University of Mosul

College of Science

Department of Physics

Third Stage

Lecture 10

Geometric Optics

2024 – 2025

Lecture 10: Combination of Thin Lenses

Preparation

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Calculate the magnification. Because M is positive and less than 1, the image is upright and smaller than the object:

$$M = -\left(\frac{-3.33\text{ cm}}{5.00\text{ cm}}\right) = +0.666$$

REMARKS Notice that in every case the image is virtual, hence on the same side of the lens as the object. Further, the image is smaller than the object. For a diverging lens and a real object, this is *always* the case, as can be proven mathematically.

QUESTION 23.8 Can a diverging lens be used as a magnifying glass? Explain.

EXERCISE 23.8 Repeat the calculation, finding the position of the image and the magnification if the object is 20.0 cm from the lens.

ANSWERS $q = -6.67$ cm, $M = 0.334$

Combinations of Thin Lenses

Many useful optical devices require two lenses. Handling problems involving two lenses is not much different from dealing with a single-lens problem twice. First, the image produced by the first lens is calculated as though the second lens were not present. The light then approaches the second lens *as if* it had come from the image formed by the first lens. Hence, **the image formed by the first lens is treated as the object for the second lens**. The image formed by the second lens is the final image of the system. If the image formed by the first lens lies on the back side of the second lens, the image is treated as a virtual object for the second lens, so p is negative. The same procedure can be extended to a system of three or more lenses. The overall magnification of a system of thin lenses is the *product* of the magnifications of the separate lenses. It's also possible to combine thin lenses and mirrors as shown in Example 23.10.

EXAMPLE 23.9 Two Lenses in a Row

GOAL Calculate geometric quantities for a sequential pair of lenses.

PROBLEM Two converging lenses are placed 20.0 cm apart, as shown in Figure 23.28a (page 812), with an object 30.0 cm in front of lens 1 on the left. **(a)** If lens 1 has a focal length of 10.0 cm, locate the image formed by this lens and determine its magnification. **(b)** If lens 2 on the right has a focal length of 20.0 cm, locate the final image formed and find the total magnification of the system.

STRATEGY We apply the thin-lens equation to each lens. The image formed by lens 1 is treated as the object for lens 2. Also, we use the fact that the total magnification of the system is the product of the magnifications produced by the separate lenses.

SOLUTION

(a) Locate the image and determine the magnification of lens 1.

See the ray diagram, Figure 23.28b. Apply the thin-lens equation to lens 1:

$$\frac{1}{30.0\text{ cm}} + \frac{1}{q} = \frac{1}{10.0\text{ cm}}$$

Solve for q, which is positive and hence to the right of the first lens:

$$q = +15.0\text{ cm}$$

Compute the magnification of lens 1:

$$M_1 = -\frac{q}{p} = -\frac{15.0\text{ cm}}{30.0\text{ cm}} = -0.500$$

(b) Locate the final image and find the total magnification.

The image formed by lens 1 becomes the object for lens 2. Compute the object distance for lens 2:

$$p = 20.0\text{ cm} - 15.0\text{ cm} = 5.00\text{ cm}$$

(Continued)

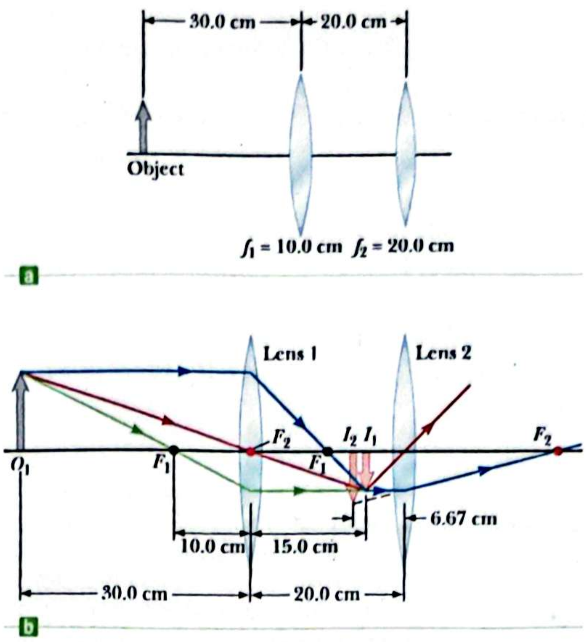

Figure 23.28 (Example 23.9)

Once again apply the thin-lens equation to lens 2 to locate the final image:

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q} = \frac{1}{20.0 \text{ cm}}$$

$$q = -6.67 \text{ cm}$$

Calculate the magnification of lens 2:

$$M_2 = -\frac{q}{p} = -\frac{(-6.67 \text{ cm})}{5.00 \text{ cm}} = +1.33$$

Multiply the two magnifications to get the overall magnification of the system:

$$M = M_1 M_2 = (-0.500)(1.33) = -0.665$$

REMARKS The negative sign for M indicates that the final image is inverted and smaller than the object because the absolute value of M is less than 1. Because q is negative, the final image is virtual.

