University of Mosul

College of Science

Department of Physics

Third Stage

Lecture 6

Geometric Optics

2024 – 2025

Lecture 6: Concave Mirrors

Preparation

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which remains in contact with the ground. ■

23.2 Images Formed by Concave Mirrors

A **spherical mirror,** as its name implies, has the shape of a segment of a sphere. Figure 23.7 shows a spherical mirror with a silvered inner, concave surface; this type of mirror is called a **concave mirror.** The mirror has radius of curvature *R*, and its center of curvature is at point *C*. Point *V* is the center of the spherical segment, and a line drawn from *C* to *V* is called the **principal axis** of the mirror.

Now consider a point source of light placed at point *O* in Figure 23.7b, on the principal axis and outside point *C*. Several diverging rays originating at *O* are shown. After reflecting from the mirror, these rays converge to meet at *I*, called the **image point**. The rays then continue and diverge from *I* as if there were an object there. As a result, a real image is formed. **Whenever reflected light actually passes through a point, the image formed there is real.**

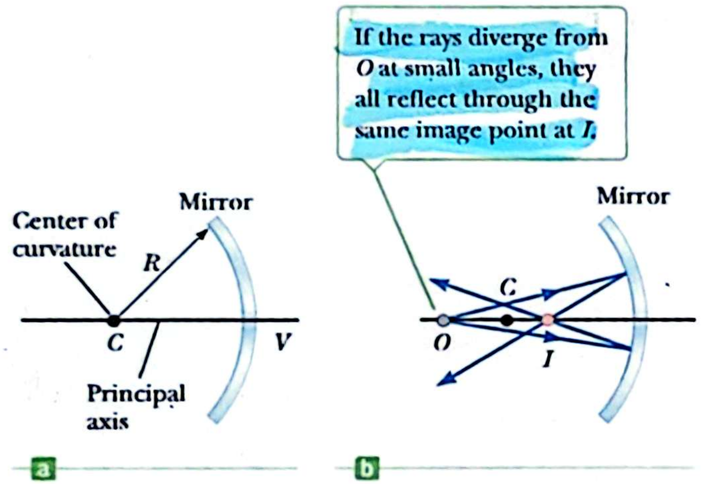


Figure 23.7 (a) A concave mirror of radius *R*. The center of curvature, *C*, is located on the principal axis. (b) A point object placed at *O* in front of a concave spherical mirror of radius *R*, where *O* is any point on the principal axis farther than *R* from the surface of the mirror, forms a real image at *I*.

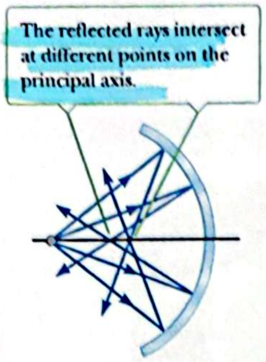


Figure 23.8 A spherical concave mirror inhibits *spherical aberration* when light rays make large angles with the principal axis.

We often assume all rays that diverge from the object make small angles with the principal axis. All such rays reflect through the image point, as in Figure 23.7b. Rays that make a large angle with the principal axis, as in Figure 23.8, converge to other points on the principal axis, producing a blurred image. This effect, called **spherical aberration**, is present to some extent with any spherical mirror and will be discussed in Section 23.7.

We can use the geometry shown in Figure 23.9 to calculate the image distance q from the object distance p and radius of curvature R. By convention, these distances are measured from point V. The figure shows two rays of light leaving the tip of the object. One ray passes through the center of curvature, C, of the mirror, hitting the mirror head-on (perpendicular to the mirror surface) and reflecting back on itself. The second ray strikes the mirror at point V and reflects as shown, obeying the law of reflection. The image of the tip of the arrow is at the point where the two rays intersect. From the largest triangle in Figure 23.9, we see that $\tan\theta = h/p$; the light-blue triangle gives $\tan\theta = -h'/q$. The negative sign has been introduced to satisfy our convention that h' is negative when the image is inverted with respect to the object, as it is here. From Equation 23.1 and these results, we find that the magnification of the mirror is

$$M = \frac{h'}{h} = -\frac{q}{p} \qquad [23.2]$$

From two other triangles in the figure, we get

$$\tan\alpha = \frac{h}{p-R} \quad \text{and} \quad \tan\alpha = -\frac{h'}{R-q}$$

