University of Mosul

College of Science

Department of Physics

Third Stage

Lecture 3

Geometric Optics

2024 – 2025

Lecture 3: Dispersion and Prisms

Preparation

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REMARKS Despite its apparent complexity, the problem isn't that different from Example 22.2.

QUESTION 22.4 Suppose the plastic were replaced by a material with a higher index of refraction. How would the width of the beam at the information layer be affected? (a) It would remain the same. (b) It would decrease. (c) It would increase.

EXERCISE 22.4 Suppose you wish to redesign the system to decrease the initial width of the beam from 0.700 0 mm to 0.600 0 mm but leave the incident angle θ_1 and all other parameters the same as before, except the index of refraction for the plastic material (n_2) and the angle θ_2. What index of refraction should the plastic have?

ANSWER 1.79

22.4 Dispersion and Prisms

In Table 22.1 we presented values for the index of refraction of various materials. If we make careful measurements, however, we find that the index of refraction in anything but vacuum depends on the wavelength of light. The dependence of the index of refraction on wavelength is called **dispersion**. Figure 22.13 is a graphical representation of this variation in the index of refraction with wavelength. Because n is a function of wavelength, Snell's law indicates that **the angle of refraction made when light enters a material depends on the wavelength of the light.** As seen in the figure, the index of refraction for a material usually decreases with increasing wavelength. This means that violet light ($\lambda \cong 400$ nm) refracts more than red light ($\lambda \cong 650$ nm) when passing from air into a material.

To understand the effects of dispersion on light, consider what happens when light strikes a prism, as in Figure 22.14a. A ray of light of a single wavelength that is incident on the prism from the left emerges bent away from its original direction of travel by an angle δ, called the **angle of deviation**. Now suppose a beam of white light (a combination of all visible wavelengths) is incident on a prism. Because of dispersion, the different colors refract through different angles of deviation, as illustrated in Figure 22.14b, and the rays that emerge from the second face of the prism spread out in a series of colors known as a visible **spectrum**, as shown in Figure 22.15 on page 772. These colors, in order of decreasing wavelength, are red, orange, yellow, green, blue, and violet. Violet light deviates the most, red light the least, and the remaining colors in the visible spectrum fall between these extremes.

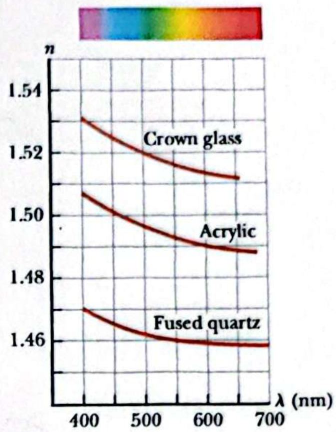


Figure 22.13 Variations of index of refraction in the visible spectrum with respect to vacuum wavelength for three materials.

Tip 22.3 Dispersion

Light of shorter wavelength, such as violet light, refracts more than light of longer wavelengths, such as red light.

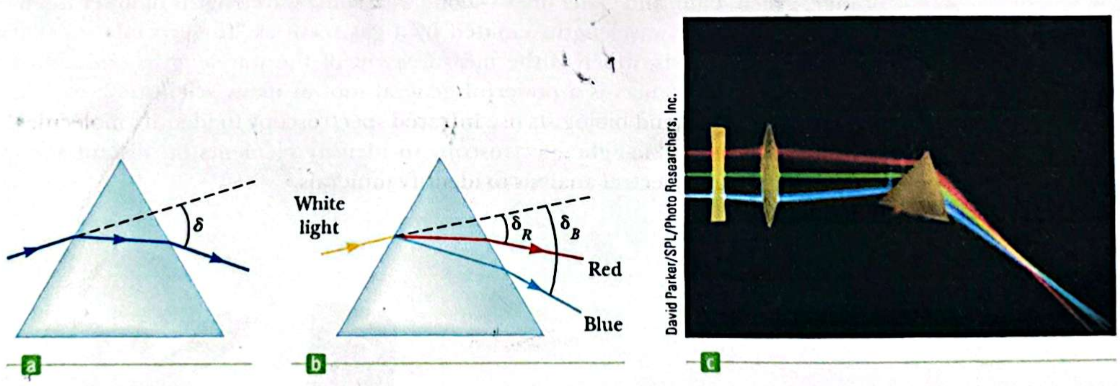
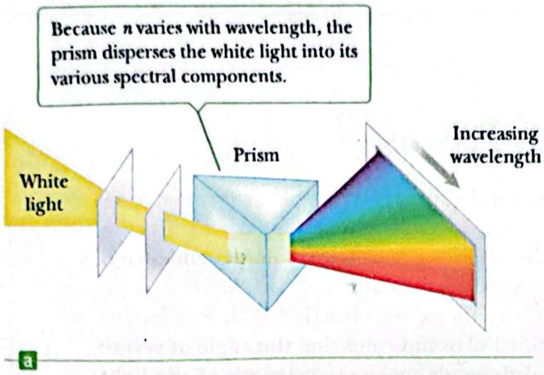