QUESTION 23.9 If lens 2 is moved so it is 40 cm away from lens 1, would the final image be upright or inverted?

EXERCISE 23.9 If the two lenses in Figure 23.28 are separated by 10.0 cm, locate the final image and find the magnification of the system. *Hint:* The object for the second lens is virtual!

ANSWERS 4.00 cm behind the second lens, $M = -0.400$

EXAMPLE 23.10 Thin Lens and a Concave Mirror

عدسة رقيقة ومرآة مقعرة

GOAL Solve a problem involving both a lens and a mirror.

PROBLEM An object is placed 20.0 cm to the right of a concave mirror with focal length 12.0 cm and 30.0 cm to the left of a converging lens with focal length 10.0 cm, as in Figure 23.29. Locate (a) the image formed by the lens alone, (b) the image created by the mirror alone, and (c) the image created by both the mirror and lens. (d) The mirror is moved so that it is 6.00 cm away from the object. Locate the image formed by the mirror and lens.

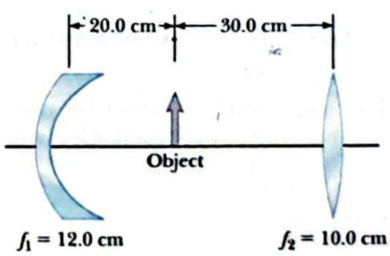

Figure 23.29 (Example 23.10)

STRATEGY Part (a) is a simple application of the thin-lens equation, Equation 23.11. Part (b) can be calculated from Equation 23.6. Using the image formed by the mirror as the object for the lens, find the image location asked for in part (c), for light that first reflects off the mirror before passing through the lens.

SOLUTION

(a) Locate the image formed by the lens alone.

Apply Equation 23.11:

$$\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f_2}$$

Substitute values and solve for the image position, q_2:

$$\frac{1}{q_2}=\frac{1}{f_2}-\frac{1}{p_2}=\frac{1}{10.0\text{ cm}}-\frac{1}{30.0\text{ cm}}=\frac{1}{15.0\text{ cm}}$$

$$q_2=15.0\text{ cm}$$

(b) Locate the image created by the mirror alone.

Apply Equation 23.6:

$$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f_1}$$

Substitute values for f_1 and p_1, which are both positive, and solve for q_1:

$$\frac{1}{q_1}=\frac{1}{f_1}-\frac{1}{p_1}=\frac{1}{12.0\text{ cm}}-\frac{1}{20.0\text{ cm}}=\frac{1}{30.0\text{ cm}}$$

$$q_1=30.0\text{ cm}$$

(c) Locate the image created by both the mirror and lens.

Apply Equation 23.11 to the image found in part (b), which becomes a real object for the lens, noticing that the image formed by the mirror is 20.0 cm from the lens:

$$\frac{1}{q_{2f}}=\frac{1}{f_2}-\frac{1}{p_{2f}}=\frac{1}{10.0\text{ cm}}-\frac{1}{20.0\text{ cm}}=\frac{1}{20.0\text{ cm}}$$

$$q_{2f}=20.0\text{ cm}$$

(d) The mirror is moved so that it is 6.00 cm to the left of the object. Locate the image formed by the mirror and lens.

The object is now much closer to the mirror. Find the new location of the image created by the mirror:

$$\frac{1}{q_1}=\frac{1}{f_1}-\frac{1}{p_1}=\frac{1}{12.0\text{ cm}}-\frac{1}{6.0\text{ cm}}=-\frac{1}{12.0\text{ cm}}$$

$$q_1=-12.0\text{ cm}$$

The image created by the mirror is virtual and therefore behind the mirror. However, it acts like a real object for the lens. Apply Equation 23.11 with $p_2 = 30.0\text{ cm} + 18.0\text{ cm} = 48.0\text{ cm}$:

$$\frac{1}{q_2}=\frac{1}{f_2}-\frac{1}{p_2}=\frac{1}{10.0\text{ cm}}-\frac{1}{48.0\text{ cm}}=\frac{19}{2.40\times 10^2\text{ cm}}$$

$$q_2=12.6\text{ cm}$$

REMARKS There are two final images created to the right of the lens, as expected. As the mirror is moved closer to the object, the final image due to both the mirror and lens moves closer to the lens. The image of part (a) is inverted; however, the image of part (c) goes through two inversions, hence is upright. The virtual mirror image of part (d) is upright, so its lens image is inverted.

