University of Mosul

College of Science

Department of Physics

Third Stage

Lecture 4

Geometric Optics

2024 – 2025

Lecture 4: Huygens’ Principle and Total Internal Reflection

Preparation

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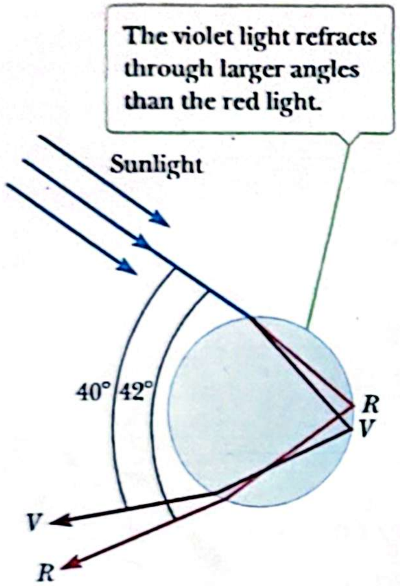


Active Figure 22.19
Refraction of sunlight by a spherical raindrop.

22.5 The Rainbow

The dispersion of light into a spectrum is demonstrated most vividly in nature through the formation of a rainbow, often seen by an observer positioned between the Sun and a rain shower. To understand how a rainbow is formed, consider Active Figure 22.19. A ray of light passing overhead strikes a drop of water in the atmosphere and is refracted and reflected as follows: It is first refracted at the front surface of the drop, with the violet light deviating the most and the red light the least. At the back surface of the drop, the light is reflected and returns to the front surface, where it again undergoes refraction as it moves from water into air. The rays leave the drop so that the angle between the incident white light and the returning violet ray is 40° and the angle between the white light and the returning red ray is 42°. This small angular difference between the returning rays causes us to see the bow as explained in the next paragraph.

Now consider an observer viewing a rainbow, as in Figure 22.20a. If a raindrop high in the sky is being observed, the red light returning from the drop can reach the observer because it is deviated the most, but the violet light passes over the

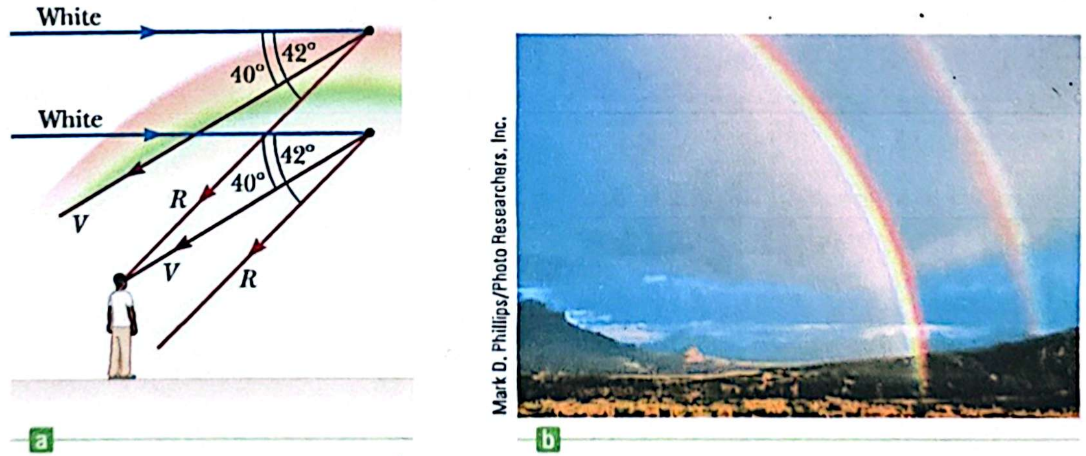


Figure 22.20 The formation of a rainbow seen by an observer standing with the Sun behind his back. (b) This photograph of a rainbow shows a distinct secondary rainbow with the colors reversed.

observer because it is deviated the least. Hence, the observer sees this drop as being red. Similarly, a drop lower in the sky would direct violet light toward the observer and appear to be violet. (The red light from this drop would strike the ground and not be seen.) The remaining colors of the spectrum would reach the observer from raindrops lying between these two extreme positions. Figure 22.20b shows a beautiful rainbow and a secondary rainbow with its colors reversed.

