University of Mosul

College of Science

Department of Physics

Third Stage

Lecture 8

Geometric Optics

2024 – 2025

Lecture 8: Images formed by Refraction

Preparation

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QUESTION 23.4 If she moves the mirror closer to her face, what happens to the image? (a) It becomes inverted and smaller. (b) It remains upright and becomes smaller. (c) It becomes inverted and larger. (d) It remains upright and becomes larger.

EXERCISE 23.4 Suppose a fun-house spherical mirror makes you appear to be one-third your normal height. If you are 1.20 m away from the mirror, find its focal length. Is the mirror concave or convex?

ANSWERS −0.600 m, convex

23.4 Images Formed by Refraction

In this section we describe how images are formed by refraction at a spherical surface. Consider two transparent media with indices of refraction n_1 and n_2, where the boundary between the two media is a spherical surface of radius R (Fig. 23.15). We assume the medium to the right has a higher index of refraction than the one to the left: $n_2 > n_1$. That would be the case for light entering a curved piece of glass from air or for light entering the water in a fishbowl from air. The rays originating at the object location O are refracted at the spherical surface and then converge to the image point I. We can begin with Snell's law of refraction and use simple geometric techniques to show that the object distance, image distance, and radius of curvature are related by the equation

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \qquad [23.7]$$

Further, the magnification of a refracting surface is

$$M = \frac{h'}{h} = -\frac{n_1 q}{n_2 p} \qquad [23.8]$$

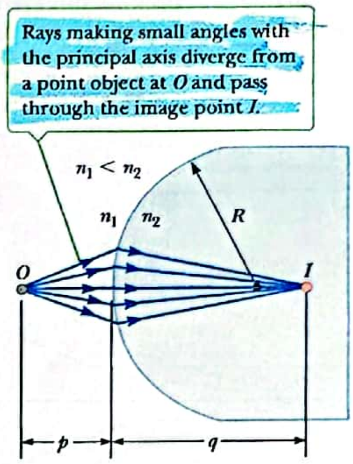


Figure 23.15 An image formed by refraction at a spherical surface.

As with mirrors, certain sign conventions hold, depending on circumstances. First note that real images are formed by refraction on the side of the surface *opposite* the side from which the light comes, in contrast to mirrors, where real images are formed on the *same* side of the reflecting surface. This makes sense because light reflects off mirrors, so any real images must form on the same side the light comes from. With a transparent medium, the rays pass through and naturally form real images on the opposite side. We define the side of the surface where light rays originate as the front side. The other side is called the back side. Because of the difference in location of real images, the refraction sign conventions for q and R are the opposite of those for reflection. For example, p, q, and R are all positive in Figure 23.15. The sign conventions for spherical refracting surfaces are summarized in Table 23.2.

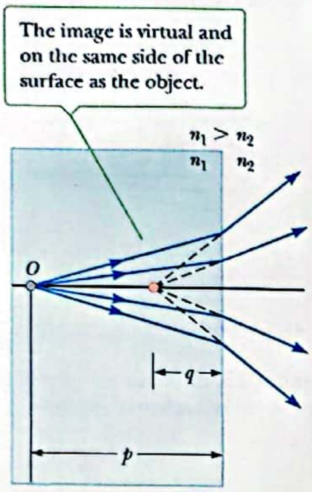
Table 23.2 Sign Conventions for Refracting Surfaces

Quantity	Symbol	In Front	In Back	Upright Image	Inverted Image
Object location	p	+	−		
Image location	q	−	+		
Radius	R	−	+		
Image height	h'			+	−

APPLYING PHYSICS 23.4 Underwater Vision BIO

Why does a person with normal vision see a blurry image if the eyes are opened underwater with no goggles or diving mask in use?

EXPLANATION The eye presents a spherical refraction surface. The eye normally functions so that light entering from the air is refracted to form an image in the retina located at the back of the eyeball. The difference in the index of refraction between water and the eye is smaller than the difference in the index of refraction between air and the eye. Consequently, light entering the eye from the water doesn't undergo as much refraction as does light entering from the air, and the image is formed behind the retina. A diving mask or swimming goggles have no optical action of their own; they are simply flat pieces of glass or plastic in a rubber mount. They do, however, provide a region of air adjacent to the eyes so that the correct refraction relationship is established and images will be in focus. ■



Active Figure 23.16
The image formed by a flat refracting surface.

