = g_1$$
 .....(121)

$$\alpha_2 = 1 - \frac{d}{2f_2} = 1 - \frac{d}{R_2} = g_2$$
 .....(122)

For stability, we thus have the requirement that:

$$0 < \left(1 - \frac{d}{R_1}\right) \left(1 - \frac{d}{R_2}\right) < 1$$
 or  $0 < g_1 g_2 < 1$  .....(123)

This condition can be expressed in the form of stability diagram, as shown in Fig. (31), the clear regions are the regions where eq. (123) is not satisfied and  $g_1g_2 > 1$ . For such conditions the relationships between  $R_1$ ,  $R_2$  and d will therefore not lead to stability, or, the cavity is unstable. For the shaded regions, eq. (123) is satisfied,  $g_1g_2 < 1$  and the cavity is stable.

Three particular points in Fig. (31) are of special interest. They represent cavities depicted in Fig. (31).

$$R1=R2=d/2$$
 (symmetric concentric) 
$$R1=R2=d$$
 (confocal) ......(124) 
$$R1=R2=\alpha$$
 (plane parallel)

All three of these cavities are on the edge of stability in the diagram and can become "lossy" for slight deviations into the shaded regions. Thus it would be wise to design those cavities so that  $g_1$ ,  $g_2$  move slightly into the stable zones indicated in Fig. (30).

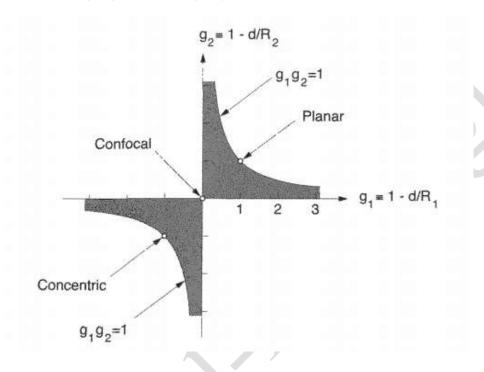
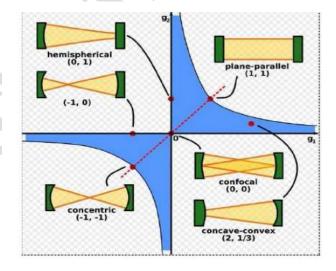


Figure (30): Stability diagram for two mirrors with radii of curvature  $R_1$  and  $R_2$ 



# Properties of Specific Two Mirrors Laser Cavities

### **Plane Parallel Cavity:**

- The plane parallel cavity or planar cavity is shown in Fig. (30), is on the edge of stability  $(g_1g_2=1)$ . If either of the mirrors had just a slightly concave figure then it would move toward the center of the diagram, it would become a long radius cavity. This cavity is essentially unstable because *it is nearly impossible to keep two flat mirrors aligned perfectly normal to the cavity axis*. The only *advantages* of this cavity would be that:
- it could access a completely cylindrically shaped volume between the mirrors and hence would match perfectly with a cylindrical gain medium, Thus, used in pulsed lasers which need the maximum energy.
- No focusing of the laser radiation inside the optical cavity. In high power lasers such focusing can cause electric breakdown, or damage to the optical elements.

### And the disadvantages are:

- High diffraction losses.
- Very high sensitivity to misalignment. Thus, very difficult to operate.

**Symmetric Confocal Cavity**: It has the radii of curvature of both mirrors equal to the separation between the mirrors, that is  $R_1=R_2=R=d$  and hence  $g_1=g_2=g=0$ . It is used because the focal length of each mirror is R=d and so the two focal coincide at the center of the cavity.

The confocal cavity can be seen from stability diagram of Fig. (31) to be on the edge of instability. A slight change in the mirror separation can move it into the unstable region of the diagram. This could cause a low gain laser to cease operation for a small range of mirror separations. It has the smallest overall beam waist when averaged over the entire mode volume within the cavity. That does not mean that it has the smallest beam waist, just the smallest average waist.

**Near Concentric (Spherical) Cavity**: It is on the opposite end of the stability diagram from the planar cavity and it It's also very sensitive to alignment. The cavity gives a large beam waist at the mirrors but a very small beam waist at the center of the cavity. Such a cavity is not very useful for extracting energy from a typical gain medium because the beam diameter at the center of the cavity is small and hence would not be taking advantage of all of the energy shared within the gain regions.

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