from which we find that

$$\frac{h'}{h} = -\frac{R-q}{p-R} \qquad [23.3]$$

If we compare Equation 23.2 with Equation 23.3, we see that

$$\frac{R-q}{p-R} = \frac{q}{p}$$

Simple algebra reduces this equation to

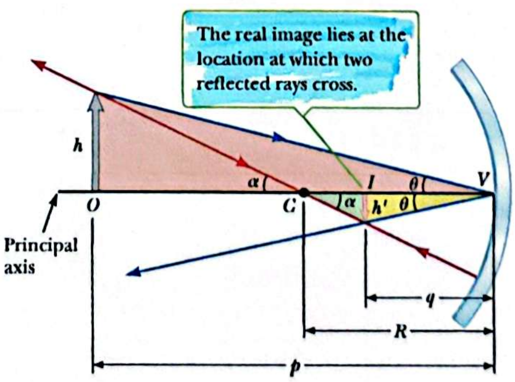
Mirror equation ▶

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \qquad [23.4]$$

This expression is called the **mirror equation**.

If the object is very far from the mirror—if the object distance p is great enough compared with R that p can be said to approach infinity—then $1/p \approx 0$, and we see from Equation 23.4 that $q \approx R/2$. In other words, when the object is very far from

Figure 23.9 The image formed by a spherical concave mirror, where the object at O lies outside the center of curvature, C.



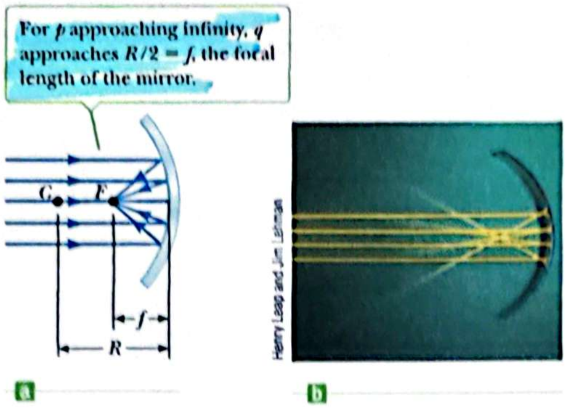


Figure 23.10 (a) Light rays from a distant object ($p = \infty$) reflect from a concave mirror through the focal point F. (b) A photograph of the reflection of parallel rays from a concave mirror.

the mirror, the image point is halfway between the center of curvature and the center of the mirror, as in Figure 23.10a. The incoming rays are essentially parallel in that figure because the source is assumed to be very far from the mirror. In this special case we call the image point the **focal point** F and the image distance the **focal length** f, where

$$f = \frac{R}{2} \qquad [23.5]$$

◀ Focal length

The mirror equation can therefore be expressed in terms of the focal length:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad [23.6]$$

Note that rays from objects at infinity are always focused at the focal point.

Tip 23.2 Focal Point ≠ Focus Point

The focal point is *not* the point at which light rays focus to form an image. The focal point of a mirror is determined *solely* by its curvature; it doesn't depend on the location of any object.

23.3 Convex Mirrors and Sign Conventions

Figure 23.11 shows the formation of an image by a **convex mirror**, which is silvered so that light is reflected from the outer, convex surface. It is sometimes called a **diverging mirror** because the rays from any point on the object diverge after reflection, as though they were coming from some point behind the mirror. The image in Figure 23.11 is virtual rather than real because it lies behind the mirror at the point the reflected rays appear to originate. In general, the image formed by a convex mirror is upright, virtual, and smaller than the object.

We won't derive any equations for convex spherical mirrors. If we did, we would find that the equations developed for concave mirrors can be used with convex mirrors if particular sign conventions are used. We call the region in which light

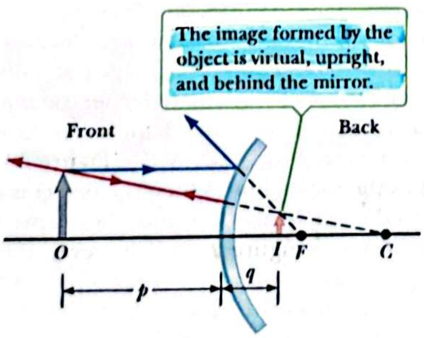


Figure 23.11 Formation of an image by a spherical, convex mirror.