QUESTION 23.10 Is it possible to have a virtual object for a mirror? Explain, giving an example.

EXERCISE 23.10 The same mirror and lens are repositioned so that the mirror is 24.0 cm to the left of the lens and the object is 20.0 cm to the right of the lens. Locate the image of (a) the lens alone, (b) the first image formed by the mirror, and (c) the final, second image formed by the lens.

ANSWERS (a) 20.0 cm to the left of the lens (b) 6.00 cm behind the mirror (c) 15.0 cm to the right of the lens

23.7 Lens and Mirror Aberrations

One of the basic problems of systems containing mirrors and lenses is the imperfect quality of the images, which is largely the result of defects in shape and form. The simple theory of mirrors and lenses assumes rays make small angles with the principal axis and all rays reaching the lens or mirror from a point source are focused at a single point, producing a sharp image. This is not always true in the real world. Where the approximations used in this theory do not hold, imperfect images are formed.

If one wishes to analyze image formation precisely, it is necessary to trace each ray, using Snell's law, at each refracting surface. This procedure shows that there is no single point image; instead, the image is blurred. The departures of real (imperfect) images from the ideal predicted by the simple theory are called **aberrations.** Two common types of aberrations are spherical aberration and chromatic aberration.

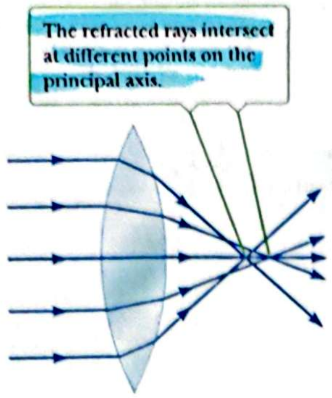

Figure 23.30 Spherical aberration used by a converging lens. Does a diverging lens use spherical aberration?

Spherical Aberration

Spherical aberration results from the fact that the focal points of light rays passing far from the principal axis of a spherical lens (or mirror) are different from the focal points of rays with the same wavelength passing near the axis. Figure 23.30 illustrates spherical aberration for parallel rays passing through a converging lens. Rays near the middle of the lens are imaged farther from the lens than rays at the edges. Hence, there is no single focal length for a spherical lens.

Most cameras are equipped with an adjustable aperture to control the light intensity and, when possible, reduce spherical aberration. (An aperture is an opening that controls the amount of light transmitted through the lens.) As the aperture size is reduced, sharper images are produced because only the central portion of the lens is exposed to the incident light when the aperture is very small. At the same time, however, progressively less light is imaged. To compensate for this loss, a longer exposure time is used. An example of the results obtained with small apertures is the sharp image produced by a pinhole camera, with an aperture size of approximately 0.1 mm.

In the case of mirrors used for very distant objects, one can eliminate, or at least minimize, spherical aberration by employing a parabolic rather than spherical surface. Parabolic surfaces are not used in many applications, however, because they are very expensive to make with high-quality optics. Parallel light rays incident on such a surface focus at a common point. Parabolic reflecting surfaces are used in many astronomical telescopes to enhance the image quality. They are also used in flashlights, in which a nearly parallel light beam is produced from a small lamp placed at the focus of the reflecting surface.

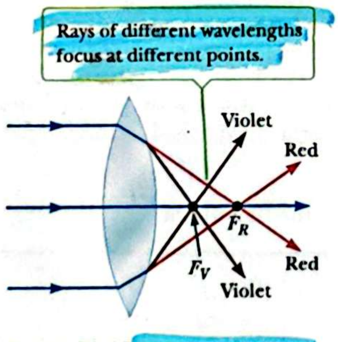

Figure 23.31 Chromatic aberration produced by a converging lens.

Chromatic Aberration

Different wavelengths of light refracted by a lens focus at different points, which gives rise to chromatic aberration. In Chapter 22 we described how the index of refraction of a material varies with wavelength. When white light passes through a lens, for example, violet light rays are refracted more than red light rays (see Fig. 23.31), so the focal length for red light is greater than for violet light. Other wavelengths (not shown in the figure) would have intermediate focal points. Chromatic aberration for a diverging lens is opposite that for a converging lens. Chromatic aberration can be greatly reduced by a combination of converging and diverging lenses.