Figure 22.14 (a) A prism refracts a light ray and deviates the light through the angle δ. (b) When light is incident on a prism, the blue light is bent more than the red light. (c) Light of different colors passes through a prism and two lenses. Note that as the light passes through the prism, different wavelengths are refracted at different angles.



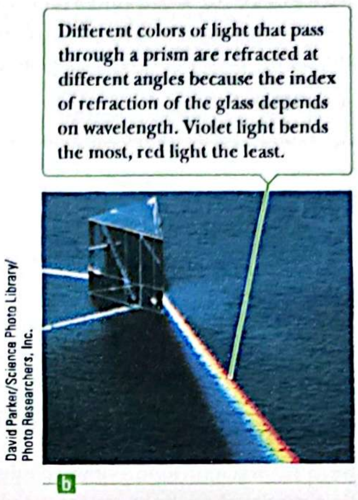


Figure 22.15 (a) Dispersion of white light by a prism. (b) White light enters a glass prism at the upper left.

Prisms are often used in an instrument known as a **prism spectrometer**, the essential elements of which are shown in Figure 22.16a. This instrument is commonly used to study the wavelengths emitted by a light source, such as a sodium vapor lamp. Light from the source is sent through a narrow, adjustable slit and lens to produce a parallel, or collimated, beam. The light then passes through the prism and is dispersed into a spectrum. The refracted light is observed through a telescope. The experimenter sees different colored images of the slit through the eyepiece of the telescope. The telescope can be moved or the prism can be rotated to view the various wavelengths, which have different angles of deviation. Figure 22.16b shows one type of prism spectrometer used in undergraduate laboratories.

APPLICATION
Identifying Gases with a Spectrometer

All hot, low-pressure gases emit their own characteristic spectra, so one use of a prism spectrometer is to identify gases. For example, sodium emits only two wavelengths in the visible spectrum: two closely spaced yellow lines. (The bright linelike images of the slit seen in a spectroscope are called *spectral lines.*) A gas emitting these, and only these, colors can be identified as sodium. Likewise, mercury vapor has its own characteristic spectrum, consisting of four prominent wavelengths—orange, green, blue, and violet lines—along with some wavelengths of lower intensity. The particular wavelengths emitted by a gas serve as "fingerprints" of that gas. Spectral analysis, which is the measurement of the wavelengths emitted or absorbed by a substance, is a powerful general tool in many scientific areas. As examples, chemists and biologists use infrared spectroscopy to identify molecules, astronomers use visible-light spectroscopy to identify elements on distant stars, and geologists use spectral analysis to identify minerals.

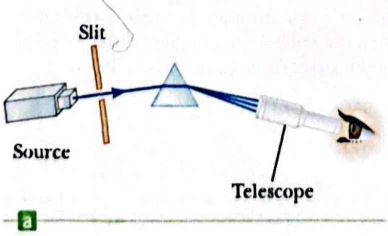


Figure 22.16 (a) A diagram of a prism spectrometer. The colors in the spectrum are viewed through a telescope. (b) A prism spectrometer with interchangeable components.

APPLYING PHYSICS 22.3 Dispersion

When a beam of light enters a glass prism, which has nonparallel sides, the rainbow of color exiting the prism is a testimonial to the dispersion occurring in the glass. Suppose a beam of light enters a slab of material with parallel sides. When the beam exits the other side, traveling in the same direction as the original beam, is there any evidence of dispersion?

