University of Mosul

College of Science

Department of Physics

Third Stage

Lecture 2

Geometric Optics

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Lecture 2: The Law of Refraction

Preparation

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22.3 The Law of Refraction

When light passes from one transparent medium to another, it's refracted because the speed of light is different in the two media.[1] The **index of refraction, n,** of a medium is defined as the ratio c/v;

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \qquad [22.4]$$

From this definition, we see that the index of refraction is a dimensionless number that is greater than or equal to 1 because v is always less than c. Further, n is equal to one for vacuum. Table 22.1 lists the indices of refraction for various substances.

As light travels from one medium to another, its frequency doesn't change. To see why, consider Figure 22.8. Wave fronts pass an observer at point A in medium 1 with a certain frequency and are incident on the boundary between medium 1 and medium 2. The frequency at which the wave fronts pass an observer at point B in medium 2 must equal the frequency at which they arrive at point A. If not, the wave fronts would either pile up at the boundary or be destroyed or created at the boundary. Because neither of these events occurs, the frequency must remain the same as a light ray passes from one medium into another.

Therefore, because the relation $v = f\lambda$ must be valid in both media and because $f_1 = f_2 = f$, we see that

$$v_1 = f\lambda_1 \text{ and } v_2 = f\lambda_2$$

Because $v_1 \neq v_2$, it follows that $\lambda_1 \neq \lambda_2$. A relationship between the index of refraction and the wavelength can be obtained by dividing these two equations and making use of the definition of the index of refraction given by Equation 22.4:

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \qquad [22.5]$$

which gives

$$\lambda_1 n_1 = \lambda_2 n_2 \qquad [22.6]$$

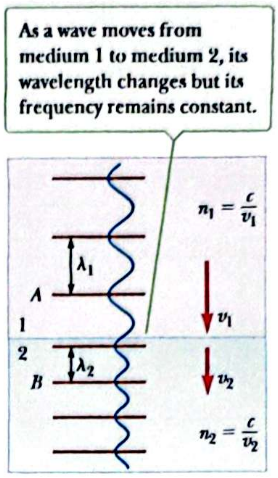


Figure 22.8 A wave travels from medium 1 to medium 2, in which it moves with lower speed.

Tip 22.1 An Inverse Relationship

The index of refraction is *inversely* proportional to the wave speed. Therefore, as the wave speed v decreases, the index of refraction, n, *increases*.

Tip 22.2 The Frequency Remains the Same

The *frequency* of a wave does *not* change as the wave passes from one medium to another. Both the wave speed and the wavelength *do* change, but the frequency remains the same.

Table 22.1 Indices of Refraction for Various Substances, Measured with Light of Vacuum Wavelength λ_0 = 589 mn

Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		**Liquids at 20°C**	
Diamond (C)	2.419	Benzene	1.501
Fluorite (CaF_2)	1.434	Carbon disulfide	1.628
Fused quartz (SiO_2)	1.458	Carbon tetrachloride	1.461
Glass, crown	1.52	Ethyl alcohol	1.361
Glass, flint	1.66	Glycerine	1.473
Ice (H_2O) (at 0°C)	1.309	Water	1.333
Polystyrene	1.49		
Sodium chloride (NaCl)	1.544	**Gases at 0°C, 1 atm**	
Zircon	1.923	Air	1.000 293
		Carbon dioxide	1.000 45

[1]The speed of light varies between media because the time lags caused by the absorption and reemission of light as it travels from atom to atom depend on the particular electronic structure of the atoms constituting each material.

Let medium 1 be the vacuum so that $n_1 = 1$. It follows from Equation 22.6 that the index of refraction of any medium can be expressed as the ratio

$$n = \frac{\lambda_0}{\lambda_n} \qquad [22.7]$$

where λ_0 is the wavelength of light in vacuum and λ_n is the wavelength in the medium having index of refraction n. Figure 22.9 is a schematic representation of this reduction in wavelength when light passes from a vacuum into a transparent medium.

We are now in a position to express Equation 22.3 in an alternate form. If we substitute Equation 22.5 into Equation 22.3, we get

Snell's law of refraction ▶

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad [22.8]$$

The experimental discovery of this relationship is usually credited to Willebrørd Snell (1591–1626) and is therefore known as **Snell's law of refraction.**

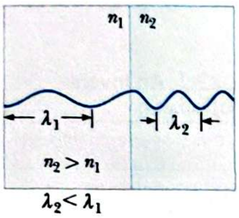


Figure 22.9 A schematic diagram of the *reduction* in wavelength when light travels from a medium with a low index of refraction to one with a higher index of refraction.

Quick Quiz

22.3 A material has an index of refraction that increases continuously from top to bottom. Of the three paths shown in Figure 22.10, which path will a light ray follow as it passes through the material?