22.6 Huygens' Principle

The laws of reflection and refraction can be deduced using a geometric method proposed by Huygens in 1678. Huygens assumed light is a form of wave motion rather than a stream of particles. He had no knowledge of the nature of light or of its electromagnetic character. Nevertheless, his simplified wave model is adequate for understanding many practical aspects of the propagation of light.

Huygens' principle is a geometric construction for determining at some instant the position of a new wave front from knowledge of the wave front that preceded it. (A wave front is a surface passing through those points of a wave which have the same phase and amplitude. For instance, a wave front could be a surface passing through the crests of waves.) **In Huygens' construction, all points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, that propagate in the forward direction with speeds characteristic of waves in that medium. After some time has elapsed, the new position of the wave front is the surface tangent to the wavelets.**

◀ Huygens' principle

Figure 22.21 illustrates two simple examples of Huygens' construction. First, consider a plane wave moving through free space, as in Figure 22.21a. At $t = 0$, the wave front is indicated by the plane labeled AA'. In Huygens' construction, each point on this wave front is considered a point source. For clarity, only a few points on AA' are shown. With these points as sources for the wavelets, we draw circles of radius $c\,\Delta t$, where c is the speed of light in vacuum and Δt is the period of propagation from one wave front to the next. The surface drawn tangent to these wavelets is the plane BB', which is parallel to AA'. In a similar manner, Figure 22.21b shows Huygens' construction for an outgoing spherical wave.

Huygens' Principle Applied to Reflection and Refraction

The laws of reflection and refraction were stated earlier in the chapter without proof. We now derive these laws using Huygens' principle. Figure 22.22a (page 776) illustrates the law of reflection. The line AA' represents a wave front of the incident light. As ray 3 travels from A' to C, ray 1 reflects from A and produces a spherical wavelet of radius AD. (Recall that the radius of a Huygens wavelet is $v\,\Delta t$.)

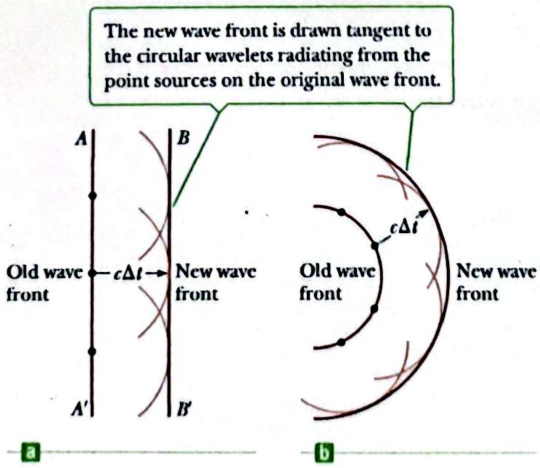


Figure 22.21 Huygens' constructions for (a) a plane wave propagating to the right and (b) a spherical wave.

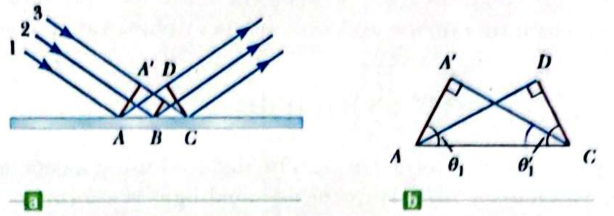


Figure 22.22 (a) Huygens' construction for proving the law of reflection. (b) Triangle ADC is congruent to triangle $AA'C$.

Because the two wavelets having radii $A'C$ and AD are in the same medium, they have the same speed v, so $AD = A'C$. Meanwhile, the spherical wavelet centered at B has spread only half as far as the one centered at A because ray 2 strikes the surface later than ray 1.

From Huygens' principle, we find that the reflected wave front is CD, a line tangent to all the outgoing spherical wavelets. The remainder of our analysis depends on geometry, as summarized in Figure 22.22b. Note that the right triangles ADC and $AA'C$ are congruent because they have the same hypotenuse, AC, and because $AD = A'C$. From the figure, we have

$$\sin\theta_1 = \frac{A'C}{AC} \quad \text{and} \quad \sin\theta_1' = \frac{AD}{AC}$$

The right-hand sides are equal, so $\sin\theta_1 = \sin\theta_1'$, and it follows that $\theta_1 = \theta_1'$, which is the law of reflection.