EXPLANATION Due to dispersion, light at the violet end of the spectrum exhibits a larger angle of refraction on entering the glass than light at the red end. All colors of light return to their original direction of propagation as they refract back out into the air. As a result, the outgoing beam is white. The net shift in the position of the violet light along the edge of the slab is larger than the shift of the red light, however, so one edge of the outgoing beam has a bluish tinge to it (it appears blue rather than violet because the eye is not very sensitive to violet light), whereas the other edge has a reddish tinge. This effect is indicated in Figure 22.17. The colored edges of the outgoing beam of white light are evidence of dispersion. ■

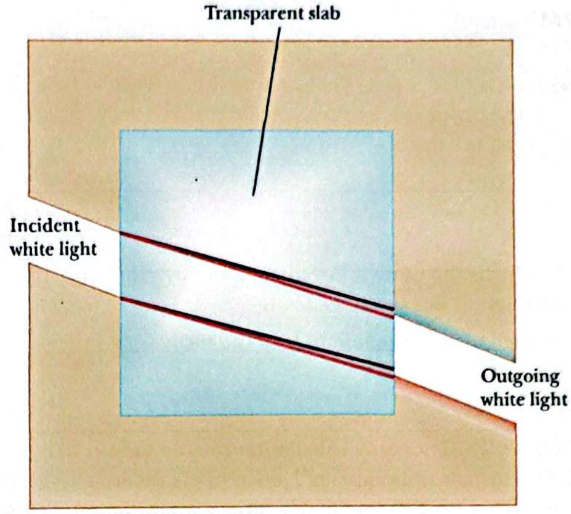


Figure 22.17 (Applying Physics 22.3)

EXAMPLE 22.5 Light Through a Prism

GOAL Calculate the consequences of dispersion.

PROBLEM A beam of light is incident on a prism of a certain glass at an angle of $\theta_1 = 30.0°$, as shown in Figure 22.18. If the index of refraction of the glass for violet light is 1.80, find **(a)** θ_2, the angle of refraction at the air–glass interface, **(b)** ϕ_2, the angle of incidence at the glass–air interface, and **(c)** ϕ_1, the angle of refraction when the violet light exits the prism. **(d)** What is the value of Δy, the amount by which the violet light is displaced vertically?

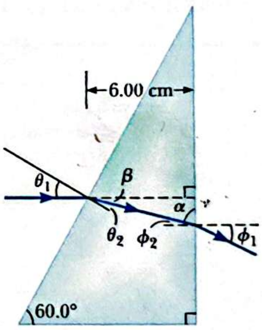


Figure 22.18 (Example 22.5)

STRATEGY This problem requires Snell's law to find the refraction angles and some elementary geometry and trigonometry based on Figure 22.18.

SOLUTION

(a) Find θ_2, the angle of refraction at the air–glass interface.

Use Snell's law to find the first angle of refraction:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow (1.00)\sin 30.0 = (1.80)\sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{0.500}{1.80}\right) = 16.1°$$

(b) Find ϕ_2, the angle of incidence at the glass–air interface.

Compute the angle β:

$$\beta = 30.0° - \theta_2 = 30.0° - 16.1° = 13.9°$$

Compute the angle α using the fact that the sum of the interior angles of a triangle equals 180°:

$$180° = 13.9° + 90° + \alpha \rightarrow \alpha = 76.1°$$

The incident angle ϕ_2 at the glass–air interface is complementary to α:

$$\phi_2 = 90° - \alpha = 90° - 76.1° = 13.9°$$

(Continued)

(c) Find ϕ_1, the angle of refraction when the violet light exits the prism.

$n_2 \sin\phi_2 = n_1 \sin\phi_1$

Apply Snell's law:

$$\phi_1 = \left(\frac{1}{n_1}\right)\sin^{-1}(n_2 \sin\phi_2)$$

$$= \left(\frac{1}{1.00}\right)\sin^{-1}[(1.80)\sin 13.9°] = 25.6°$$

(d) What is the value of Δy, the amount by which the violet light is displaced vertically?

Use the tangent function to find the vertical displacement:

$$\tan\beta = \frac{\Delta y}{\Delta x} \quad\rightarrow\quad \Delta y = \Delta x \tan\beta$$

$$\Delta y = (6.00\text{ cm})\tan(13.9°) = 1.48\text{ cm}$$

REMARKS The same calculation for red light is left as an exercise. The violet light is bent more and displaced farther down the face of the prism. Notice that a theorem in geometry about parallel lines and the angles created by a transverse line give $\phi_2 = \beta$ immediately, which would have saved some calculation. In general, however, this tactic might not be available.

QUESTION 22.5 On passing through the prism, will yellow light bend through a larger angle or smaller angle than the violet light? (a) Yellow light bends through a larger angle. (b) Yellow light bends through a smaller angle. (c) The angles are the same.

EXERCISE 22.5 Repeat parts (a) through (d) of the example for red light passing through the prism, given that the index of refraction for red light is 1.72.

ANSWERS (a) 16.9° (b) 13.1° (c) 22.9° (d) 1.40 cm