University of Mosul

College of Science

Department of Physics

Third Stage

Lecture 7

Geometric Optics

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Lecture 7: Convex Mirrors

Preparation

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Figure 23.14 A convex side-view mirror on a vehicle produces an upright image that is smaller than the object. The smaller image means that the object is closer than its apparent distance as observed in the mirror.

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mirror is as a side-view mirror on a car (Fig. 23.14). This kind of mirror is usually placed on the passenger side of the car and carries the warning "Objects are closer than they appear." Without such warning, a driver might think she is looking into a flat mirror, which doesn't alter the size of the image. She could be fooled into believing that a truck is far away because it looks small, when it's actually a large semi very close behind her, but diminished in size because of the image formation characteristics of the convex mirror.

■ APPLYING PHYSICS 23.2 Concave Versus Convex

A virtual image can be anywhere behind a concave mirror. Why is there a maximum distance at which the image can exist behind a *convex* mirror?

EXPLANATION Consider the concave mirror first and imagine two different light rays leaving any object and striking the mirror. If the object is at the focal point, the light rays reflecting from the mirror will be parallel to the mirror axis. They can be interpreted as forming a virtual image infinitely far away behind the mirror. As the object is brought closer to the mirror, the reflected rays will diverge through larger and larger angles, resulting in their extensions converging closer and closer to the back of the mirror. When the object is brought right up to the mirror, the image is right behind the mirror. When the object is much closer to the mirror than the focal length, the mirrors acts like a flat mirror and the image is just as far behind the mirror as the object is in front of it. The image can therefore be anywhere from infinitely far away to right at the surface of the mirror. For the convex mirror, an object at infinity produces a virtual image at the focal point. As the object is brought closer, the reflected rays diverge more sharply and the image moves closer to the mirror. As a result, the virtual image is restricted to the region between the mirror and the focal point. ■

■ APPLYING PHYSICS 23.3 Reversible Waves

Large trucks often have a sign on the back saying, "If you can't see my mirror, I can't see you." Explain this sign.

EXPLANATION The trucking companies are making use of the principle of the reversibility of light rays. For an image of you to be formed in the driver's mirror, there must be a pathway for rays of light to reach the mirror, allowing the driver to see your image. If you can't see the mirror, this pathway doesn't exist. ■

■ EXAMPLE 23.2 Images Formed by a Concave Mirror

GOAL Calculate properties of a concave mirror.

PROBLEM Assume a certain concave, spherical mirror has a focal length of 10.0 cm. **(a)** Locate the image and find the magnification for an object distance of 25.0 cm. Determine whether the image is real or virtual, inverted or upright, and larger or smaller. Do the same for object distances of **(b)** 10.0 cm and **(c)** 5.00 cm.

STRATEGY For each part, substitute into the mirror and magnification equations. Part (b) involves a limiting process because the answers are infinite.

SOLUTION

(a) Find the image position for an object distance of 25.0 cm. Calculate the magnification and describe the image.

Use the mirror equation to find the image distance:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

Substitute and solve for q. According to Table 23.1, p and f are positive.

$$\frac{1}{25.0\text{ cm}}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}$$

$$q = 16.7\text{ cm}$$

Because q is positive, the image is in front of the mirror and is real. The magnification is given by substituting into Equation 23.2:

$$M=-\frac{q}{p}=-\frac{16.7\text{ cm}}{25.0\text{ cm}}=-0.668$$

The image is smaller than the object because $|M|<1$, and it is inverted because M is negative. (See Fig. 23.13a.)

(b) Locate the image distance when the object distance is 10.0 cm. Calculate the magnification and describe the image.

The object is at the focal point. Substitute $p = 10.0$ cm and $f = 10.0$ cm into the mirror equation:

$$\frac{1}{10.0\text{ cm}}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}$$

$$\frac{1}{q}=0 \quad\rightarrow\quad q=\infty$$

Because $M = -q/p$, the magnification is also infinite.

(c) Locate the image distance when the object distance is 5.00 cm. Calculate the magnification and describe the image.

Once again, substitute into the mirror equation:

$$\frac{1}{5.00\text{ cm}}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}$$

$$\frac{1}{q}=\frac{1}{10.0\text{ cm}}-\frac{1}{5.00\text{ cm}}=-\frac{1}{10.0\text{ cm}}$$

$$q=-10.0\text{ cm}$$

The image is virtual (behind the mirror) because q is negative. Use Equation 23.2 to calculate the magnification:

$$M=-\frac{q}{p}=-\left(\frac{-10.0\text{ cm}}{5.00\text{ cm}}\right)=2.00$$

The image is larger (magnified by a factor of 2) because $|M|>1$, and upright because M is positive. (See Fig. 23.13b.)

REMARKS Note the characteristics of an image formed by a concave, spherical mirror. When the object is outside the focal point, the image is inverted and real; at the focal point, the image is formed at infinity; inside the focal point, the image is upright and virtual.

QUESTION 23.2 What location does the image approach as the object gets arbitrarily far away from the mirror? (a) infinity (b) the focal point (c) the radius of curvature of the mirror (d) the mirror itself

EXERCISE 23.2 If the object distance is 20.0 cm, find the image distance and the magnification of the mirror.

ANSWER $q = 20.0$ cm, $M = -1.00$

EXAMPLE 23.3 Images Formed by a Convex Mirror

GOAL Calculate properties of a convex mirror.

