University of Mosul

College of Science

Department of Physics

Third Stage

Lecture 9

Geometric Optics

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Lecture 9: Thin Lenses

Preparation

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Because an upright image and an inverted image are seen when the image of a tree is observed in a reflecting pool of water, the observer unconsciously calls on this past experience and concludes that the sky is reflected by a pool of water in front of the tree.

23.6 Thin Lenses

A typical **thin lens** consists of a piece of glass or plastic, ground so that each of its two refracting surfaces is a segment of either a sphere or a plane. Lenses are commonly used to form images by refraction in optical instruments, such as cameras, telescopes, and microscopes. The equation that relates object and image distances for a lens is virtually identical to the mirror equation derived earlier, and the method used to derive it is also similar.

Figure 23.21 shows some representative shapes of lenses. Notice that we have placed these lenses in two groups. Those in Figure 23.21a are thicker at the center than at the rim, and those in Figure 23.21b are thinner at the center than at the rim. The lenses in the first group are examples of **converging lenses**, and those in the second group are **diverging lenses**. The reason for these names will become apparent shortly.

As we did for mirrors, it is convenient to define a point called the **focal point** for a lens. For example, in Figure 23.22a, a group of rays parallel to the axis passes through the focal point F after being converged by the lens. The distance from the focal point to the lens is called the **focal length f. The focal length is the image distance that corresponds to an infinite object distance.** Recall that we are considering the lens to be very thin. As a result, it makes no difference whether we take the focal length to be the distance from the focal point to the surface of the lens or the distance from the focal point to the center of the lens because the difference between these two lengths is negligible. A thin lens has *two* focal points, as illustrated in Figure 23.22, one on each side of the lens. One focal point corresponds to parallel rays traveling from the left and the other corresponds to parallel rays traveling from the right.

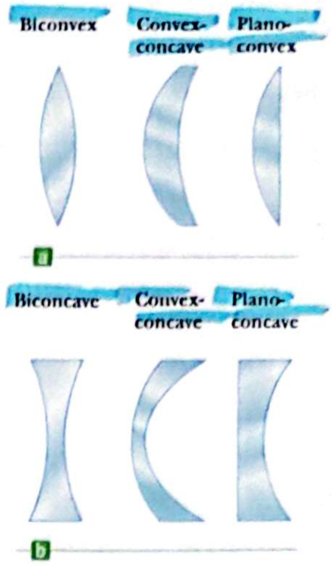


Figure 23.21 Various lens shapes. (a) Converging lenses have positive focal lengths and are thickest at the middle. (b) Diverging lenses have negative focal lengths and are thickest at the edges.

Photos courtesy of Henry Leap and Jim Lehman

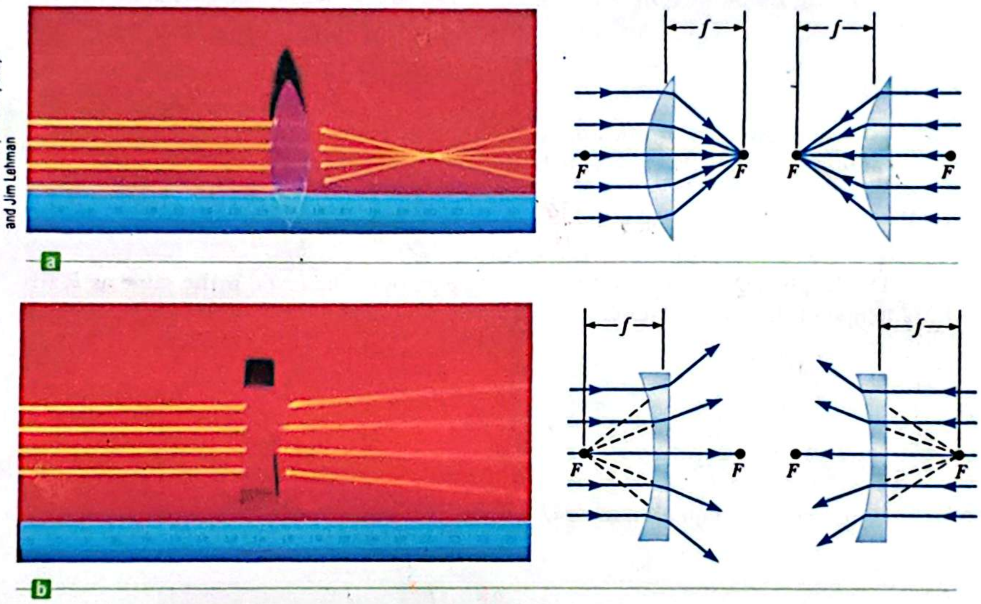


Figure 23.22 (*Left*) Photographs of the effects of converging and diverging lenses on parallel rays. (*Right*) The focal points of the (a) biconvex lens and (b) biconcave lens.