Table 23.1 Sign Conventions for Mirrors

Quantity	Symbol	In Front	In Back	Upright Image	Inverted Image
Object location	p	+	−		
Image location	q	+	−		
Focal length	f	+	−		
Image height	h'			+	−
Magnification	M			+	−

Tip 23.3 Positive Is Where the Light Is

The quantities p, q, and f are all positive when they are located where the light is—in front of the mirror as indicated in Figure 23.12.

rays move the *front side* of the mirror, and the other side, where virtual images are formed, the *back side*. For example, in Figures 23.9 and 23.11, the side to the left of the mirror is the front side and the side to the right is the back side. Figure 23.12 is helpful for understanding the rules for object and image distances, and Table 23.1 summarizes the sign conventions for all the necessary quantities. Notice that when the quantities p, q, and f (and R) are located where the light is—in front of the mirror—they are positive, whereas when they are located behind the mirror (where the light isn't), they are negative.

Ray Diagrams for Mirrors

We can conveniently determine the positions and sizes of images formed by mirrors by constructing *ray diagrams* similar to the ones we have been using. This kind of graphical construction tells us the overall nature of the image and can be used to check parameters calculated from the mirror and magnification equations. Making a ray diagram requires knowing the position of the object and the location of the center of curvature. To locate the image, three rays are constructed (rather than only the two we have been constructing so far), as shown by the examples in Active Figure 23.13. All three rays start from the same object point; for these examples, the tip of the arrow was chosen. For the concave mirrors in Active Figures 23.13a and 23.13b, the rays are drawn as follows:

1. Ray 1 is drawn parallel to the principal axis and is reflected back through the focal point F.
2. Ray 2 is drawn through the focal point and is reflected parallel to the principal axis.
3. Ray 3 is drawn through the center of curvature, C, and is reflected back on itself.

Note that rays actually go in all directions from the object; we choose to follow those moving in a direction that simplifies our drawing.

The intersection of any *two* of these rays at a point locates the image. The third ray serves as a check of our construction. The image point obtained in this fashion must always agree with the value of q calculated from the mirror formula.

In the case of a concave mirror, note what happens as the object is moved closer to the mirror. The real, inverted image in Active Figure 23.13a moves to the left as the object approaches the focal point. When the object is at the focal point, the image is infinitely far to the left. When the object lies between the focal point and the mirror surface, as in Active Figure 23.13b, however, the image is virtual and upright.

With the convex mirror shown in Active Figure 23.13c, the image of a real object is always virtual and upright. As the object distance increases, the virtual image shrinks and approaches the focal point as p approaches infinity. You should construct a ray diagram to verify these statements.

The image-forming characteristics of curved mirrors obviously determine their uses. For example, suppose you want to design a mirror that will help people shave

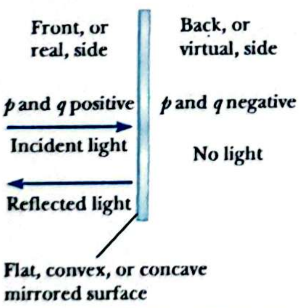
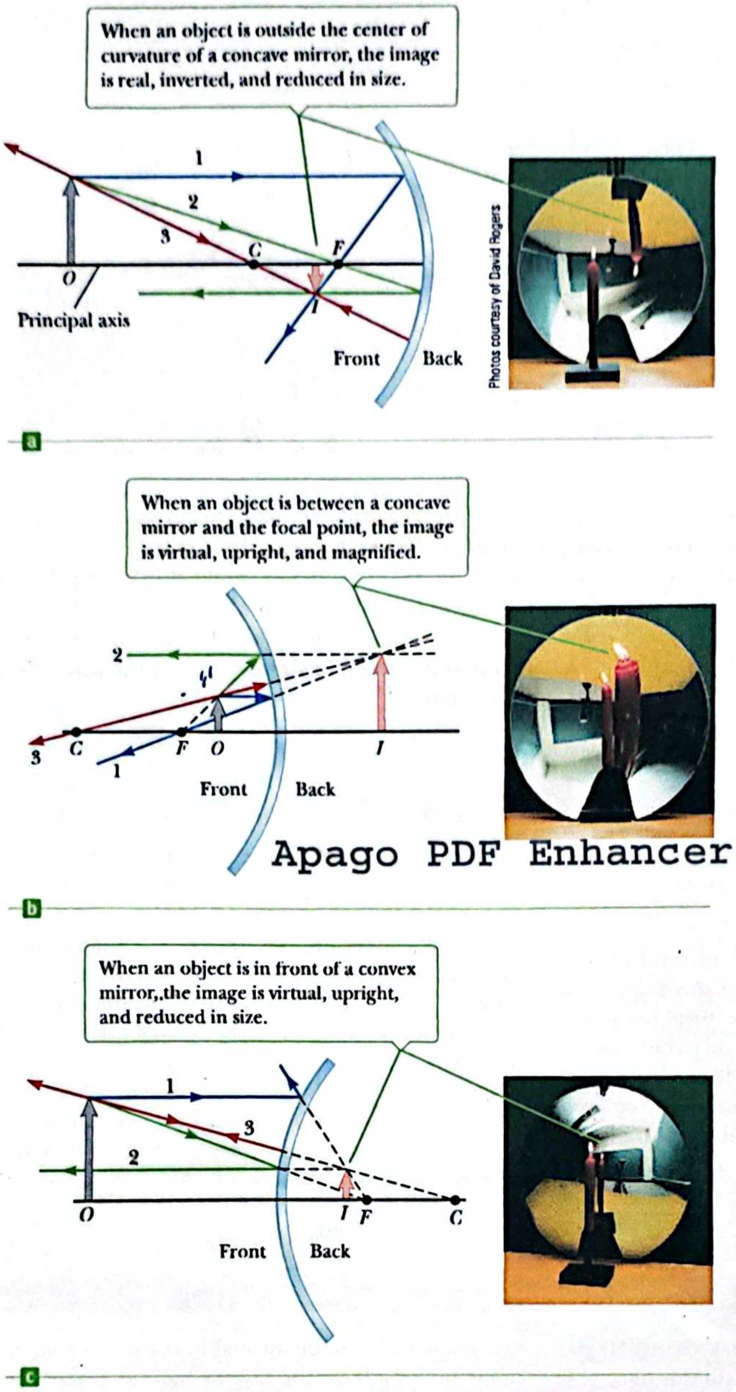