Huygens' principle and Figure 22.23a can be used to derive Snell's law of refraction. In the time interval Δt, ray 1 moves from A to B and ray 2 moves from A' to C. The radius of the outgoing spherical wavelet centered at A is equal to $v_2\,\Delta t$. The distance $A'C$ is equal to $v_1\,\Delta t$. Geometric considerations show that angle $A'AC$ equals θ_1 and angle ACB equals θ_2. From triangles $AA'C$ and ACB, we find that

$$\sin\theta_1 = \frac{v_1\Delta t}{AC} \quad \text{and} \quad \sin\theta_2 = \frac{v_2\Delta t}{AC}$$

If we divide the first equation by the second, we get

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

From Equation 22.4, though, we know that $v_1 = c/n_1$ and $v_2 = c/n_2$. Therefore,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

and it follows that

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

which is the law of refraction.

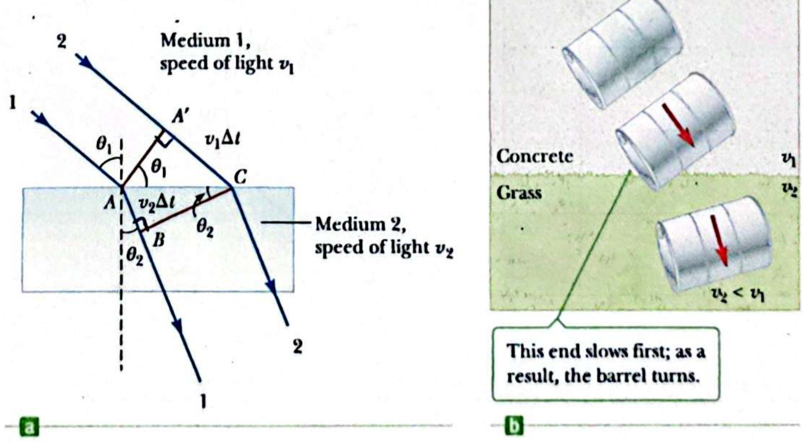


Figure 22.23 (a) Huygens' construction for proving the law of refraction. (b) Overhead view of a barrel rolling from concrete onto grass.

A mechanical analog of refraction is shown in Figure 22.23b. When the left end of the rolling barrel reaches the grass, it slows down, while the right end remains on the concrete and moves at its original speed. This difference in speeds causes the barrel to pivot, changing the direction of its motion.

22.7 Total Internal Reflection

An interesting effect called ***total internal reflection*** can occur when light encounters the boundary between a medium with a *higher* index of refraction and one with a *lower* index of refraction. Consider a light beam traveling in medium 1 and meeting the boundary between medium 1 and medium 2, where n_1 is greater than n_2 (Active Fig. 22.24). Possible directions of the beam are indicated by rays 1 through 5. Note that the refracted rays are bent away from the normal because n_1 is greater than n_2. At some particular angle of incidence θ_c, called the **critical angle**, the refracted light ray moves parallel to the boundary so that $\theta_2 = 90°$ (Active Fig. 22.24b). *For angles of incidence greater than* θ_c, the beam is entirely reflected at the boundary, as is ray 5 in Active Figure 22.24a. This ray is reflected as though it had struck a perfectly reflecting surface. It and all rays like it obey the law of reflection: the angle of incidence equals the angle of reflection.