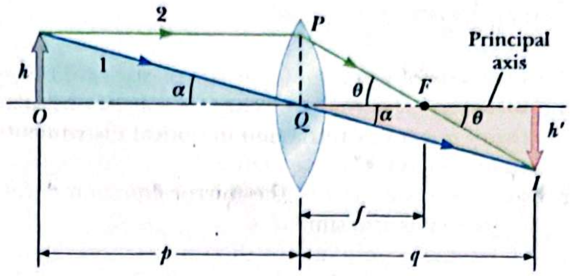


Figure 23.23 A geometric construction for developing the thin-lens equation.

Rays parallel to the axis diverge after passing through a lens of biconcave shape, shown in Figure 23.22b. In this case the focal point is defined to be the point where the diverged rays appear to originate, labeled F in the figure. Figures 23.22a and 23.22b indicate why the names *converging* and *diverging* are applied to these lenses.

Now consider a ray of light passing through the center of a lens. Such a ray is labeled ray 1 in Figure 23.23. For a thin lens, a ray passing through the center is undeflected. Ray 2 in the same figure is parallel to the principal axis of the lens (the horizontal axis passing through O), and as a result it passes through the focal point F after refraction. Rays 1 and 2 intersect at the point that is the tip of the image arrow.

We first note that the tangent of the angle α can be found by using the blue and gold shaded triangles in Figure 23.23:

$$\tan\alpha = \frac{h}{p} \quad \text{or} \quad \tan\alpha = -\frac{h'}{q}$$

From this result, we find that

$$M = \frac{h'}{h} = -\frac{q}{p} \qquad [23.10]$$

The equation for magnification by a lens is the same as the equation for magnification by a mirror. We also note from Figure 23.23 that

$$\tan\theta = \frac{PQ}{f} \quad \text{or} \quad \tan\theta = -\frac{h'}{q-f}$$

The height PQ used in the first of these equations, however, is the same as h, the height of the object. Therefore,

$$\frac{h}{f} = -\frac{h'}{q-f}$$

$$\frac{h'}{h} = -\frac{q-f}{f}$$

Using the latter equation in combination with Equation 23.10 gives

$$\frac{q}{p} = \frac{q-f}{f}$$

which reduces to

Thin-lens equation ▶

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad [23.11]$$

This equation, called the **thin-lens equation**, can be used with both converging and diverging lenses if we adhere to a set of sign conventions. Figure 23.24 is useful

Table 23.3 Sign Conventions for Thin Lenses

Quantity	Symbol	In Front	In Back	Convergent	Divergent
Object location	p	+	−		
Image location	q	−	+		
Lens radii	R_1, R_2	−	+		
Focal length	f			+	−

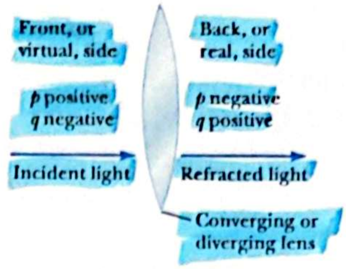


Figure 23.24 A diagram for obtaining the signs of p and q for a thin lens or a refracting surface.

for obtaining the signs of p and q, and Table 23.3 gives the complete sign conventions for lenses. Note that a converging lens has a positive focal length under this convention and a diverging lens has a negative focal length. Hence, the names *positive* and *negative* are often given to these lenses.

The focal length for a lens in air is related to the curvatures of its front and back surfaces and to the index of refraction n of the lens material by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad [23.12]$$

◀ Lens-maker's equation

where R_1 is the radius of curvature of the front surface of the lens and R_2 is the radius of curvature of the back surface. (As with mirrors, we arbitrarily call the side from which the light approaches the *front* of the lens.) Table 23.3 gives the sign conventions for R_1 and R_2. Equation 23.12, called the **lens-maker's equation**, enables us to calculate the focal length from the known properties of the lens.