22.4 As light travels from a vacuum ($n = 1$) to a medium such as glass ($n > 1$), which of the following properties remains the same, the (a) wavelength, (b) wave speed, or (c) frequency?

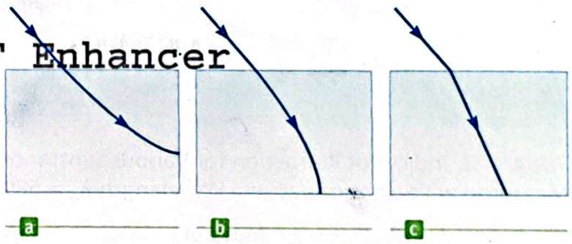
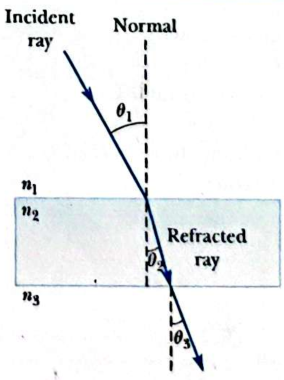


Figure 22.10 (Quick Quiz 22.3)

EXAMPLE 22.2 Angle of Refraction for Glass

GOAL Apply Snell's law.

PROBLEM A light ray of wavelength 589 nm (produced by a sodium lamp) traveling through air is incident on a smooth, flat slab of crown glass at an angle θ_1 of 30.0° to the normal, as sketched in Figure 22.11. (a) Find the angle of refraction, θ_2. (b) At what angle θ_3 does the ray leave the glass as it re-enters the air? (c) How does the answer for θ_3 change if the ray enters water below the slab instead of the air?



STRATEGY Substitute quantities into Snell's law and solve for the unknown angles of refraction.

Figure 22.11 (Example 22.2) Refraction of light by glass.

SOLUTION

(a) Find the angle of refraction, θ_2.

Solve Snell's law (Eq. 22.8) for $\sin\theta_2$:

$$(1) \quad \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

From Table 22.1, find $n_1 = 1.00$ for air and $n_2 = 1.52$ for crown glass. Substitute these values into Equation (1) and take the inverse sine of both sides:

$$\sin\theta_2 = \left(\frac{1.00}{1.52}\right)(\sin 30.0°) = 0.329$$

$$\theta_2 = \sin^{-1}(0.329) = 19.2°$$

(b) At what angle θ_3 does the ray leave the glass as it re-enters the air?

Write Equation (1), replacing θ_3 with θ_2 and θ_1 with θ_2:

$$(2)\quad \sin\theta_3 = \frac{n_2}{n_3}\sin\theta_2 = \frac{1.52}{1.00}\sin(19.2°) = 0.500$$

Take the inverse sine of both sides to find θ_3:

$$\theta_3 = \sin^{-1}(0.500) = 30.0°$$

(c) How does the answer for θ_3 change if the ray enters water below the slab instead of air?

Write Equation (2) and substitute a different value for n_3:

$$\sin\theta_3 = \frac{n_2}{n_3}\sin\theta_2 = \frac{1.52}{1.33}\sin(19.2°) = 0.376$$

$$\theta_3 = \sin^{-1}(0.376) = 22.1°$$

REMARKS Notice that the light ray bends toward the normal when it enters a material of a higher index of refraction, and away from the normal when entering a material with a lower index of refraction. In passing through a slab of material with parallel surfaces, for example from air to glass and back to air, the final direction of the ray is parallel to the direction of the incident ray. The only effect in that case is a lateral displacement of the light ray.

QUESTION 22.2 If the glass is replaced by a transparent material with smaller index of refraction, will the refraction angle θ_2 be (a) smaller, (b) larger, or (c) unchanged?

EXERCISE 22.2 Suppose a light ray in air ($n = 1.00$) enters a cube of material ($n = 2.50$) at a 45.0° angle with respect to the normal and then exits the bottom of the cube into water ($n = 1.33$). At what angle to the normal does the ray leave the slab?

ANSWER 32.1°

EXAMPLE 22.3 Light in Fused Quartz

GOAL Use the index of refraction to determine the effect of a medium on light's speed and wavelength.

PROBLEM Light of wavelength 589 nm in vacuum passes through a piece of fused quartz of index of refraction $n = 1.458$. (a) Find the speed of light in fused quartz. (b) What is the wavelength of this light in fused quartz? (c) What is the frequency of the light in fused quartz?

STRATEGY Substitute values into Equations 22.4 and 22.7.

SOLUTION

(a) Find the speed of light in fused quartz.

Obtain the speed from Equation 22.4:

$$v = \frac{c}{n} = \frac{3.00 \times 10^8\ \text{m/s}}{1.458} = 2.06 \times 10^8\ \text{m/s}$$

(b) What is the wavelength of this light in fused quartz?