We can use Snell's law to find the critical angle. When $\theta_1 = \theta_c$ and $\theta_2 = 90°$, Snell's law (Eq. 22.8) gives

$$n_1 \sin\theta_c = n_2 \sin 90° = n_2$$

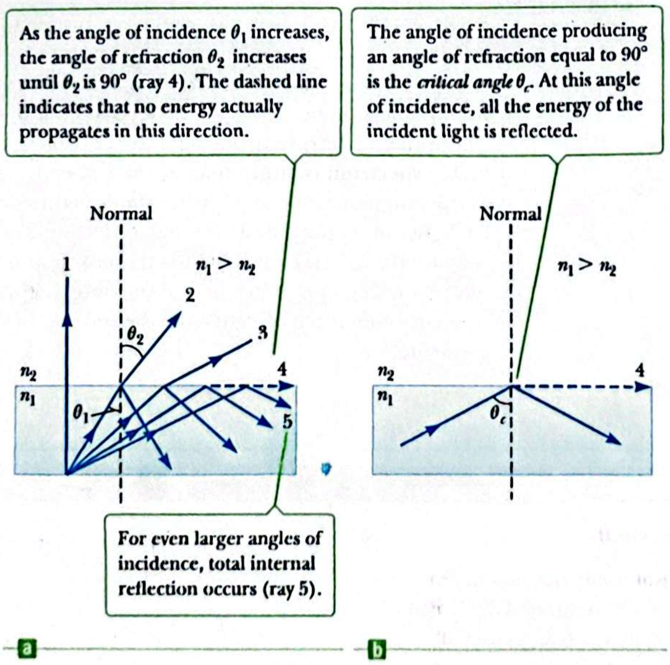
$$\sin\theta_c = \frac{n_2}{n_1} \quad \text{for } n_1 > n_2 \qquad [22.9]$$

Equation 22.9 can be used only when n_1 is greater than n_2 because **total internal reflection occurs only when light is incident on the boundary of a medium having a lower index of refraction than the medium in which it's traveling.** If n_1 were less than n_2, Equation 22.9 would give $\sin\theta_c > 1$, which is an absurd result because the sine of an angle can never be greater than 1.

When medium 2 is air, the critical angle is small for substances with large indices of refraction, such as diamond, where $n = 2.42$ and $\theta_c = 24.0°$. By comparison,


Courtesy of Henry Leap and Jim Lehman

This photograph shows nonparallel light rays entering a glass prism. The bottom two rays undergo total internal reflection at the longest side of the prism. The top three rays are refracted at the longest side as they leave the prism.



Active Figure 22.24
(a) Rays from a medium with index of refraction n_1 travel to a medium with index of refraction n_2, where $n_1 > n_2$. (b) Ray 4 is singled out.

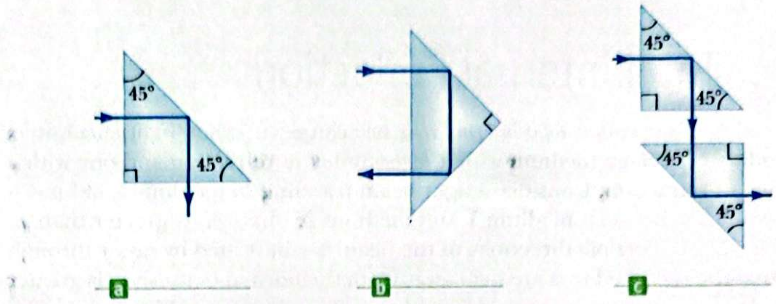


Figure 22.25 Internal reflection in a prism. (a) The ray is deviated by 90°. (b) The direction of the ray is reversed. (c) Two prisms used as a periscope.

for crown glass, $n = 1.52$ and $\theta_c = 41.0°$. This property, combined with proper faceting, causes a diamond to sparkle brilliantly.

APPLICATION
Submarine Periscopes

A prism and the phenomenon of total internal reflection can alter the direction of travel of a light beam. Figure 22.25 illustrates two such possibilities. In one case the light beam is deflected by 90° (Fig. 22.25a), and in the second case the path of the beam is reversed (Fig. 22.25b). A common application of total internal reflection is a submarine periscope. In this device two prisms are arranged as in Figure 22.25c so that an incident beam of light follows the path shown and the user can "see around corners."

APPLYING PHYSICS 22.4 Total Internal Reflection and Dispersion

A beam of white light is incident on the curved edge of a semicircular piece of glass, as shown in Figure 22.26. The light enters the curved surface along the normal, so it shows no refraction. It encounters the straight side of the glass at the center of curvature of the curved side and refracts into the air. The incoming beam is moved clockwise (so that the angle θ increases) such that the beam always enters along the normal to the curved side and encounters the straight side at the center of curvature of the curved side. Why does the refracted beam become redder as it approaches a direction parallel to the straight side?