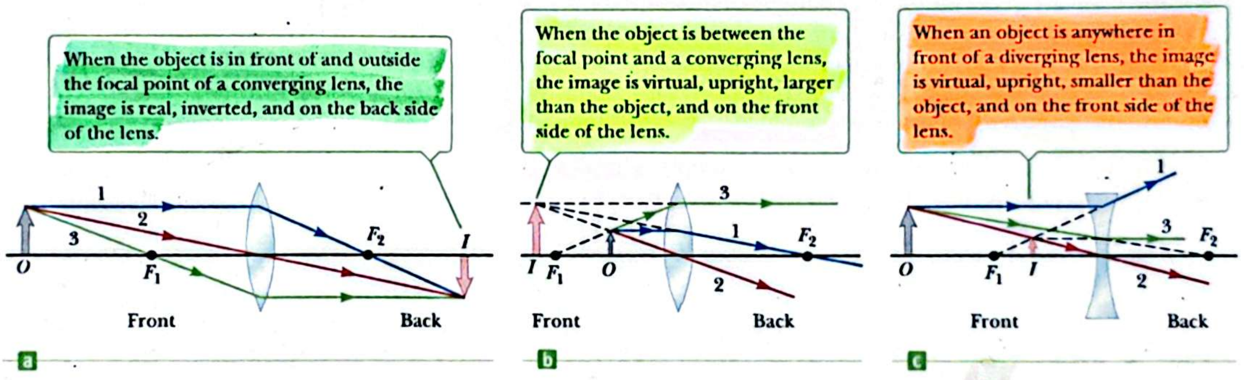
Ray Diagrams for Thin Lenses

Ray diagrams are essential for understanding the overall image formation by a thin lens or a system of lenses. They should also help clarify the sign conventions already discussed. Active Figure 23.25 illustrates this method for three single-lens situations. To locate the image formed by a converging lens (Active Figs. 23.25a and 23.25b), the following three rays are drawn from the top of the object:

1. The first ray is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through (or appears to come from) one of the focal points.
2. The second ray is drawn through the center of the lens. This ray continues in a straight line.
3. The third ray is drawn through the other focal point and emerges from the lens parallel to the principal axis.

Tip 23.4 Positive Is Again Where the Light Is

For lenses, p and q are positive where the light is, where the object or image is real. For real objects, the light originates with the object in front of the lens, so p is positive there as indicated in Figure 23.24. If the image forms in back of the lens, q is positive there, as well.



Active Figure 23.25
Ray diagrams for locating the image formed by a thin lens.

Tip 23.5 We Choose Only a Few Rays

Although our ray diagrams in Figure 23.25 only show three rays leaving an object, an infinite number of rays can be drawn between the object and its image.

A similar construction is used to locate the image formed by a diverging lens, as shown in Active Figure 23.25c. The point of intersection of *any two* of the rays in these diagrams can be used to locate the image. The third ray serves as a check on construction.

For the converging lens in Active Figure 23.25a, where the object is *outside* the front focal point ($p > f$), the ray diagram shows that the image is real and inverted. When the real object is *inside* the front focal point ($p < f$), as in Active Figure 23.25b, the image is virtual and upright. For the diverging lens of Active Figure 23.25c, the image is virtual and upright.

Quick Quiz

23.4 A clear plastic sandwich bag filled with water can act as a crude converging lens in air. If the bag is filled with air and placed under water, is the effective lens (a) converging or (b) diverging?

23.5 In Active Figure 23.25a the blue object arrow is replaced by one that is much taller than the lens. How many rays from the object will strike the lens?

23.6 An object is placed to the left of a converging lens. Which of the following statements are true, and which are false? (a) The image is always to the right of the lens. (b) The image can be upright or inverted. (c) The image is always smaller or the same size as the object.

Your success in working lens or mirror problems will be determined largely by whether you make sign errors when substituting into the lens or mirror equations. The only way to ensure you don't make sign errors is to become adept at using the sign conventions. The best ways to do so are to work a multitude of problems on your own and to construct confirming ray diagrams. Watching an instructor or reading the example problems is no substitute for practice.

APPLYING PHYSICS 23.5 Vision and Diving Masks BIO

Diving masks often have a lens built into the glass faceplate for divers who don't have perfect vision. This lens allows the individual to dive without the necessity of glasses because the faceplate performs the necessary refraction to produce clear vision. Normal glasses have lenses that are curved on both the front and rear surfaces. The lenses in a diving-mask faceplate often have curved surfaces only on the inside of the glass. Why is this design desirable?