Use Equation 22.7 to calculate the wavelength:

$$\lambda_n = \frac{\lambda_0}{n} = \frac{589\ \text{nm}}{1.458} = 404\ \text{nm}$$

(c) What is the frequency of the light in fused quartz?

The frequency in quartz is the same as in vacuum. Solve $c = f\lambda$ for the frequency:

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{589 \times 10^{-9}\ \text{m}} = 5.09 \times 10^{14}\ \text{Hz}$$

REMARKS It's interesting to note that the speed of light in vacuum, 3.00×10^8 m/s, is an upper limit for the speed of material objects. In our treatment of relativity in Chapter 26, we will find that this upper limit is consistent with experimental observations. However, it's possible for a particle moving in a medium to have a speed that exceeds the speed of light in

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that medium. For example, it's theoretically possible for a particle to travel through fused quartz at a speed greater than 2.06×10^8 m/s, but it must still have a speed less than 3.00×10^8 m/s.

QUESTION 22.3 True or False: If light with wavelength λ in glass passes into water with index n_w, the new wavelength of the light is λ/n_w.

of index of refraction $n = 1.5442$

EXERCISE 22.3 Light with wavelength 589 nm passes through crystalline sodium chloride. In this medium, find (a) the speed of light, (b) the wavelength, and (c) the frequency of the light.

ANSWER (a) 1.94×10^8 m/s (b) 381 nm (c) 5.09×10^{14} Hz

EXAMPLE 22.4 Refraction of Laser Light in a Digital Videodisc (DVD)

GOAL Apply Snell's law together with geometric constraints.

PROBLEM A DVD is a video recording consisting of a spiral track about 1.0 μm wide with digital information. (See Fig. 22.12a.) The digital information consists of a series of pits that are "read" by a laser beam sharply focused on a track in the information layer. If the width a of the beam at the information layer must equal 1.0 μm to distinguish individual tracks and the width w of the beam as it enters the plastic is 0.700 0 mm, find the angle θ_1 at which the conical beam should enter the plastic. (See Fig. 22.12b.) Assume the plastic has a thickness $t = 1.20$ mm and an index of refraction $n = 1.55$. Note that this system is relatively immune to small dust particles degrading the video quality because particles would have to be as large as 0.700 mm to obscure the beam at the point where it enters the plastic.

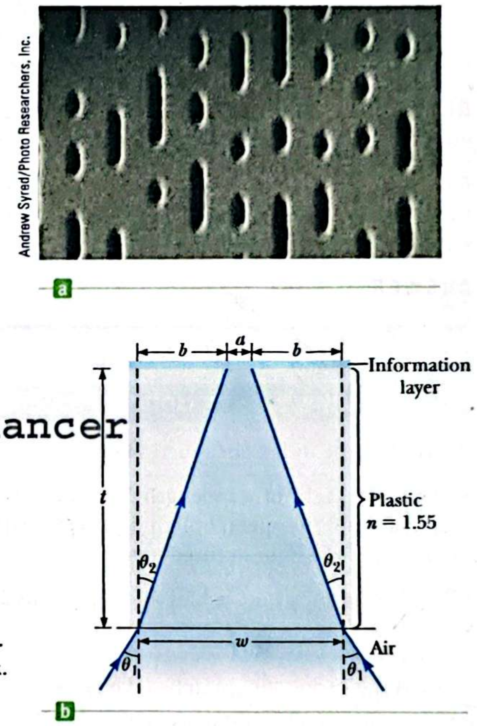

Figure 22.12 (Example 22.4) A micrograph of a DVD surface showing tracks and pits along each track. (b) Cross section of a cone-shaped laser beam used to read a DVD.

STRATEGY Use right-triangle trigonometry to determine the angle θ_2 and then apply Snell's law to obtain the angle θ_1.

SOLUTION

From the top and bottom of Figure 22.12b, obtain an equation relating w, b, and a:

$$w = 2b + a$$

Solve this equation for b and substitute given values:

$$b = \frac{w - a}{2} = \frac{700.0 \times 10^{-6}\,\text{m} - 1.0 \times 10^{-6}\,\text{m}}{2} = 349.5\ \mu\text{m}$$

Now use the tangent function to find θ_2:

$$\tan\theta_2 = \frac{b}{t} = \frac{349.5\ \mu\text{m}}{1.20 \times 10^3\ \mu\text{m}} \rightarrow \theta_2 = 16.2^\circ$$

Finally, use Snell's law to find θ_1:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_1 = \frac{n_2 \sin\theta_2}{n_1} = \frac{1.55 \sin 16.2^\circ}{1.00} = 0.432$$

$$\theta_1 = \sin^{-1}(0.433) = 25.6^\circ$$