Flat Refracting Surfaces

If the refracting surface is flat, then R approaches infinity and Equation 23.7 reduces to

$$\frac{n_1}{p} = -\frac{n_2}{q}$$

$$q = -\frac{n_2}{n_1}p \qquad [23.9]$$

From Equation 23.9, we see that the sign of q is opposite that of p. Consequently, the image formed by a flat refracting surface is on the same side of the surface as the object. This statement is illustrated in Active Figure 23.16 for the situation in which n_1 is greater than n_2, where a virtual image is formed between the object and the surface. Note that the refracted ray bends *away* from the normal in this case because $n_1 > n_2$.

Quick Quiz

23.2 A person spearfishing from a boat sees a fish located 3 m from the boat at an apparent depth of 1 m. To spear the fish, should the person aim (a) at, (b) above, or (c) below the image of the fish?

23.3 True or False: (a) The image of an object placed in front of a concave mirror is always upright. (b) The height of the image of an object placed in front of a concave mirror must be smaller than or equal to the height of the object. (c) The image of an object placed in front of a convex mirror is always upright and smaller than the object.

EXAMPLE 23.5 Gaze into the Crystal Ball

GOAL Calculate the properties of an image created by a spherical lens.

PROBLEM A coin 2.00 cm in diameter is embedded in a solid glass ball of radius 30.0 cm (Fig. 23.17). The index of refraction of the ball is 1.50, and the coin is 20.0 cm from the surface. Find the position of the image of the coin and the height of the coin's image.

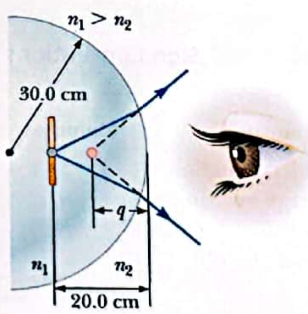


Figure 23.17 (Example 23.5) A coin embedded in a glass ball forms a virtual image between the coin and the surface of the glass.

STRATEGY Because the rays are moving from a medium of high index of refraction (the glass ball) to a medium of lower index of refraction (air), the rays originating at the coin are refracted away from the normal at the surface and diverge outward. The image is formed in the glass and is virtual. Substitute into Equations 23.7 and 23.8 for the image position and magnification, respectively.

SOLUTION

Apply Equation 23.7 and take $n_1 = 1.50$, $n_2 = 1.00$, $p = 20.0$ cm, and $R = -30.0$ cm:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

$$\frac{1.50}{20.0\text{ cm}} + \frac{1.00}{q} = \frac{1.00 - 1.50}{-30.0\text{ cm}}$$

Solve for q:

$$q = -17.1\text{ cm}$$

To find the image height, use Equation 23.8 for the magnification:

$$M = -\frac{n_1 q}{n_2 p} = -\frac{1.50(-17.1\text{ cm})}{1.00(20.0\text{ cm})} = \frac{h'}{h}$$

$$h' = 1.28h = (1.28)(2.00\text{ cm}) = 2.56\text{ cm}$$

REMARKS The negative sign on q indicates that the image is in the same medium as the object (the side of incident light), in agreement with our ray diagram, and therefore must be virtual. The positive value for M means that the image is upright.

QUESTION 23.5 How would the final answer be affected if the ball and observer were immersed in water? (a) It would be smaller. (b) It would be larger. (c) There would be no change.

EXERCISE 23.5 A coin is embedded 20.0 cm from the surface of a similar ball of transparent substance having radius 30.0 cm and unknown composition. If the coin's image is virtual and located 15.0 cm from the surface, find the (a) index of refraction of the substance and (b) magnification.

ANSWERS (a) 2.00 (b) 1.50

EXAMPLE 23.6 The One That Got Away

GOAL Calculate the properties of an image created by a flat refractive surface.