SOLUTION The main reason for curving only the inner surface of the lens in the diving-mask faceplate is to enable the diver to see clearly while underwater and in the air. If there were curved surfaces on both the front and the back of the diving lens, there would be two refractions. The lens could be designed so that these two refractions would give clear vision while the diver is in air. When the diver went underwater, however, the refraction between the water and the glass at the first interface would differ because the index of refraction of water is different from that of air. Consequently, the diver's vision wouldn't be clear underwater. ■

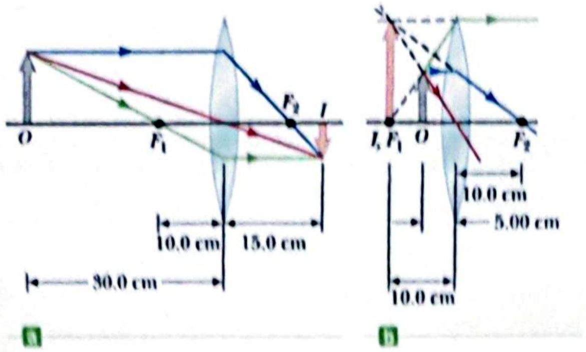
EXAMPLE 23.7 Images Formed by a Converging Lens

GOAL Calculate geometric quantities associated with a converging lens.

PROBLEM A converging lens of focal length 10.0 cm forms images of an object situated at various distances. (a) If the object is placed 30.0 cm from the lens, locate the image, state whether it's real or virtual, and find its magnification. (b) Repeat the problem when the object is at 10.0 cm and (c) again when the object is 5.00 cm from the lens.

STRATEGY All three problems require only substitution into the thin-lens equation and the associated magnification equation, Equations 23.10 and 23.11, respectively. The conventions of Table 23.3 must be followed.

Figure 23.26 (Example 23.7)



SOLUTION

(a) Find the image distance and describe the image when the object is placed at 30.0 cm.

The ray diagram is shown in Figure 23.26a. Substitute values into the thin-lens equation to locate the image:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{30.0\text{ cm}} + \frac{1}{q} = \frac{1}{10.0\text{ cm}}$$

Solve for q, the image distance. It's positive, so the image is real and on the far side of the lens:

$$q = +15.0\text{ cm}$$

The magnification of the lens is obtained from Equation 23.10. M is negative and less than 1 in absolute value, so the image is inverted and smaller than the object:

$$M = -\frac{q}{p} = -\frac{15.0\text{ cm}}{30.0\text{ cm}} = -0.500$$

(b) Repeat the problem, when the object is placed at 10.0 cm.

object is placed at 10.0 cm

Locate the image by substituting into the thin-lens equation:

$$\frac{1}{10.0\text{ cm}} + \frac{1}{q} = \frac{1}{10.0\text{ cm}} \rightarrow \frac{1}{q} = 0$$

This equation is satisfied only in the limit as q becomes infinite. Similarly, M becomes infinite, as well.

$$q \rightarrow \infty$$

(c) Repeat the problem when the object is placed 5.00 cm from the lens.

See the ray diagram shown in Figure 23.26b. Substitute into the thin-lens equation to locate the image:

$$\frac{1}{5.00\text{ cm}} + \frac{1}{q} = \frac{1}{10.0\text{ cm}}$$

Solve for q, which is negative, meaning the image is on the same side as the object and is virtual:

$$q = -10.0\text{ cm}$$

Substitute the values of p and q into the magnification equation. M is positive and larger than 1, so the image is upright and double the object size:

$$M = -\frac{q}{p} = -\left(\frac{-10.0\text{ cm}}{5.00\text{ cm}}\right) = +2.00$$

REMARKS The ability of a lens to magnify objects led to the inventions of reading glasses, microscopes, and telescopes.

QUESTION 23.7 If the lens is used to form an image of the Sun on a screen, how far from the lens should the screen be located?

EXERCISE 23.7 Suppose the image of an object is upright and magnified 1.75 times when the object is placed 15.0 cm from a lens. Find (a) the location of the image and (b) the focal length of the lens.

ANSWERS (a) −26.3 cm (virtual, on the same side as the object) (b) 34.9 cm