EXPLANATION When the outgoing beam approaches the direction parallel to the straight side, the incident angle is approaching the critical angle for total internal reflection. Dispersion occurs as the light passes out of the glass. The index of refraction for light at the violet end of the visible spectrum is larger than at the red end. As a result, as the outgoing beam approaches the straight side, the violet light undergoes total internal reflection, followed by the other colors. The red light is the last to undergo total internal reflection, so just before the outgoing light disappears, it's composed of light from the red end of the visible spectrum. ■

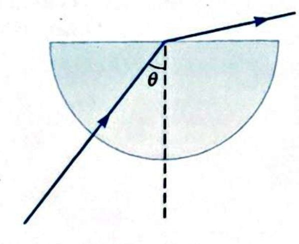


Figure 22.26 (Applying Physics 22.4)

EXAMPLE 22.6 A View from the Fish's Eye

GOAL Apply the concept of total internal reflection.

PROBLEM (a) Find the critical angle for a water–air boundary. (b) Use the result of part (a) to predict what a fish will see (Fig. 22.27) if it looks up toward the water surface at angles of 40.0°, 48.6°, and 60.0°.

STRATEGY After finding the critical angle by substitution, use that the path of a light ray is reversible: at a given angle, wherever a light beam can go is also where a light beam can come from, along the same path.

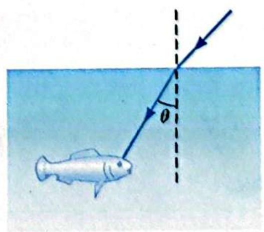


Figure 22.27 (Example 22.6) A fish looks upward toward the water's surface.

SOLUTION

(a) Find the critical angle for a water–air boundary.

Substitute into Equation 22.9 to find the critical angle:

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1.00}{1.333} = 0.750$$

$$\theta_c = \sin^{-1}(0.750) = 48.6°$$

(b) Predict what a fish will see if it looks up toward the water surface at angles of 40.0°, 48.6°, and 60.0°.

At an angle of 40.0°, a beam of light from underwater will be refracted at the surface and enter the air above. Because the path of a light ray is reversible (Snell's law works both going and coming), light from above can follow the same path and be perceived by the fish. At an angle of 48.6°, the critical angle for water, light from underwater is bent so that it travels along the surface. So light following the same path in reverse can reach the fish only by skimming along the water surface before being refracted toward the fish's eye. At angles greater than the critical angle of 48.6°, a beam of light shot toward the surface will be completely reflected down toward the bottom of the pool. Reversing the path, the fish sees a reflection of some object on the bottom.

QUESTION 22.6 If the water is replaced by a transparent fluid with a higher index of refraction, is the critical angle of the fluid–air boundary (a) larger, (b) smaller, or (c) the same as for water?

EXERCISE 22.6 Suppose a layer of oil with $n = 1.50$ coats the surface of the water. What is the critical angle for total internal reflection for light traveling in the oil layer and encountering the oil–water boundary?

ANSWER 62.7°

Fiber Optics

Another interesting application of total internal reflection is the use of solid glass or transparent plastic rods to "pipe" light from one place to another. As indicated in Figure 22.28, light is confined to traveling within the rods, even around gentle curves, as a result of successive internal reflections. Such a light pipe can be quite flexible if thin fibers are used rather than thick rods. If a bundle of parallel fibers is used to construct an optical transmission line, images can be transferred from one point to another.

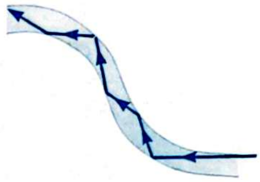


Figure 22.28 Light travels in a curved transparent rod by multiple internal reflections.