Figure 23.12 A diagram describing the signs of p and q for convex and concave mirrors.



Active Figure 23.13
Ray diagrams for spherical mirrors and corresponding photographs of the images of candles.

or apply cosmetics. For this, you need a concave mirror that puts the user inside the focal point, such as the mirror in Active Figure 23.13b. With that mirror, the image is upright and greatly enlarged. In contrast, suppose the primary purpose of a mirror is to observe a large field of view. In that case you need a convex mirror such as the one in Active Figure 23.13c. The diminished size of the image means that a fairly large field of view is seen in the mirror. Mirrors like this one are often placed in stores to help employees watch for shoplifters. A second use of such a

Figure 23.14 A convex side-view mirror on a vehicle produces an upright image that is smaller than the object. The smaller image means that the object is closer than its apparent distance as observed in the mirror.

mirror is as a side-view mirror on a car (Fig. 23.14). This kind of mirror is usually placed on the passenger side of the car and carries the warning "Objects are closer than they appear." Without such warning, a driver might think she is looking into a flat mirror, which doesn't alter the size of the image. She could be fooled into believing that a truck is far away because it looks small, when it's actually a large semi very close behind her, but diminished in size because of the image formation characteristics of the convex mirror.

■ APPLYING PHYSICS 23.2 | Concave Versus Convex

A virtual image can be anywhere behind a concave mirror. Why is there a maximum distance at which the image can exist behind a *convex* mirror?

EXPLANATION Consider the concave mirror first and imagine two different light rays leaving the object and striking the mirror. If the object is at the focal point, the light rays reflecting from the mirror will be parallel to the mirror axis. They can be interpreted as forming a virtual image infinitely far away behind the mirror. As the object is brought closer to the mirror, the reflected rays will diverge through larger and larger angles, resulting in their extensions converging closer and closer to the back of the mirror. When the object is brought right up to the mirror, the image is right behind the mirror. When the object is much closer to the mirror than the focal length, the mirrors acts like a flat mirror and the image is just as far behind the mirror as the object is in front of it. The image can therefore be anywhere from infinitely far away to right at the surface of the mirror. For the convex mirror, an object at infinity produces a virtual image at the focal point. As the object is brought closer, the reflected rays diverge more sharply and the image moves closer to the mirror. As a result, the virtual image is restricted to the region between the mirror and the focal point. ■

■ APPLYING PHYSICS 23.3 | Reversible Waves

Large trucks often have a sign on the back saying, "If you can't see my mirror, I can't see you." Explain this sign.

EXPLANATION The trucking companies are making use of the principle of the reversibility of light rays. For an image of you to be formed in the driver's mirror, there must be a pathway for rays of light to reach the mirror, allowing the driver to see your image. If you can't see the mirror, this pathway doesn't exist. ■

■ EXAMPLE 23.2 | Images Formed by a Concave Mirror

GOAL Calculate properties of a concave mirror.