PROBLEM An object 3.00 cm high is placed 20.0 cm from a convex mirror with a focal length of magnitude 8.00 cm. Find (a) the position of the image, (b) the magnification of the mirror, and (c) the height of the image.

STRATEGY This problem again requires only substitution into the mirror and magnification equations. Multiplying the object height by the magnification gives the image height.

SOLUTION

(a) Find the position of the image.

Because the mirror is convex, its focal length is negative. Substitute into the mirror equation:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{20.0\text{ cm}}+\frac{1}{q}=\frac{1}{-8.00\text{ cm}}$$

Solve for q:

$$q = -5.71\text{ cm}$$

(b) Find the magnification of the mirror.

Substitute into Equation 23.2:

$$M=-\frac{q}{p}=-\left(\frac{-5.71\text{ cm}}{20.0\text{ cm}}\right)=0.286$$

(c) Find the height of the image.

Multiply the object height by the magnification:

$$h' = hM = (3.00\text{ cm})(0.286) = 0.858\text{ cm}$$

REMARKS The negative value of q indicates the image is virtual, or behind the mirror, as in Figure 23.13c. The image is upright because M is positive.

QUESTION 23.3 True or False: A convex mirror can produce only virtual images.

EXERCISE 23.3 Suppose the object is moved so that it is 4.00 cm from the same mirror. Repeat parts (a) through (c).

ANSWERS (a) −2.67 cm (b) 0.668 (c) 2.00 cm; the image is upright and virtual.

EXAMPLE 23.4 The Face in the Mirror

GOAL Find a focal length from a magnification and an object distance.

PROBLEM When a woman stands with her face 40.0 cm from a cosmetic mirror, the upright image is twice as tall as her face. What is the focal length of the mirror?

STRATEGY To find f in this example, we must first find q, the image distance. Because the problem states that the image is upright, the magnification must be positive (in this case, $M = +2$), and because $M = -q/p$, we can determine q.

SOLUTION

Obtain q from the magnification equation:

$$M=-\frac{q}{p}=2$$

$$q=-2p=-2(40.0\text{ cm})=-80.0\text{ cm}$$

Because q is negative, the image is on the opposite side of the mirror and hence is virtual. Substitute q and p into the mirror equation and solve for f:

$$\frac{1}{40.0\text{ cm}}-\frac{1}{80.0\text{ cm}}=\frac{1}{f}$$

$$f = 80.0\text{ cm}$$

REMARKS The positive sign for the focal length tells us that the mirror is concave, a fact we already knew because the mirror magnified the object. (A convex mirror would have produced a smaller image.)

QUESTION 23.4 If she moves the mirror closer to her face, what happens to the image? (a) It becomes inverted and smaller. (b) It remains upright and becomes smaller. (c) It becomes inverted and larger. (d) It remains upright and becomes larger.

EXERCISE 23.4 Suppose a fun-house spherical mirror makes you appear to be one-third your normal height. If you are 1.20 m away from the mirror, find its focal length. Is the mirror concave or convex?

ANSWERS −0.600 m, convex

23.4 Images Formed by Refraction

In this section we describe how images are formed by refraction at a spherical surface. Consider two transparent media with indices of refraction n_1 and n_2, where the boundary between the two media is a spherical surface of radius R (Fig. 23.15). We assume the medium to the right has a higher index of refraction than the one to the left: $n_2 > n_1$. That would be the case for light entering a curved piece of glass from air or for light entering the water in a fishbowl from air. The rays originating at the object location O are refracted at the spherical surface and then converge to the image point I. We can begin with Snell's law of refraction and use simple geometric techniques to show that the object distance, image distance, and radius of curvature are related by the equation

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \qquad [23.7]$$

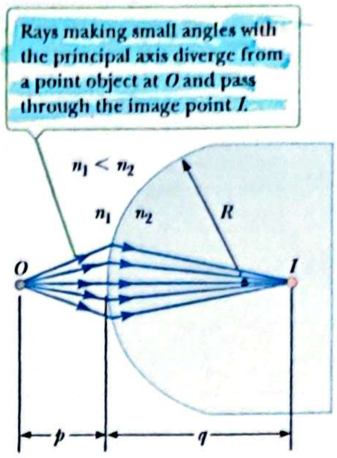

Figure 23.15 An image formed by refraction at a spherical surface.

Further, the magnification of a refracting surface is

$$M = \frac{h'}{h} = -\frac{n_1 q}{n_2 p} \qquad [23.8]$$

As with mirrors, certain sign conventions hold, depending on circumstances. First note that real images are formed by refraction on the side of the surface *opposite* the side from which the light comes, in contrast to mirrors, where real images are formed on the *same* side of the reflecting surface. This makes sense because light reflects off mirrors, so any real images must form on the same side the light comes from. With a transparent medium, the rays pass through and naturally form real images on the opposite side. We define the side of the surface where light rays originate as the front side. The other side is called the back side. Because of the difference in location of real images, the refraction sign conventions for q and R are the opposite of those for reflection. For example, p, q, and R are all positive in Figure 23.15. The sign conventions for spherical refracting surfaces are summarized in Table 23.2.

Table 23.2 Sign Conventions for Refracting Surfaces

Quantity	Symbol	In Front	In Back	Upright Image	Inverted Image
Object location	p	+	−		
Image location	q	−	+		
Radius	R	−	+		
Image height	h'			+	−