Very little light intensity is lost in these fibers as a result of reflections on the sides. Any loss of intensity is due essentially to reflections from the two ends and absorption by the fiber material. Fiber-optic devices are particularly useful for viewing images produced at inaccessible locations. Physicians often use fiber-optic cables to aid in the diagnosis and correction of certain medical problems without the intrusion of major surgery. For example, a fiber-optic cable can be threaded through the esophagus and into the stomach to look for ulcers. In this application the cable consists of two fiber-optic lines: one to transmit a beam of light into the stomach for illumination and the other to allow the light to be transmitted out of the stomach. The resulting image can, in some cases, be viewed directly by the physician, but more often is displayed on a television monitor or saved in digital form. In a similar way, fiber-optic cables can be used to examine the colon or to help physicians perform surgery without the need for large incisions.

BIO APPLICATION

Fiber Optics in Medical Diagnosis and Surgery

The field of fiber optics has revolutionized the entire communications industry. Billions of kilometers of optical fiber have been installed in the United States to carry high-speed Internet traffic, radio and television signals, and telephone calls. The fibers can carry much higher volumes of telephone calls and other forms of communication than electrical wires because of the higher frequency of the infrared light used to carry the information on optical fibers. Optical fibers are also preferable to copper wires because they are insulators and don't pick up stray electric and magnetic fields or electronic "noise."

APPLICATION

Fiber Optics in Telecommunications

(*Left*) Strands of glass optical fibers are used to carry voice, video, and data signals in telecommunication networks. (*Right*) A bundle of optical fibers is illuminated by a laser.

Dennis O'Clair/Tony Stone Images/Getty Images

Hank Morgan/Photo Researchers, Inc.

APPLYING PHYSICS 22.5 Design of an Optical Fiber

An optical fiber consists of a transparent core surrounded by cladding, which is a material with a lower index of refraction than the core (Fig. 22.29). A cone of angles, called the acceptance cone, is at the entrance to the fiber. Incoming light at angles within this cone will be transmitted through the fiber, whereas light entering the core from angles outside the cone will not be transmitted. The figure shows a light ray entering the fiber just within the acceptance cone and undergoing total internal reflection at the interface between the core and the cladding. If it is technologically difficult to produce light so that it enters the fiber from a small range of angles, how could you adjust the indices of refraction of the core and cladding to increase the size of the acceptance cone? Would you design the indices to be farther apart or closer together?

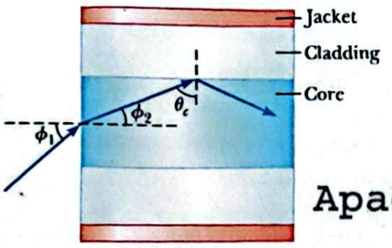


Figure 22.29 (Applying Physics 22.5)

EXPLANATION The acceptance cone would become larger if the critical angle (θ_c in the figure) could be made smaller. This adjustment can be done by making the index of refraction of the cladding material smaller so that the indices of refraction of the core and cladding material would be farther apart. ■

SUMMARY

22.1 The Nature of Light

Light has a dual nature. In some experiments it acts like a wave, in others like a particle, called a photon by Einstein. The energy of a photon is proportional to its frequency,

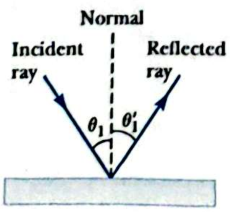
$$E = hf \quad [22.1]$$

where $h = 6.63 \times 10^{-34}$ J · s is *Planck's constant*.

22.2 Reflection and Refraction

In the reflection of light off a flat, smooth surface, the angle of incidence, θ_1, with respect to a line perpendicular to the surface is equal to the angle of reflection, θ_1':

$$\theta_1' = \theta_1 \quad [22.2]$$



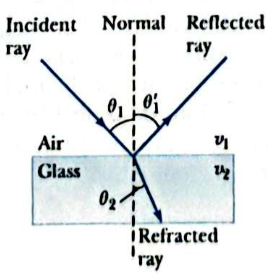
The wave under reflection model.

Light that passes into a transparent medium is bent at the boundary and is said to be *refracted*. The angle of refraction is the angle the ray makes with respect to a line perpendicular to the surface after it has entered the new medium.

22.3 The Law of Refraction

The **index of refraction** of a material, n, is defined as

$$n = \frac{c}{v} \quad [22.4]$$



The wave under refraction model.