PROBLEM A small fish is swimming at a depth d below the surface of a pond (Fig. 23.18). (a) What is the *apparent depth* of the fish as viewed from directly overhead? (b) If the fish is 12 cm long, how long is its image?

STRATEGY In this example the refracting surface is flat, so R is infinite. Hence, we can use Equation 23.9 to determine the location of the image, which is the apparent location of the fish.

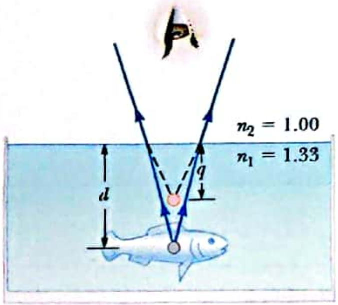


Figure 23.18 (Example 23.6) The apparent depth q of the fish is less than the true depth d.

SOLUTION

(a) Find the apparent depth of the fish.

Substitute $n_1 = 1.33$ for water and $p = d$ into Equation 23.9:

$$q = -\frac{n_2}{n_1}p = -\frac{1}{1.33}d = -0.752d$$

(b) What is the size of the fish's image?

Use Equation 23.9 to eliminate q from the Equation 23.8, the magnification equation:

$$M = \frac{h'}{h} = -\frac{n_1 q}{n_2 p} = -\frac{n_1\left(-\frac{n_2}{n_1}p\right)}{n_2 p} = 1$$

$$h' = h = 12\text{ cm}$$

REMARKS Again, because q is negative, the image is virtual, as indicated in Figure 23.18. The apparent depth is approximately three-fourths the actual depth. For instance, if $d = 4.0$ m, then $q = -3.0$ m.

QUESTION 23.6 Suppose a similar experiment is carried out with an object immersed in oil ($n = 1.5$) the same distance below the surface. How does the apparent depth of the object compare with its apparent depth when immersed in water? (a) The apparent depth is unchanged. (b) The apparent depth is larger. (c) The apparent depth is smaller.

(Continued)

EXERCISE 23.6 A spear fisherman estimates that a trout is 1.5 m below the water's surface. What is the actual depth of the fish?

ANSWER 2.0 m

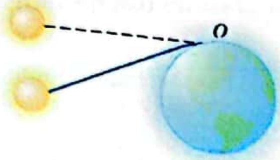


Figure 23.19 Because light is refracted by Earth's atmosphere, an observer at O sees the Sun even though it has fallen below the horizon.

23.5 Atmospheric Refraction

Images formed by refraction in our atmosphere lead to some interesting phenomena. One such phenomenon that occurs daily is the visibility of the Sun at dusk even though it has passed below the horizon. Figure 23.19 shows why it occurs. Rays of light from the Sun strike Earth's atmosphere (represented by the shaded area around the planet) and are bent as they pass into a medium that has an index of refraction different from that of the almost empty space in which they have been traveling. The bending in this situation differs somewhat from the bending we have considered previously in that it is gradual and continuous as the light moves through the atmosphere toward an observer at point O. This is because the light moves through layers of air that have a continuously changing index of refraction. When the rays reach the observer, the eye follows them back along the direction from which they appear to have come (indicated by the dashed path in the figure). The end result is that the Sun appears to be above the horizon even after it has fallen below it.

The **mirage** is another phenomenon of nature produced by refraction in the atmosphere. A mirage can be observed when the ground is so hot that the air directly above it is warmer than the air at higher elevations. The desert is a region in which such circumstances prevail, but mirages are also seen on heated roadways during the summer. The layers of air at different heights above Earth have different densities and different refractive indices. The effect these differences can have is pictured in Figure 23.20a. The observer sees the sky and a tree in two different ways. One group of light rays reaches the observer by the straight-line path A, and the eye traces these rays back to see the tree in the normal fashion. In addition, a second group of rays travels along the curved path B. These rays are directed toward the ground and are then bent as a result of refraction. As a consequence, the observer also sees an inverted image of the tree and the background of the sky as he traces the rays back to the point at which they appear to have originated.